On the Inconsistency of the Standard Model in Exactly Four Dimensions

Ervin Goldfain

Global Institute for Research, Education and Scholarship (GIRES).

Email: ervingoldfain@gmail.com

Abstract

The triviality of the Higgs sector in exactly four spacetime dimensions is a serious obstruction to the ultraviolet completion of the Standard Model for Particle Physics (SM). In contrast with the Asymptotic Safety program of Quantum Gravity, this brief report argues that *minimal spacetime fractality* above the SM scale, a) is a natural extension of Renormalization Group dynamics and b), it can successfully explain away the triviality problem.

Key words: Higgs triviality, Standard Model, UV completion, Asymptotic Safety, , minimal fractal manifold (MFM), dimensional flow.

List of abbreviations

SM = Standard Model

UV = ultraviolet sector of field theory

QFT = Quantum Field Theory

EFT = Effective Field Theory

RG = Renormalization Group

LHC = Large Hadron Collider.

QG = Quantum Gravity

EWSB = electroweak symmetry breaking

AS = Asymptotic Safety

MFM = minimal fractal manifold

I. Introduction

The LHC discovery in 2012 of a scalar particle consistent with the Higgs boson has completed the particle content of the SM while simultaneously boosting long-standing questions about its UV consistency. Among these, the triviality of scalar field theories in four spacetime dimensions remains an unsettled concern. Unlike the hierarchy problem or naturalness arguments, which depend on aesthetic or model-dependent criteria, triviality reflects a mathematically established limitation of interacting scalar QFT. Rigorous results from constructive field theory indeed prove that the Higgs self-coupling must vanish in the UV if *locality*, *unitarity*, and *a continuum limit* are preserved [1, 3].

It has been known for several decades that interacting ϕ^4 theory in exactly four spacetime dimensions lacks a nontrivial continuum limit. Perturbative RG analyses reveal a positive beta function for the quartic coupling, leading either to a *Landau pole at finite energy* or to a *vanishing renormalized coupling* in the UV. More importantly, results from constructive field theory and lattice studies [2] have established that this behavior *is not* an artifact of the perturbation methodology but a genuine property of the theory. As a result, the Higgs sector of SM cannot be UV complete under the standard assumptions of locality, unitarity, and fixed spacetime dimensionality [1 - 3].

The usual response to triviality is to interpret the SM as an *effective field theory* (EFT) valid below a cutoff scale. While phenomenologically adequate, this viewpoint does not address the deeper question of whether a consistent UV completion exists within the framework of local QFT in four spacetime dimensions. In our view, triviality is not merely a technical inconvenience but an unavoidable sign that at least one foundational assumption must be revisited.

Several strategies have been proposed to evade this conclusion. Among them, *Asymptotic Safety* (AS) posits the existence of a non-Gaussian UV fixed point that renders the theory predictive at arbitrarily high energies. Although this idea may bear fruit in the context of Quantum Gravity (QG), its applicability to scalar field theories in exactly four dimensions remains questionable [4, 5].

In line with our previous publications, we argue below that an unexplored alternative approach is to abandon the (intuitively driven) assumption that spacetime dimension is fixed and integer-valued at all scales. We point out that *dimensional flow*—where the effective spacetime dimension becomes scale-dependent—provides an “Occam Razor” resolution to the triviality problem. Within this framework, the hypotheses underlying triviality theorems are no longer relevant in the continuum limit, while standard four-dimensional physics is recovered at accessible energies. We emphasize that this approach does not modify the low-energy physics of SM but instead extends the RG framework to include the dynamic behavior of spacetime itself. We find that, while AS in scalar sectors remains a far-fetched conjecture, allowing spacetime dimension to run with the observation scale provides a straightforward solution to Higgs triviality. The bottom line of our approach is that *spacetime is inherently a fractal structure endowed with arbitrarily small but nonvanishing deviations from four dimensions,* that is, $\varepsilon = 4 - d << 1$ [6 – 8]

2. Triviality of the Higgs sector

The Higgs sector of SM is a relativistic scalar theory with quartic interaction:

$$\mathcal{L}_H = |\partial H|^2 - m^2 |H|^2 - \lambda |H|^4$$

In exactly four spacetime dimensions, this theory belongs to the universality class of the φ^4 models, whose RG behavior is rigorously known. The one-loop RG equation of this theory reads

$$\beta_\lambda = \mu \frac{d\lambda}{d\mu} = \frac{3}{8\pi^2}\lambda^2 + \mathcal{O}(\lambda^3).$$

with solution

$$\lambda(\mu) = \frac{\lambda(\mu_0)}{1 - \frac{3\lambda(\mu_0)}{8\pi^2}\ln(\mu/\mu_0)}.$$

