Twin Prime Formation Through a Proper Alliance of Layers of Odd Composite Sequences

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Note: In this paper, the term "sequence" refers to the odd composite sequence.

Abstract

This paper develops a **Prime Pair Framework** for studying **twin primes** using structured sequences of **odd composites.** The base sequence

$$O_1 = 9 + 6r_1,\ r_1 \neq \frac{(1+2x)^2 - 13}{6} \text{ or } \frac{(1+2x)^2 - 7}{6}$$

provides intervals containing two odd integers, which may form twin primes if both are prime. When the general sequences

$$O_a = (2a+1)^2 + (2+4a)r_a\ (a = \{2,3,4,\dots,k\})$$

and $O_1$are in proper alliance, the conditions are formed through **lower**, **upper and double twin prime formation mechanisms.**

Defination

Odd Composite

We define the odd composite numbers as follows:

$$O_C = (2a+1)^2 + (2+4a)r,\ a \geq 1,\ r \geq 0$$

The formula given in OEIS A083487, $T(n,k) = 2nk + n + k(1 \leq k \leq n)$ does not provide a way to distinguish odd composites layer by layer.

So we can change the formula as follow.

$$2nk + n + k(1 \leq k \leq n)$$

This formula is odd composite index

$$2\big(T(n,k)\big) + 1 = 2(2nk + n + k) + 1 = 4nk + 2n + 2k + 1$$

Let $k = a$

$$due\ to\ k \leq n, we\ can\ rewrite\ n = k + r = a + r(r = \{0,1,2,\dots\})$$

$$4nk + 2n + 2k + 1 = 4(a+r)a + 2(a+r) + 2a + 1$$

$$= 4a^2 + 4ar + 2a + 2a + 2r + 1$$

$$= (2a+1)^2 + (2+4a)r$$

So the formula become

$$O_C = (2a + 1)^2 + (2 + 4a)r, a \geq 1, r \geq 0$$

This formula can generate as the following odd composite sequence layers related to a.

Sequence Examples

For $a = 1$:

$$O_1 = (2 \times 1 + 1)^2 + (2 + 4 \times 1)r = 9 + 6r$$

Thus, the sequence for a=1 is:

$$Sequence\ \mathrm{O}_1 = \{9, 15, 21, 27, \dots\}$$

This sequence O_1 has exactly two odd integers because the formula generates terms of the form $6r$ and the interval length is 6.

For example

between 9 and 15 we find the odd integers 11 and 13

between 15 and 21 we find the odd integers 17 and 19

between 21 and 27 we find the odd integers 23 and 25

However, these two odd integers may be both primes, both odd composites, or one prime and one odd composite.

Importantly, the terms of the sequence itself — 9, 15, 21, 27, ... — are all odd composites.

Therefore, the sequence O_1 is designated as the **base odd composite sequence** for the purpose of searching for twin primes.

For a = 2:

$$\mathrm{O}_2 = (2 \times 2 + 1)^2 + (2 + 4 \times 2)\mathrm{r} = 25 + 10\mathrm{r}$$

⇒ Sequence O_2= {25, 35, 45, 55, ...}

O_2 sequence has 4 odd integer at each interval and the interval length is 10.

For a = 3:

$$\mathrm{O}_3 = (2 \times 3 + 1)^2 + (2 + 4 \times 3)\mathrm{r} = 49 + 14\mathrm{r}$$

⇒ O_3Sequence = {49, 63, 77, 91, ...}

O_3 sequence has 6 odd integers at each interval and the interval length is 14.

Fig. 1

Integer	S_1	S2	S3	S4	S5	Integer	S_1	S2	S3	S4	S5	Integer	S_1	S2	S3	S4	S5	Integer	S_1	S2	S3	S4	S5
1						52						100						154					
2						53						101						155		155			
3						54						102						156					
4						55		55				103						157					
5						56						104						158					
6						57	57					105	105	105	105			159	159				
7						58						106						160					
8						59						107						161			161		
9	9					60						108						162					
10						61						109						163					
11						62						110						164					
12						63	63		63			111	111					165	165	165			
13						64						112						166					
14						65		65				113						167					
15	15					66						114						168					
16						67						115		115				169					
17						68						116						170					
18						69	69					117	117			117		171	171			171	
19						70						118						172					
20						71						119			119			173					
21	21					72						120						174					
22						73						121					121						
23						74						122											
24						75	75	75				123	123										
25		25				76						124											
26						77			77			125		125									
27	27					78						126											
28						79						127											
29						80						128											
30						81	81			81		129	129										
31						82						130											
32						83						131											
33	33					84						132											
34						85		85				133			133								
35		35				86						134											
36						87	87					135	135	135		135							
37						88						136											
38						89						137											
39	39					90						138											
40						91			91			139											
41						92						140											
42						93	93					141	141										
43						94						142											
44						95		95				143											
45	45	45				96						144											
46						97						145		145									
47						98						146											
48						99	99			99		147	147		147								
49			49									148											
50												149											
51	51											150											
												151											
												152											
												153	153			153							

Maximum Sequence for Twin Prime Checking

To determine the largest sequence S_k that must be checked when searching for twin primes, we proceed as follows:

the base sequence:

$$S_1 = 9 + 6r_1$$

the general quadratic form for kth:

$$S_k = (2\mathrm{k}+1)^2 + (2+4k)r_k$$

Let $r_k = 0$

$$9 + 6\mathrm{r}_1 = (2\mathrm{k}+1)^2$$

Rearranging gives:

$$2\mathrm{k} + 1 = \sqrt{9 + 6\mathrm{r}_1}$$

Solving for k:

$$\mathrm{k} = \left\lfloor \frac{\sqrt{9+6\mathrm{r}_1} - 1}{2} \right\rfloor$$

The value of k represents the maximum number of the sequence $O_\mathrm{k}$that needs to be checked.

