Dihedral angles between the polygonal faces in various regular & uniform polyhedra: Platonic and Archimedean solids

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Abstract

This paper presents a set of practically oriented formulas and tables compiled from geometric data of various uniform polyhedra, listing the dihedral angles between adjacent faces sharing a common edge. These tables are specifically intended to support the physical and computational construction of convex uniform polyhedral shells composed of different regular polygonal faces. By using the tabulated dihedral angles, wire-frame and shell models of polyhedra can be constructed efficiently by successively joining adjacent planar faces at their common edges with the correct angular orientation. The presented data are particularly useful for applications in geometric modeling, structural design, educational model fabrication, and the development of computational algorithms for polyhedral assembly. Overall, the tables provide a convenient and reliable reference for the practical realization and analysis of convex uniform polyhedral structures.

Keywords: Dihedral angles, regular n-gonal faces, HCR's formulas, Platonic solids, Archimedean solids

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1. Introduction

In this paper, the sets of analytic formulas and tables have been derived and prepared by the author using his data tables of convex uniform polyhedra [1], specifically all 5 Platonic solids [2,3] and 13 Archimedean solids [4] for determining the dihedral angles between any two adjacent regular polygonal faces for different convex uniform polyhedral shells. These are very useful for the construction and preparing the wire-frame models of the convex uniform polyhedral shells having different regular polygonal faces. A polyhedral shell can be easily constructed/framed by continuously fixing all its adjacent (flat) faces each two as a pair at their common edge at an angle equal to the dihedral angle between them.

2. Dihedral angles for Platonic and Archimedean solids

2.1. Dihedral angle between two regular polygonal faces with a common edge: The dihedral angle is the angle of inclination between any two adjacent faces, having a common edge in a convex uniform polyhedron, measured normal to their common edge. Mathematically, the dihedral angle (θ_{mne}) between any two adjacent regular m-gonal & n-gonal faces, having a common edge, at the normal distances H_m & H_n respectively from the centre of a uniform polyhedron with edge length a is given by the generalized formula as follows

$$\boldsymbol{\theta_{mne}} = \tan^{-1}\left\{\frac{H_m}{\left(\frac{a}{2}\cot\frac{\pi}{m}\right)}\right\} + \tan^{-1}\left\{\frac{H_n}{\left(\frac{a}{2}\cot\frac{\pi}{n}\right)}\right\} \qquad \forall\ m,n \in N\ \&\ \ m,n \geq 3 \qquad \ldots\ldots..(1)$$

Proof: Consider two adjacent regular m-gonal and n-gonal faces sharing a common edge PQ of length a such that their normal distances from the centre O of a convex uniform polyhedron are H_m & H_n respectively. The perpendiculars OM and ON dropped to m-gonal and n-gonal faces intersect them at their centres M and N respectively. Let r_m and r_n be the respective radii of inscribed circles that touch regular m-gonal and n-gonal faces at the common point A i.e. mid-point of common edge PQ (as shown in Figure 1 below).

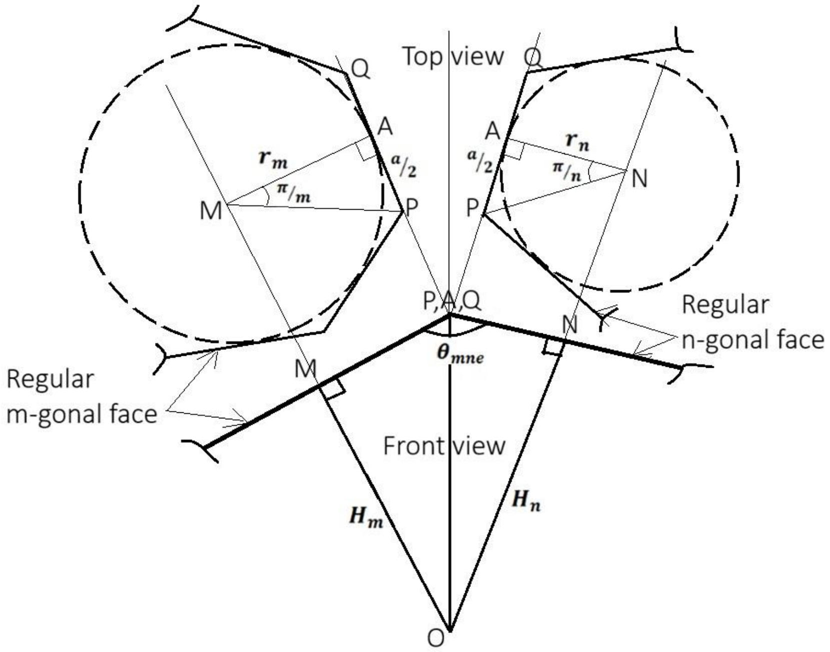

Figure 1: H_m and H_n are normal distances, from the centre O of a convex uniform polyhedron, of regular m-gonal and n-gonal faces with a common edge PQ, that are in the plane of paper in Top view, and ⊥ plane of paper in Front view.