This solution leads to a couple of problematic scenarios, namely,

1. *Landau pole* at finite $\mu = \Lambda$ determined by the condition

$$1 - \frac{3\lambda(\mu_0)}{8\pi^2}\ln\left(\frac{\mu}{\mu_0}\right) = 0$$

2. The continuous spacetime limit $\Lambda \to \infty$ necessarily implies $\lambda(\mu_0) \to 0$.

The second scenario, typically associated with the triviality problem, tells us that a nontrivial continuum limit of ϕ^4 fails to exist in exactly four dimensions. That is to say that, in ordinary four-dimensional spacetime, the interacting scalar theory necessarily becomes free. As previously mentioned, triviality is not a *perturbative artifact*, it is also supported by rigorous analysis and lattice results [1 – 3].

2. Triviality implications for SM

It is well known that the Higgs sector is not an optional feature of SM because:

- It generates fermion and vector boson masses,
- It unitarizes longitudinal $W_L W_L$ scattering
- It fixes the process of spontaneous electroweak symmetry breaking (EWSB).

If the Higgs self-coupling vanishes in the UV, $\lambda(\mu) \to 0$ as $\Lambda \to \infty$, then:

1. The Higgs boson becomes non-interacting (a behavior also called Gaussian),
2. The vacuum expectation value becomes unstable,
3. EWSB loses dynamical meaning,
4. SM cannot be defined as a UV-complete QFT.

That the Higgs sector of SM cannot survive as a mathematically consistent theory in exactly four spacetime dimensions is a *fundamental obstruction*, not a model-building inconvenience.

3. Why "UV cutoff" arguments do not evade triviality

A common evasion to triviality is that SM is only an effective field theory (EFT) below some cutoff scale. In our view, this interpretation is open to debate and logically insufficient because:

- Triviality does not say the SM breaks at high energy,
- It says no interacting continuum limit exists at all in four spacetime dimensions (4D).

An EFT must still arise as a low-energy limit of a high-energy theory. If that theory lives in 4D and is local and unitary, triviality is prone to resurface in one way or another.

The natural question is then: *What must change to allow a nontrivial UV completion?* Next section contains a brief survey of potential solutions that have been put forward over the last decades.

4. Proposals on how to solve the triviality problem

4.1 Supersymmetry (SUSY)

SUSY addresses the triviality problem by embedding the Higgs sector into a larger, and more constrained framework that prevents it from becoming free as the energy scale goes up. Specifically, some SUSY models claim to provide a symmetry-based origin for the Higgs self-coupling that keeps it perturbative and well-behaved. However, no experimental evidence for SUSY exists at the time of writing.

4.2 Composite Higgs / Technicolor

Technicolor and Composite Higgs models address the triviality problem by removing the need for a fundamental, elementary scalar field—which is deemed the source of the issue—and replacing it with a strongly interacting sector.

Technicolor assumes that the Higgs is a *bound state* of new fermions (techni-fermions) held together by a new gauge interaction. This new interaction is asymptotically free at high energy, similar to Quantum Chromodynamics (QCD). Instead of the SM potential with an elementary scalar, Technicolor posits that electroweak symmetry is broken dynamically by a fermion condensate analogous to the action of Cooper pairs in superconductivity.

By contrast, *composite Higgs models* postulate that a light Higgs boson exists but is a *composite pseudo-Nambu-Goldstone boson*. Its mass is protected by a global symmetry, making it naturally light and avoiding the infinite self-coupling issues of fundamental scalars.

Original Technicolor models were largely ruled out by the LHC discovery of the 125 GeV Higgs.

4.3 Asymptotic Safety

Asymptotic Safety (AS) offers a non-perturbative solution to the triviality problem by replacing the Landau pole with a nontrivial UV fixed point. Couplings are allowed to reach a stable, non-zero value at infinite cutoffs (continuum four-dimensional spacetime), preserving the theory's interacting nature. In particular, by establishing this UV fixed point, the theory becomes UV-complete, meaning it can be extended to arbitrarily high energy scales without encountering the mathematical inconsistencies typical for SM [4, 5].

AS is confronted with several limitations related to:

- *Gravity Dependence*: The mechanism often relies on the inclusion of QG effects, which act as a stabilizing "anti-screening" force for gauge and Yukawa couplings.

- *Model Specificity*: While promising, finding a specific fixed point that accounts for all SM couplings remains an active area of research and debate.
- No rigorous proof for scalar sectors exist in AS.