$a = 2nk + n + k(1 \leq k \leq n) = \{4, 7, 10, \dots\}$ can be skipped, because:

Let $a = 1$:

$$O_1 = 9 + 6r_1$$

When $a = 4$:

$$O_4 = (1 + 2 \times 4)^2 + (2 + 4 \times 4)r_4 = 81 + 18r_4$$

Equating:

$$O_1 = O_4 \ \Rightarrow \ 9 + 6r_1 = 81 + 18r_4$$

Thus:

$$r_1 = \frac{72 + 18r_4}{6} = 12 + 3r_4$$

Therefore:

$$O_4 \subseteq O_1$$

Twin prime formation

Depending on the tested value of r_1, the occurrence of twin primes can be classified into three distinct cases:

1. **Lower formation** — when the twin prime pair arises below the tested r_1.
2. **Upper formation** — when the twin prime pair arises above the tested r_1.
3. **Double twin prime formation** — when both lower and upper twin prime pairs occur simultaneously around the tested r_1.

Lower Formation Theorem

To search for twin primes, we begin with the base formula

$$O_1 = 9 + 6\mathrm{r}_1(\{9, 15, 21, 27, \ldots\}, \mathrm{r}_1 \neq \frac{(1+2x)^2 - 13}{6}, x = \{1,2,3, \ldots\})$$

If other odd composite sequences form a proper alliance with this base sequence under the lower formation conditions, then the numbers

$$\mathrm{O}_1 + 2 = 11 + 6\mathrm{r}_1 \text{and } \mathrm{O}_1 + 4 = 13 + 6\mathrm{r}_1$$

are twin primes.

The alliance condition is expressed as:

$$0 \leq [2 + 4\mathrm{a}]\left\{\frac{9 + 6\mathrm{r}_1 - (2\mathrm{a}+1)^2}{2 + 4\mathrm{a}}\right\} \leq 4\mathrm{a} - 4$$

where

$$\mathrm{a} \in \{2, 3, 4, \ldots, \mathrm{k}\},\ \mathrm{k} = \left\lfloor\frac{\sqrt{9 + 6\mathrm{r}_1} - 1}{2}\right\rfloor$$

Proof of Lower Formation Condition

To search for twin primes, we begin with the base formula

$$O_1 = 9 + 6\mathrm{r}_1$$

Lemma 1. For every integer $r_1 \geq 0$, the interval between $9 + 6r_1$and $9 + 6(r_1 + 1)$contains exactly two odd integers, namely $11 + 6r_1$and $13 + 6r_1$.

Proof

$$9 + 6r_1, 9 + 6(r_1 + 1) = 15 + 6r_1$$

$9 + 6r_1$ *and* $15 + 6r_1$ are odd composite integers,so the odd integers between this interval are

$11 + 6r_1$and $13 + 6r_1$. However, it is uncertain whether those two odd integers are prime or odd composite. ■

Condition for Lower Formation

The general sequence:

$$\mathrm{O}_\mathrm{a} = (2\mathrm{a} + 1)^2 + (2 + 4\mathrm{a})\mathrm{r}_\mathrm{a}$$

By setting $\mathrm{O}_1 = \mathrm{O}_\mathrm{a}$, we obtain:

$$9 + 6\mathrm{r}_1 = (2\mathrm{a} + 1)^2 + (2 + 4\mathrm{a})\mathrm{r}_\mathrm{a}$$

which gives:

$$\mathrm{r_a} = \frac{9 + 6\mathrm{r_1} - (2\mathrm{a} + 1)^2}{2 + 4\mathrm{a}}$$

Other odd composite sequences such as $O_2, O_3, \dots, O_k$ contain more interval length compared to the base sequence O_1.

- In O_2, there are 4 odd integers between each step and the interval length is 10.
- In O_3, there are 6 odd integers between each step and the interval length is 14.
- More generally, in O_a, there are $2a$ odd integers between each interval and the interval length is 2+4a.

If O_1 and the other sequences $O_2, O_3, \dots, O_k$ form a proper alliance, then the Lower Formation condition arises.

For example:

- Since the O_1 sequence has an interval length of 6, the maximum allowable extra interval length for the O_2 sequence is $10 - 6 = 4$. Thus, the extra interval length of O_2 must lie between 0 and 4. Only under this condition will the odd integers $11 + 6r_1$ and $13 + 6r_1$, lying between $9 + 6r_1$ and $15 + 6r_1$, remain non-composite when tested at O_2.
- In O_3, the calculation gives $14 - 6 = 8$.
- In general, for O_a, the bound becomes $(2 + 4a) - 6 = 4a - 4$.