In right ΔMAP (Fig. 1, top view)

$$\tan\angle AMP = \frac{AP}{AM} \Rightarrow \tan\frac{\pi}{m} = \frac{\left(\frac{a}{2}\right)}{AM} \qquad \left(\because \angle AMP = \frac{2\pi}{2m} = \frac{\pi}{m},\ AP = \frac{PQ}{2} = \frac{a}{2}\right)$$

$$\Rightarrow AM = \frac{a}{2}\cot\frac{\pi}{m} \qquad \ldots\ldots\ldots..(2)$$

Similarly, in right ΔNAP (Fig. 1, top view)

$$\tan\angle ANP = \frac{AP}{AN} \Rightarrow \tan\frac{\pi}{n} = \frac{\left(\frac{a}{2}\right)}{AN} \qquad \left(\because \angle AMP = \frac{2\pi}{2n} = \frac{\pi}{n},\ AP = \frac{PQ}{2} = \frac{a}{2}\right)$$

$$\Rightarrow AN = \frac{a}{2}\cot\frac{\pi}{n} \qquad \ldots\ldots\ldots..(3)$$

In right ΔOMA (Fig. 1, front view)

$$\tan\angle OAM = \frac{OM}{AM} = \frac{H_m}{\frac{a}{2}\cot\frac{\pi}{m}} \qquad (\text{Setting value of } AM \text{ from Eq. (2)})$$

$$\Rightarrow \angle OAM = \tan^{-1}\left(\frac{H_m}{\frac{a}{2}\cot\frac{\pi}{m}}\right) \qquad \ldots\ldots\ldots..(4)$$

Similarly, in right ΔONA (Fig. 1, front view)

$$\tan\angle OAN = \frac{ON}{AN} = \frac{H_n}{\frac{a}{2}\cot\frac{\pi}{n}} \qquad (\text{Setting value of } AN \text{ from Eq. (3)})$$

$$\Rightarrow \ \angle OAN = \tan^{-1}\left(\frac{H_n}{\frac{a}{2}\cot\frac{\pi}{n}}\right) \qquad \dots\dots\dots..(5)$$

Now, the dihedral angle, θ_{mne} between the adjacent regular m-gonal and n-gonal faces of a convex uniform polyhedron is given as follows

$$\theta_{mne} = \angle MAN = \angle OAM + \angle OAN \qquad \text{(From above Fig. 1)}$$

$$\theta_{mne} = \tan^{-1}\left\{\frac{H_m}{\left(\frac{a}{2}\cot\frac{\pi}{m}\right)}\right\} + \tan^{-1}\left\{\frac{H_n}{\left(\frac{a}{2}\cot\frac{\pi}{n}\right)}\right\} \qquad \big(\text{Setting values from Eq. (4) \& (5)}\big)$$

The above generalized formula i.e. Eq.(1) is applicable to any convex uniform polyhedron to compute the dihedral angle between any two adjacent regular m-gonal and n-gonal faces with a common edge of length a by specifying the numbers of sides m & n along with the corresponding normal distances H_m & H_n . It is worth noticing that the dihedral angle doesn't depend edge length a but depends solely on the geometric configuration of the uniform polyhedron, specifically on the types and arrangement of its regular polygonal faces.

2.2. Dihedral angle between two vertically opposite regular polygonal faces with a common vertex but no common edge: Mathematically, the dihedral angle (θ_{mnvo}) between two vertically opposite regular m-gonal & n-gonal faces, having a common vertex but no common edge, at the normal distances H_m & H_n respectively from the centre of a uniform polyhedron with edge length a is given by the generalized formula as follows

$$\boldsymbol{\theta_{mnvo} = \sin^{-1}\left(\frac{H_m}{R_o}\right) + \sin^{-1}\left(\frac{H_n}{R_o}\right)} \qquad \forall\ m,n \in N \ \& \ m,n \geq 3 \qquad \dots\dots..(6)$$

Proof: Consider two vertically opposite regular m-gonal and n-gonal faces sharing a common vertex A but no common edge such that their normal distances from the centre O of a convex uniform polyhedron are H_m & H_n respectively. The perpendiculars OM and ON dropped to m-gonal and n-gonal faces intersect them at their centres M and N respectively. Let R_o be the radius of circumscribed sphere touching all the vertices of given convex uniform polyhedron [1] (as shown in Figure 2 below).