4.4 Higher-dimensional operators (ϕ^6, ϕ^8)

At least in principle, the inclusion of higher-dimensional operators like ϕ^6 or ϕ^8 with positive couplings allows for a stable vacuum even if the quartic coupling turns negative at high scales. This can significantly reduce the impact of triviality, which assumes that only SM interactions exist. These operators are "irrelevant" in the RG sense, meaning their effects are typically suppressed by powers of the heavy physics scale. However, they can still produce observable effects at low energy, such as corrections to the *precision parameters*, the latter being measurable quantities used to parameterize potential "new physics" contributions to the electroweak sector. Although unproven to date, higher dimensional operators may, if certain conditions are met, offset the Higgs triviality entirely or bring it under control.

Next section introduces the idea of *continuous dimensional deviation* lying at the foundation of the *minimal fractal manifold (MFM)* and providing a clear-cut solution to the triviality problem [6 – 8].

5. Emergence of a nontrivial fixed point in scalar field theory

Consider the Higgs potential of field theory written as,

$$V(\varphi)=\lambda(|\varphi|^2-\frac{1}{2}\mathrm{v}^2)^2 \tag{1}$$

where v stands for the vacuum expectation value of the Higgs boson and φ is considered a real scalar field for simplicity. (1) can be cast in the form

$$V_H(\varphi)=V(\varphi)-\frac{1}{4}\lambda v^4=-\lambda v^2\varphi^2+\lambda\varphi^4 \tag{2a}$$

(2a) can be associated with the partition function of Statistical Physics based upon the functional integral

$$Z[j]=\int D\varphi\exp(-S[\varphi]) \tag{2b}$$

where the action has the form,

$$S[\varphi]=\frac{1}{2}\int d\vec{x}\varphi(\vec{x})[r-\nabla^2]\varphi(\vec{x})+\frac{1}{4}u\int d\vec{x}\varphi^4(\vec{x})-\int d\vec{x}\,j(\vec{x})\varphi(\vec{x}) \tag{3}$$

Here, $j(\vec{x})$ plays the role of an external current and the coefficient r has the dimensions of [mass] ^ 2. Side by side comparison of (2a) and (3) leads to the identification,

$$r=m^2 \Leftrightarrow 2\lambda v^2 \tag{4}$$

$$\lambda \Leftrightarrow -\frac{1}{4}u \tag{5}$$

The RG analysis of (3) starts by splitting the field into its long and short wavelengths components according to

$$\varphi(\vec{k})=\varphi_{long}(\vec{k})+\varphi_{short}(\vec{k}) \tag{6}$$

$$\varphi_{long}(\vec{k})=\begin{cases}\varphi(\vec{k}), & 0<k<\Lambda/b \\ 0, & \Lambda/b<k<\Lambda\end{cases} \tag{7}$$

$$\varphi_{short}(\vec{k}) = \begin{cases} 0\ ,\ 0 < k < \Lambda/b \\ \varphi(\vec{k}), \quad \Lambda/b < k < \Lambda \end{cases} \tag{8}$$

where b is a scaling factor and $\Lambda < \Lambda_{UV}$ denotes the upper energy scale of RG calculations. The RG flow of parameters (r, λ, j) is described by the β-functions of the theory. To compute the β-functions, one considers an infinitesimal momentum shell integration defined by

$$b = \exp(\delta l) \approx 1 + \delta l \tag{9}$$

and the RG flow equations in near four-dimensional spacetime read,

$$\beta_r = \frac{dr}{dl} = 2r + 3K_4\Lambda^4 \frac{u}{r+\Lambda^2} + O(u^2) \tag{10}$$

$$\beta_u = \frac{du}{dl} = \varepsilon u - 9K_4\Lambda^4 \frac{u^2}{(r+\Lambda^2)^2} + O(u^2) \tag{11}$$

in which $K_4 = (8\pi^2)^{-1}$. Equations (10) and (11) have a trivial (Gaussian) fixed point solution defined as

$$\beta_r = \beta_u = 0 \Rightarrow r^* = u^* = 0 \tag{12}$$

To analyze the behavior of RG flows near (12), one proceeds by linearizing (10) and (11) and solving the corresponding eigenvalue equation. The pair of solutions satisfying the eigenvalue equation is given by,

$$\lambda_1^a = 2 > 0 \tag{13}$$

$$\lambda_2^a = \varepsilon = 4 - d \tag{14}$$

It is seen that (14) is positive (relevant) for $d < 4$ but negative (irrelevant) for $d > 4$. Since the Gaussian fixed point (12) corresponds to a vanishing quartic coefficient (4)-(5), it follows from this analysis that Higgs sector is *unstable* in less than $d = 4$ dimensions but turns *stable* in $d > 4$ dimensions. A non-trivial fixed-point (called the *Wilson-Fisher* or *WF point*) of RG equations (10) - (11) emerges if one considers the small dimensional deviation $\varepsilon = 4 - d << 1$ as a *tunable parameter.* Expanding the RG equations to quadratic order yields