So the extra interval length

$$(2 + 4a)(r_a - \lfloor r_a \rfloor) = (2 + 4a)\{r_a\}$$

Therefore, we impose the **proper alliance condition**:

$$0 \leq (2 + 4a)\{r_a\} \leq 4a - 4$$

In summary: the Lower Formation condition ensures that when the extra interval length stays within the bound 0 and $4a - 4$.

Substituting the expression for $\mathrm{r_a}$:

$$0 \leq [2 + 4\mathrm{a}]\left\{\frac{9 + 6\mathrm{r_1} - (2\mathrm{a} + 1)^2}{2 + 4\mathrm{a}}\right\} \leq 4\mathrm{a} - 4$$

Therefore, the condition requires that when the proper alliance holds for $a \in \{2, 3, 4, \dots, k\}$, the odd integers $11 + 6r_1$ and $13 + 6r_1$ form a twin prime pair under the Lower Formation Theorem. ■

Lower Twin Prime Formation Analysis

Fig.2

test r_1	50					
$9+6r_1$	309					
sequence O_a	1	2	3	5	6	8
initial sequence$((1+2a)^2)$	9	25	49	121	169	289
sequence different(2+4a)	6	10	14	22	26	34
maximun limit(4a-4)	0	4	8	16	20	28
r_a	50	28.4	18.6	8.55	5.38	0.59
$\lfloor r_a \rfloor$	50	28	18	8	5	0
$\{r_a\}$	0	0.4	0.57	0.55	0.38	0.59
(2+4a){ra}	0	4	8	12	10	20
upper formation	pass	fail	pass	pass	pass	pass
lower formation	pass	pass	pass	pass	pass	pass

O_a	1	2	3	5	6	8
289						8,0
290						
291	1,47					
292						
293						
294						
295		2,27				
296						
297	1,48			5,8		
298						
299					6,5	
300						
301			3,18			
302						
303	1,49					
304						
305		2,28				
306			Extra interval length			
307						
308	Test r_1					
309	1,50					
310						
311						
312	Twin prime under lower formation					
313						
314						
315	1,51	2,29	3,19			
316						
317						
318						
319				4,9		
320						
321	1,52					
322						
323						8,1
324						
325		2,30			6,6	
327	1,53					

At $r_1 = 50$

$$k = \left\lfloor \frac{\sqrt{9+6r_1}-1}{2} \right\rfloor = \left\lfloor \frac{\sqrt{9+6\times 50}-1}{2} \right\rfloor = \left\lfloor \frac{\sqrt{309}-1}{2} \right\rfloor = 8$$

So we check values of $a = 2,3,4,5,6,7,8$.

Since $a = 4$is equivalent to the set of $a = 1$ and $a = 7$is equivalent to the set of $a = 5$.

Therefore, the necessary values to check are:

$$a = 2,3,5,6,8$$

Maximum limit(4a-4)={4,8,16,20,28)

$(2 + 4a)\{r_a\} = \{4,8,12,10,20\}$

$$0 \leq (2 + 4a)\{r_a\} \leq 4a - 4$$

Thus, the sequence demonstrates that twin primes are formed through the lower formation mechanism in this case.

$$r_1 = 50$$

lower twin prime

- $11 + 6r_1 = 11 + (6 \times 50) = 11 + 300 = 311$
- $13 + 6r_1 = 13 + (6 \times 50) = 13 + 300 = 313$

The numbers 311 and 313 are both prime, and they differ by 2. Therefore, they form an lower twin prime pair.

Upper Formation Theorem

To search for twin primes, we begin with the base sequence

$$\mathrm{O}_1 = 9 + 6\mathrm{r}_1(\mathrm{r}_1 \neq \frac{(1+2x)^2 - 7}{6}, x = \{1,2,3, \dots\}$$

If other sequences form a proper alliance with this base sequence under the lower formation conditions, then the numbers

$$\mathrm{O}_1 - 4 = 5 + 6\,\mathrm{r}_1 \text{and } \mathrm{O}_1 - 2 = 7 + 6\,\mathrm{r}_1$$

are twin primes.

The alliance condition is expressed as:

$$6 \leq [2 + 4\mathrm{a}]\left\{\frac{9 + 6\mathrm{r}_1 - (2\mathrm{a}+1)^2}{2 + 4\mathrm{a}}\right\} \leq 2 + 4\mathrm{a}$$

where

$$\mathrm{a} \in \{2, 3, 4, \dots, \mathrm{k}\},\ \mathrm{k} = \left\lfloor \frac{\sqrt{9 + 6\mathrm{r}_1} - 1}{2} \right\rfloor$$

Proof

In our search for twin primes, we designate O_1as the base sequence:

$$O_1 = 9 + 6r_1$$

Lemma 2 For every integer $r_1 \geq 0$, the interval between $9 + 6(r_1 - 1)$and $9 + 6r_1$contains exactly two odd integers, namely $5 + 6r_1$and $7 + 6r_1$.