$$\frac{dr}{dl} \approx 2r + au - bur \tag{15}$$

$$\frac{du}{dl} \approx \varepsilon u - 3bu^2 \tag{16}$$

with $a = 3K_4\Lambda^2$ and $b = 3K_4$. The WF point derived from (15) - (16) is located at,

$$\boxed{\left|u^*\right| = 4\lambda^* = \frac{1}{3b}\varepsilon} \tag{17}$$

$$\boxed{r^* = (m^*)^2 = -\frac{a}{6b}\varepsilon} \tag{18}$$

The pair of eigenvalues associated with the WF point are found to be,

$$\lambda_1^{WF} = 2 - \frac{\varepsilon}{3} > 0 \tag{19}$$

$$\lambda_2^{WF} = -\varepsilon < 0 \tag{20}$$

which makes the WF point *stable* in less than four dimensions ($d < 4$). The flows corresponding to the Gaussian and WF fixed points are displayed in Figs. 1 - 2 below. The key point of this analysis is that, according to (17) – (18), both mass ^ 2 term and coupling parameter of scalar field *arise from the continuous and nonvanishing dimensional deviation* ε, whose value runs with the energy scale.

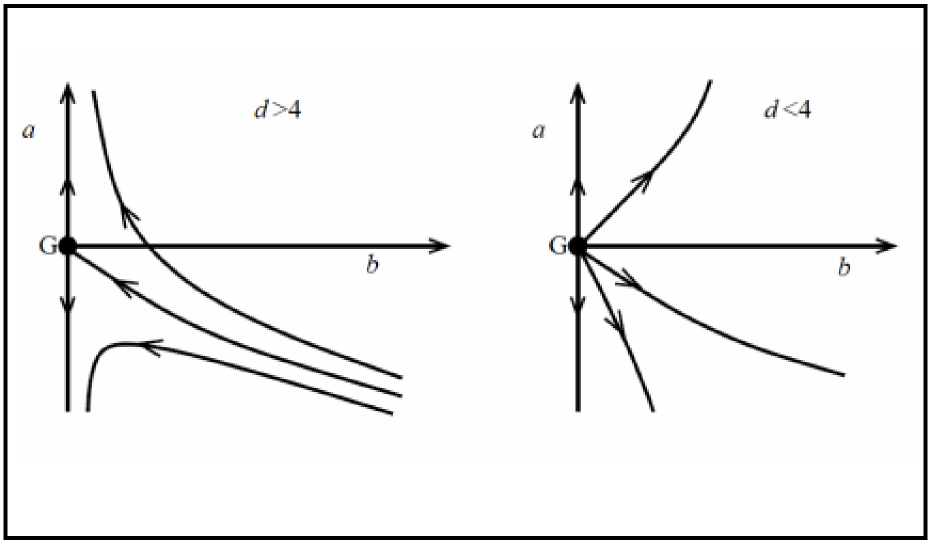

Fig.1: The Gaussian fixed point of scalar field theory

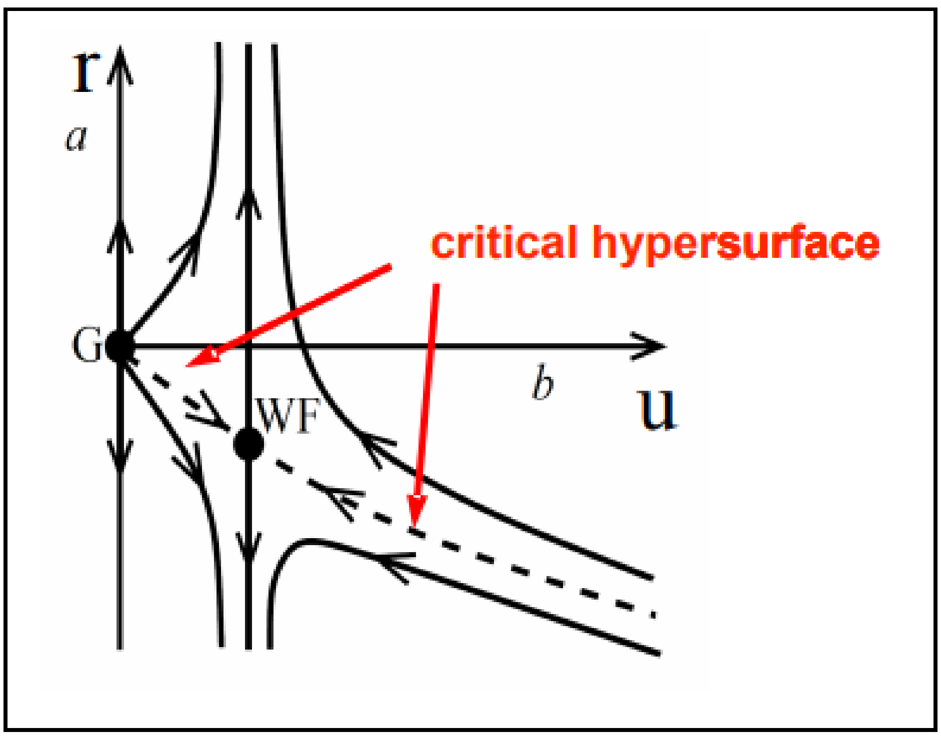

Fig. **2**: The Wilson-Fisher fixed point of scalar field theory

It is therefore apparent that, if the dimensional parameter is $\varepsilon = 4 - d << 1$, the WF point imparts a *non-zero mass* and *non-zero self-coupling* for the scalar field, automatically solving the triviality problem. Moreover, allowing high-energy fluctuations of dimensional parameter to split and bifurcate the WF point in a self-similar pattern, recovers the *full composition* of SM [9]. Remarkably, the *bifurcation scenario* of the Higgs sector naturally accounts for

a) the fine-tuning and tachyonic mass problems,

b) the existence of three generations of fermions,

c) chiral symmetry breaking in electroweak channels [10],

d) the existence of the Dark Matter/Cantor Dust bifurcation branch sandwiched between gauge boson and neutrino branches,

e) the theoretical basis for the sum-of-squares relationship, mass of the top quark and the scaling behavior of particle masses and mixing angles [11].

6. Conclusions

The triviality of scalar field theories in exactly four spacetime dimensions represents a fundamental obstruction to the UV completion of the SM Higgs sector. Unlike phenomenological shortcomings that can be addressed by extending particle content or invoking symmetry principles, triviality is rooted in rigorous mathematical results that leave little room for reinterpretation. When locality, unitarity, and the existence of a continuum limit are maintained, an interacting Higgs sector cannot survive in the UV within *fixed* four-dimensional spacetime.

We have further argued that almost all proposed resolutions which preserve fixed dimensionality—most notably AS in scalar sector - remain conjectural and are not currently supported by rigorous evidence. In contrast, allowing spacetime dimensions to flow with the energy scale directly modifies the canonical scaling behavior responsible for triviality and does so without introducing new low-energy degrees of freedom or conflicting with existing experimental constraints.

From this perspective, dimensional flow should not be viewed as an exotic modification of particle physics but as a natural extension of RG dynamics. More broadly, these results suggest that the UV completion of QFT requires spacetime itself to participate dynamically in the Renormalization process. In this sense, Higgs triviality may be viewed not as a failure of SM, but as an indication that fixed four-dimensional spacetime is an *emergent, infrared concept* rather than a fundamental one.

References

[1] M. Aizenman, *Proof of the Triviality of* ϕ_d^4 *Field Theory and Some Mean-Field Features of Ising Models for* $d > 4$, Phys. Rev. Lett. **47**, 1 (1981).

[2] J. Fröhlich, *On the Triviality of* $\lambda\phi_d^4$ *Theories and the Approach to the Critical Point in* $d \geq 4$*Dimensions*, Nucl. Phys. B **200**, 281 (1982).

[3] J. Glimm and A. Jaffe, *Quantum Physics: A Functional Integral Point of View*, Springer (1987).

[4] C. Wetterich, *Exact evolution equation for the effective potential*, Phys. Lett. B **301**, 90 (1993).

[5] R. Percacci, *An Introduction to Covariant Quantum Gravity and Asymptotic Safety*, World Scientific (2017).

[6] E. Goldfain, preprint https://doi.org/10.13140/RG.2.2.16611.87844/1 (2025).

[7] E. Goldfain, preprint https://doi.org/10.13140/RG.2.2.21554.99524/2 (2023).

[8] E. Goldfain, preprint https://doi.org/10.13140/RG.2.2.27631.50084/1 (2026).

[9] E. Goldfain, preprint https://doi.org/10.13140/RG.2.2.14675.39209/1 (2024).

[10] E. Goldfain, preprint https://doi.org/10.13140/RG.2.2.16252.40321 (2025)

[11] E. Goldfain, preprint https://doi.org/10.13140/RG.2.2.33784.92163 (2025).