Proof

$$9 + 6(r_1 - 1) = 3 + 6r_1, 9 + 6r_1$$

$3 + 6r_1$ *and* $9 + 6r_1$ are odd composite integers,so the odd integers between this interval are

$5 + 6r_1$and $7 + 6r_1$. However, it is uncertain whether those two odd integers are prime or odd composite. ■

Condition for Upper Formation

For the upper twin prime to emerge, the extra interval lengths of the other sequences $(O_2, O_3, \dots, O_k)$must be at least 6and less than equal 2+4a. Only under this **proper alliance** condition will the odd integers $5 + 6r_1$and $7 + 6r_1$, lying between $3 + 6r_1$and $9 + 6r_1$, remain non-composite. If the extra interval length is less than 6, then the sequences $(O_2, O_3, \dots, O_k)$fall within the interval $[3, +6r_1 9 + 6r_1]$, and consequently $5 + 6r_1$and $7 + 6r_1$will no longer form a prime pair.

Therefore, we impose the **proper alliance condition**:

$$6 \leq (2 + 4a)\{r_a\} \leq 2 + 4a$$

This ensures that the upper formation is possible only when the extra interval length is greater than or equal to 6.

Substituting the definition of $r_a$into the inequality gives:

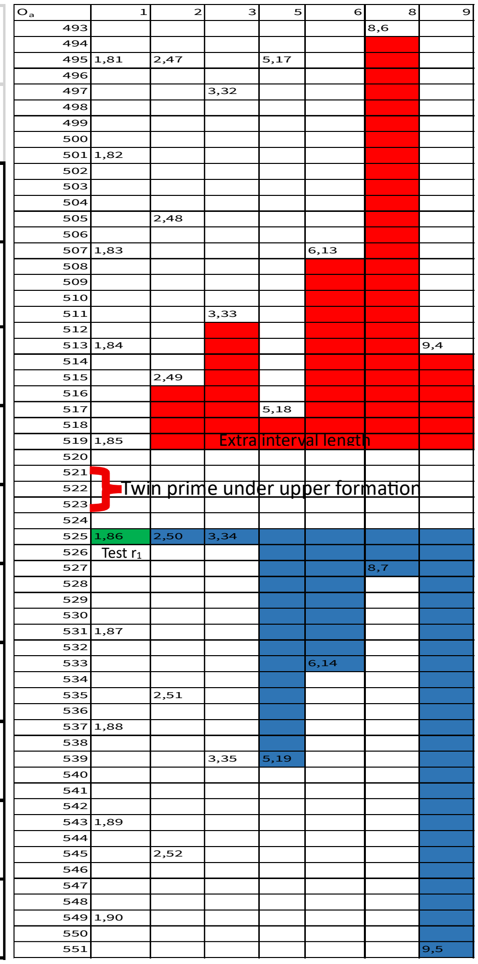
$$6 \leq (2 + 4a)\left\{\frac{9 + 6r_1 - (2a + 1)^2}{2 + 4a}\right\} \leq 2 + 4a$$

Therefore, the condition requires that when the proper alliance holds for $a \in \{2, 3, 4, \dots, k\}$, the odd integers $5 + 6r_1$and $7 + 6r_1$form a twin prime pair under the **Upper Formation Theorem.** ■

Upper Twin Prime Formation Analysis

Fig.3

test r_1	86						
$9+6r_1$	525						
sequence O_a	1	2	3	5	6	8	9
initial sequence($(1+2a)^2$)	9	25	49	121	169	289	361
sequence different(2+4a)	6	10	14	22	26	34	38
maximun limit(4a-4)	0	4	8	16	20	28	32
r_a	86	50	34	18.4	13.7	6.94	4.32
$\lfloor r_a \rfloor$	86	50	34	18	13	6	4
$\{r_a\}$	0	0	0	0.36	0.69	0.94	0.32
(2+4a){ra}	0	0	0	8	18	32	12
upper formation	pass	pass	pass	pass	pass	pass	pass
lower formation	pass	pass	pass	pass	pass	fail	pass



At $r_1 = 86$

$$k = \left\lfloor \frac{\sqrt{9+6r_1}-1}{2} \right\rfloor = \left\lfloor \frac{\sqrt{9+6\times 86}-1}{2} \right\rfloor = \left\lfloor \frac{\sqrt{325}-1}{2} \right\rfloor = 10$$

So we check values of $a = 2,3,4,5,6,7,8,9,10$.

Since $a = 4$is equivalent to the set of $a = 1$, $a = 7$is equivalent to the set of $a = 5$ and $a =$ 10is equivalent to the set of $a = 6$.

Therefore, the necessary values to check are:

$$a = 2,3,5,6,8,9$$

$(2 + 4a)\{r_a\} = \{0,0,8,18,32,12\}$

$$6 \le (2 + 4a)\{r_a\} \le 2 + 4a$$

Thus, the sequence demonstrates that twin primes are formed through the upper formation mechanism in this case.

$$r_1 = 86$$

Upper twin prime

- $5 + 6r_1 = 5 + (6 \times 86) = 5 + 516 = 521$
- $7 + 6r_1 = 7 + (6 \times 86) = 7 + 516 = 523$

The numbers 521 and 523 are both prime, and they differ by 2. Therefore, they form an lower twin prime pair.

Lower and Upper Formation Connection Theorem

If a tested value $r_1$yields a lower formation, then testing $r_1 + 1$will result in an upper formation.

Proof

Lower formation condition

$$0 \leq (2 + 4a)\{r_a\} \leq 2 + 4a - 6$$

$$0 \leq [2 + 4\mathrm{a}]\left\{\frac{9 + 6\mathrm{r}_1 - (2\mathrm{a} + 1)^2}{2 + 4\mathrm{a}}\right\} \leq 2 + 4\mathrm{a} - 6$$

For r_1+1

$$[2 + 4\mathrm{a}]\left\{\frac{9 + 6(\mathrm{r}_1 + 1) - (2\mathrm{a} + 1)^2}{2 + 4\mathrm{a}}\right\} = [2 + 4\mathrm{a}]\left\{\frac{9 + 6\mathrm{r}_1 - (2\mathrm{a} + 1)^2}{2 + 4\mathrm{a}} + \frac{6}{2 + 4a}\right\}$$

$$[2 + 4\mathrm{a}]\left\{\frac{9 + 6(\mathrm{r}_1 + 1) - (2\mathrm{a} + 1)^2}{2 + 4\mathrm{a}}\right\} = [2 + 4\mathrm{a}]\left\{\frac{9 + 6(\mathrm{r}_1 + 1) - (2\mathrm{a} + 1)^2}{2 + 4\mathrm{a}}\right\} + 6$$

So

$$0 + 6 \leq [2 + 4\mathrm{a}]\left\{\frac{9 + 6\mathrm{r}_1 - (2\mathrm{a} + 1)^2}{2 + 4\mathrm{a}}\right\} + 6 \leq 2 + 4\mathrm{a} - 6 + 6$$

$$6 \leq [2 + 4\mathrm{a}]\left\{\frac{9 + 6\mathrm{r}_1 - (2\mathrm{a} + 1)^2}{2 + 4\mathrm{a}}\right\} + 6 \leq 2 + 4\mathrm{a}\blacksquare$$

Double Twin Prime Theorem

Base Sequence

To search for double twin primes, we begin with the base sequence:

$$O_{DTP} = 45 + 30r_{DTP},\ r_{DTP} = \{0, 1, 2, 3, \dots\}$$

This sequence generates candidate numbers for double twin primes.

If other sequences form a proper alliance with this base sequence O_{DTP} under the double twin prime conditions, then:

- $41 + 30r_{DTP}$ and $43 + 30r_{DTP}$ will be upper twin primes
- $47 + 30r_{DTP}$ and $49 + 30r_{DTP}$ will be lower twin primes

The alliance condition is expressed as:

$$(2+4a)\left\{\frac{45 + 30r_{(1=2)(DTP)} - (1+2a)^2}{2+4a}\right\} = 0 \ or$$

$$6 \leq (2+4a)\left\{\frac{45 + 30r_{(1=2)(DTP)} - (1+2a)^2}{2+4a}\right\} \leq 4a - 4$$

Proof

Due to $O_{\mathrm{DTP}} \subseteq O_1$ *and b*y Lemma 1 and Lemma 2,

$O_{\mathrm{DTP}} - 4 = 41 + 30r_{DTP}$, $O_{\mathrm{DTP}} - 2 = 43 + 30r_{DTP}$, $O_{\mathrm{DTP}} + 2 = 47 + 30r_{DTP}$ and $O_{\mathrm{DTP}} + 4 = 49 + 30r_{DTP}$ are odd integers located above and below r_{DTP}. However, it is uncertain whether these odd integers are prime or odd composite.

Lemma 3

The base sequence for the Double Twin Prime theorem is:

$$O_{\mathrm{DTP}} = 45 + 30r_{\mathrm{DTP}},\ r_{\mathrm{DTP}} = \{0, 1, 2, 3, \dots\}$$

Proof

- When $6 \leq 10\{r_2\} < 10(0)$, only the **upper twin prime** can appear. This is because the next extra interval length is less than 6 (since $10 - 6 = 4 < 6$), so a lower twin prime cannot be generated.
- When $0 < 10\{r_2\} \leq 4$, only the **lower twin prime** can appear. This is because the next extra interval length is less than 6 (since $4 < 6$), so an upper twin prime cannot be generated.

Therefore, for a **double twin prime** to arise, the interval of O_a must be either **twice as large as** the interval of O_1, or **equal to it**. Since the interval length of the sequence O_1 is 6 and the interval length of the sequence O_2 is 10, the necessary condition for the existence of a double twin prime

$$O_1 = O_2 \ \Rightarrow \ 9 + 6r_1 = 25 + 10r_2$$

We begin with the condition:

$$6r_1 - 10r_2 = 16$$

This is a linear Diophantine equation, meaning we are looking for integer solutions (r_1, r_2).

Let's test small integer values:

- If $r_1 = 6$, then:

$$6(6) - 10r_2 = 16$$

$$-10r_2 = 16 - 36$$

$$r_2 = 2$$

So, one particular solution is:

$$(r_1, r_2) = (6, 2)$$

For linear Diophantine equations, the general solution is found by adding multiples of the coefficients:

$$r_1 = 6 + 10r_{DTP},\ r_2 = 2 + 6r_{DTP}, r_{DTP} \in \mathbb{Z}$$

Substitute the general solution:

- For O_1:

$$O_{DTP} = 9 + 6(6 + 5r_{DTP}) = 9 + 36 + 30r_{DTP} = 45 + 30r_{DTP}$$

- For O_2:

$$O_{DTP} = 25 + 10(2 + 3r_{DTP}) = 25 + 20 + 30r = 45 + 30r_{DTP}$$

Both expressions agree perfectly.

Thus, the base sequence for the Double Twin Prime theorem is:

$$O_{\mathrm{DTP}} = 45 + 30r_{\mathrm{DTP}},\ r_{\mathrm{DTP}} = \{0, 1, 2, 3, \dots\}$$

This sequence generates the candidate numbers that can potentially form double twin primes■

So we can calculate r_a of O_a

$$O_{\mathrm{DTP}} = O_a,\ a = \{3, 4, \dots, k\}$$

$$45 + 30r_{\mathrm{DTP}} = (1 + 2a)^2 + (2 + 4a)r_a$$

$$r_a = \frac{45 + 30r_{\mathrm{DTP}} - (1 + 2a)^2}{2 + 4a}$$

For double twin prime condition,

$$(2 + 4a)\{r_a\} = 0$$

or

$$6 \le (2 + 4a)\{r_a\} \le 4a - 4$$

$$(2+4a)\left\{\frac{45+30r_{\mathrm{DTP}}-(1+2a)^2}{2+4a}\right\}=0 \ or$$

$$6 \le (2+4a)\left\{\frac{45+30r_{\mathrm{DTP}}-(1+2a)^2}{2+4a}\right\} \le 4a-4$$

produces twin primes, both the lower and upper twin prime formations occur simultaneously.

In this situation:

- $41+30r_{DTP}$ and $43+30r_{DTP}$ will be upper twin primes
- $47+30r_{DTP}$ and $49+30r_{DTP}$ will be lower twin primes

When both conditions are satisfied at the same time, we obtain a double twin prime configuration. ■

Double Twin Prime Formation Analysis

We examine the sequence values for different a:

test r_{DTP}	5				
$45+30r_{DTP}$	195				
sequence O_a	1	2	3	5	6
initial sequence($(1+2a)^2$)	9	25	49	121	169
sequence different(2+4a)	6	10	14	22	26
maximun limit(4a-4)	0	4	8	16	20
r_a	31	17	10.4	3.36	1
$\lfloor r_a \rfloor$	31	17	10	3	1
$\{r_a\}$	0	0	0.43	0.36	0
(2+4a){ra}	0	0	6	8	0
upper formation	pass	pass	pass	pass	pass
lower formation	pass	pass	pass	pass	pass

integer	O_1	O_2	r_{DTP}	O_3	O_5	O_6
183	1,29					
184						
185		2,16				
186						
187					5,3	
188						
189	1,30			3,10		
190						
191						
192	Upper	twin prime				
193				Extra interval length		
194	Test r DTP					
195	1,31	2,17	5			6,1
196						
197						
198	Lower	twin prime				
199						
200						
201	1,32					
202						
203				3,11		
204						
205		2,18				
206						
207	1,33					
208						
209					5,4	

At $r_{DTP}=5$

$$\mathrm{k}=\left\lfloor\frac{\sqrt{45+30\mathrm{r}_{DTP}}-1}{2}\right\rfloor=\left\lfloor\frac{\sqrt{45+30\times 5}-1}{2}\right\rfloor=\left\lfloor\frac{\sqrt{195}-1}{2}\right\rfloor=6$$

So we check values of $a=3,4,5,6$.

Since $a=4$is equivalent to the set of $a=1$.

Therefore, the necessary values to check are:

$$a = 3,5,6$$

Maximum limit(4a-4)={8,16,20}

$(2+4a)\{r_a\} = \{6,8,0\}$

$$6 \leq (2+4a)\{r_a\} \leq 4a-4$$

Thus, the sequence demonstrates that twin primes are formed both the lower and upper formation mechanism in this case.

$$\mathrm{r}_{\mathrm{DTP}} = 5$$

upper twin primes

- $41 + 30r_{DTP} = 41 + 30 \times 5 = 41 + 150 = 191$
- $43 + 30r_{DTP} = 43 + 30 \times 5 = 43 + 150 = 193$

Thus, the pair $(191, 193)$are both prime and differ by 2, forming an upper twin prime pair.

lower twin primes

- $47 + 30r_{DTP} = 47 + 30 \times 5 = 47 + 150 = 197$
- $49 + 30r_{DTP} = 49 + 30 \times 5 = 49 + 150 = 199$

Thus, the pair $(197, 199)$are both prime and differ by 2, forming a lower twin prime pair.

Conclusion

Since both the upper formation $(191, 193)$and the lower formation $(197, 199)$yield valid twin primes, the base sequence $\mathrm{O}_{DTP} = 195$ produces a double twin prime configuration.

Triple Twin Prime Impossibility Theorem

$$\mathrm{O_{DTP}} - 10, \mathrm{O_{DTP}} - 8, \mathrm{O_{DTP}} - 4, \mathrm{O_{DTP}} - 2, \mathrm{O_{DTP}} + 2, \mathrm{O_{DTP}} + 6 \ (2\ upper + 1 lower)$$

Or

$$\mathrm{O_{DTP}} - 4, \mathrm{O_{DTP}} - 2, \mathrm{O_{DTP}} + 2, \mathrm{O_{DTP}} + 4, \mathrm{O_{DTP}} + 8, \mathrm{O_{DTP}} + 10 (1 upper + 2 lower)$$

When $O_1$and O_2coincide exactly, the next extra interval length is 4. Since 4 is less than the required extra interval length of 6 for another twin prime to arise, triple twin primes can never occur under any circumstances.

Proof

For a triple twin prime to exist, a double twin prime must first occur. Therefore, the base sequence for a double twin prime requires that $O_1 = O_2$exactly.

- The sequence O_1has an interval of 6.
- The sequence O_2has an interval of 10.

When we align them precisely, the extra interval is:

$$O_2 \text{ interval} - O_1 \text{ interval} = 10 - 6 = 4$$

Since 4 is less than the minimum requirement of 6 for another twin prime to form, a triple twin prime can never arise.

Infinity Candidate Analysis

When considering infinity, we must examine both the rate of change of $r_a$with respect to r_1, and the rate of change of $r_{a+1}$with respect to r_a.

$$r_a = \frac{9 + 6r_1 - (1 + 2a)^2}{2 + 4a} = \frac{9 - (1 + 2a)^2}{2 + 4a} + \frac{6}{2 + 4a} r_1$$

$$\frac{d}{dr_1} r_a = \frac{6}{2 + 4a} = \frac{3}{1 + 2a}$$

Since $O_a = O_{a+1}$, we have:

$$(1 + 2a)^2 + (2 + 4a)r_a = \big(1 + 2(a + 1)\big)^2 + \big(2 + 4(a + 1)\big)r_{a+1}$$

$$r_{a+1} = \frac{(1 + 2a)^2 + (2 + 4a)r_a - \big(1 + 2(a + 1)\big)^2}{4a + 6}$$
$$= \frac{(1 + 2a)^2 - \big(1 + 2(a + 1)\big)^2}{4a + 6} + \frac{2 + 4a}{4a + 6} r_a$$

$$\frac{d}{dr_a} r_{a+1} = \frac{2 + 4a}{6 + 4a} = \frac{1 + 2a}{3 + 2a}$$

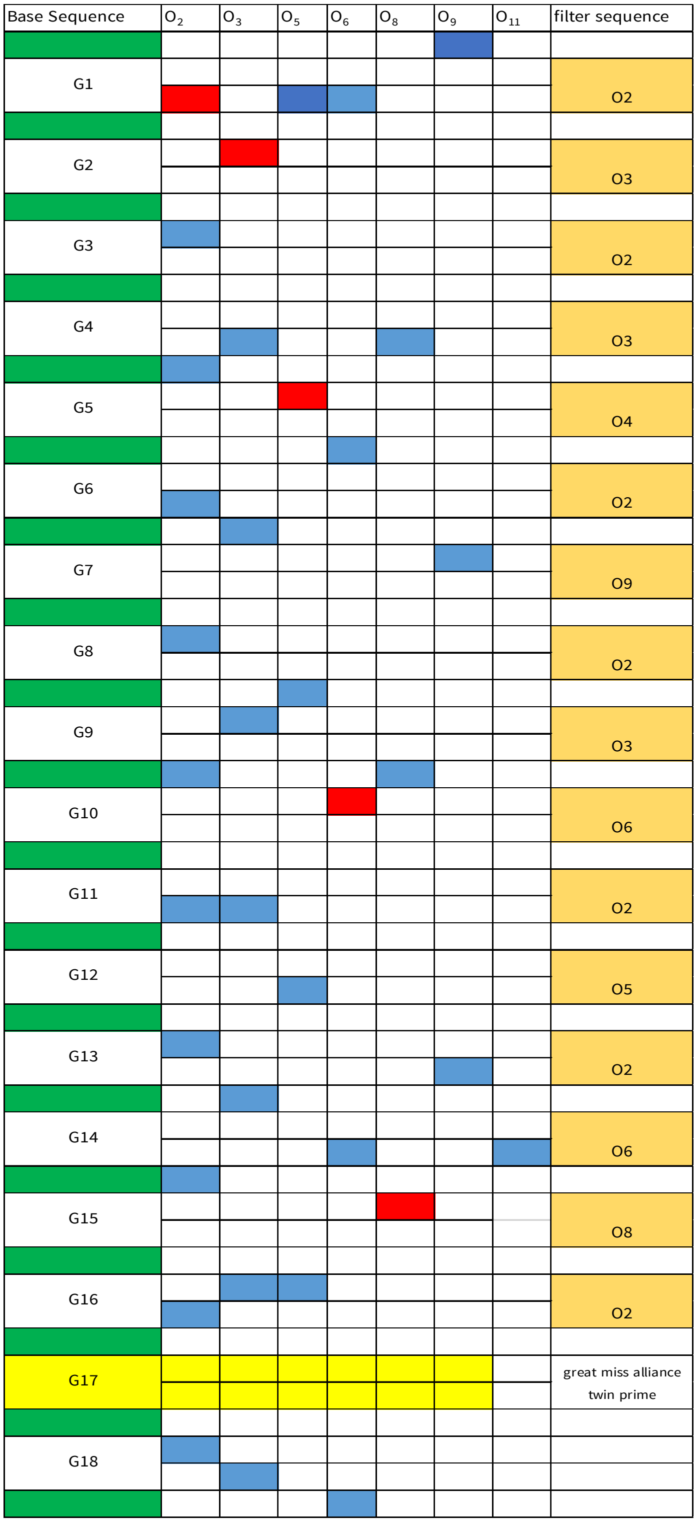
$$\frac{4a - 4}{2 + 4a}$$

Taking limits:

$$\lim_{a \to \infty} \frac{d}{dr_1} r_a = 0, \ \lim_{a \to \infty} \frac{d}{dr_a} r_{a+1} = 1$$

So the formation of twin primes is primarily **dominant at small values of a.**

Great miss alliance for infinity candidate



In the figure

The green squares represent the base sequence O_1.

The gaps G_1, G_2, G_3, ...denote the intervals between terms of the base sequence O_1, each containing two odd integers.

The red squares mark the designated positions of the corresponding sequences, chosen so as to create the maximum possible obstruction.

The blue squares indicate the positions of the corresponding sequences that align consistently with the red squares.

In the above figure, the corresponding sequences are arranged precisely so that the possibility of twin prime formation is minimized. If these placements were adjusted, the occurrence of twin primes would become closer. In actual computation, such exact alignment does not occur, and infinity can arise. However, even if such a great misalignment were to happen, the farthest possible step for twin prime formation has been explicitly calculated.

$$9 + 6r_1 = (1 + 2k)^2 + (2 + 4k)r_k (k = maximum\ sequence\ for\ twin\ prime\ searching)$$

$$9 + 6r_1 = (1 + 2k)^2 (r_k = 0)$$

$$r_1 = \frac{(1 + 2k)^2 - 9}{6}$$

$$6\{r_1\} = 0 \ or \ 4$$

In any sequence, if the starting points coincide with the base sequence O_1, they remain aligned or the extra interval length becomes 4, as shown in the above derivation.

At the beginning of our search for twin primes, we arrange the sequence to coincide with $r_1$so that the occurrence of twin primes will have the lowest probability.

The O_2sequence is aligned with the second odd integer of the first gap G_1, thereby filtering out $G_3, G_6, G_8, G_{11}, G_{13}, G_{16}$from producing twin primes.

The O_3sequence is aligned with the first odd integer of G_2, which filters out $G_4$and G_9.

The O_4sequence is already contained in O_1, so no further alignment is needed.

The O_5sequence is aligned with the first odd integer of G_5, filtering out G_{12}.

The O_6sequence is aligned with the first odd integer of G_{10}, filtering out G_{14}.

The O_7sequence is already contained in O_5, so no further alignment is required.

The O_8sequence is aligned with G_{15}.

The O_9sequence is the most extensively tested sequence. Even though this arrangement is designed to give the lowest probability of twin prime occurrence, a twin prime will still be found at G_{17}.

By the above conditions, at least one twin prime will always be found between $(1 + 2x)^2$**and** $(3 + 2x)^2 + (6 + 4x)$.

odd integer	O1	O2	O3	O5	O6	O8	O9	O11
361							9,0	
363	1,59			5,11				
365		2,34						
367								
369	1,60							
371			3,23					
373								
375	1,61	2,35						
377					6,8			
379								
381	1,62							
383								
385		2,36	3,24	5,12				
387	1,63							
389								
391						8,3		
393	1,64							
395		2,37						
397								
399	1,65		3,25				9,1	
401								
403					6,9			
405	1,66	2,38						
407				5,13				
409								
411	1,67							
413			3,26					
415		2,39						
417	1,68							
419								
421								
423	1,69							
425		2,40				8,4		
427			3,27					
429	1,70			5,14	6,10			
431								
433								
435	1,71	2,41						
437							9,2	
439								
441	1,72		3,28					
443								
445		2,42						

This figure shows the actual calculation of how a great misalliance occurs in order for the twin primes 419 and 421 to appear.

Only odd integers have been included in the computation.

Moreover, the odd composites sequences are arithmetic sequence, always change and continually shift. From the reasoning above, the inequalities described will persist even at infinity. Consequently, as the layers of odd composite sequences increase, the emergence of twin primes becomes slower, yet twin primes will continue to appear even at infinity.

Reference

OEIS A083487