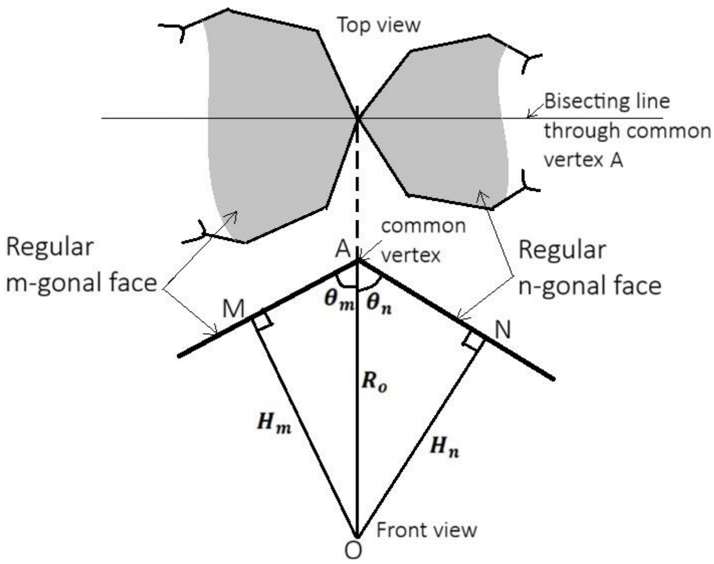

Figure 2: H_m and H_n are normal distances, from the centre O of a convex uniform polyhedron, of two vertically opposite regular m-gonal and n-gonal faces that are in the plane of paper in Top view, and ⊥ plane of paper in Front view.

In right ΔOMA (Fig. 2, front view)

$$\sin\angle MAO = \frac{OM}{OA} \Rightarrow \sin\theta_m = \frac{H_m}{R_o}$$

$$\Rightarrow \theta_m = \sin^{-1}\left(\frac{H_m}{R_o}\right) \quad \dots\dots\dots..(7)$$

In right ΔONA (Fig. 2, front view)

$$\sin\angle NAO = \frac{ON}{OA} \Rightarrow \sin\theta_n = \frac{H_n}{R_o}$$

$$\Rightarrow \theta_n = \sin^{-1}\left(\frac{H_n}{R_o}\right) \quad \dots\dots\dots..(8)$$

Now, the dihedral angle, θ_{mnvo} between two vertically opposite regular m-gonal and n-gonal faces of convex uniform polyhedron is given as follows

$$\theta_{mnvo} = \angle MAN = \angle MAO + \angle NAO = \theta_m + \theta_n \quad \text{(From above Fig. 2)}$$

$$\theta_{mnvo} = \sin^{-1}\left(\frac{H_m}{R_o}\right) + \sin^{-1}\left(\frac{H_n}{R_o}\right) \quad \left(\text{Setting values from Eq. (7) \& (8)}\right)$$

The above generalized formula i.e. Eq.(6) is applicable to any convex uniform polyhedron to compute the dihedral angle between any two vertically opposite regular m-gonal and n-gonal faces with a common vertex but no common edge by specifying circum-radius R_o with the corresponding normal distances H_m & H_n . It is worth noticing that the dihedral angle doesn't depend edge length a but depends solely on the geometric configuration of the uniform polyhedron.

2.3. Dihedral angle between regular polygonal faces bisected by an unshared edge connecting their vertices: The dihedral angle (θ_{nnbue}) between any two regular n-gonal faces that are bisected by an unshared edge connecting their vertices, and are at a normal distance H_n from the centre of a convex uniform polyhedron with edge length a and radius R_o of its circumscribed sphere, is given by the generalized formula as follows

$$\boldsymbol{\theta_{nnbue} = 2\sin^{-1}\left(\frac{\sqrt{\left(\left(\frac{2R_o}{a}\right)^2 - 1\right)\left(\left(\frac{2R_o}{a}\right)^2 \sin^2\frac{\pi}{n} - 1\right)} - 1}{\left(\frac{2R_o}{a}\right)^2 \sin\frac{\pi}{n}}\right)}$$

$$\boldsymbol{= 2\cot^{-1}\left(\frac{\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1} + \left(\frac{2H_n}{a}\right)\sin\frac{\pi}{n}}{\left(\frac{2H_n}{a}\right)\sin\frac{\pi}{n}\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1} - 1}\right)} \quad \forall\ n \in N,\ n \geq 3 \quad \dots\dots..(9)$$

Proof: Consider two regular n-gonal faces bisected by an extended unshared-edge AB, connecting their vertices A and B, of length a such that their normal distances from the centre O of a convex uniform polyhedron are equal to H_n (as shown in Figure 3 (a) below). Let R_o be the radius of circumscribed sphere that touches all the vertices of convex uniform polyhedron (see Figure 3 (b), below).

In right ΔMSA (Fig. 3(b)),

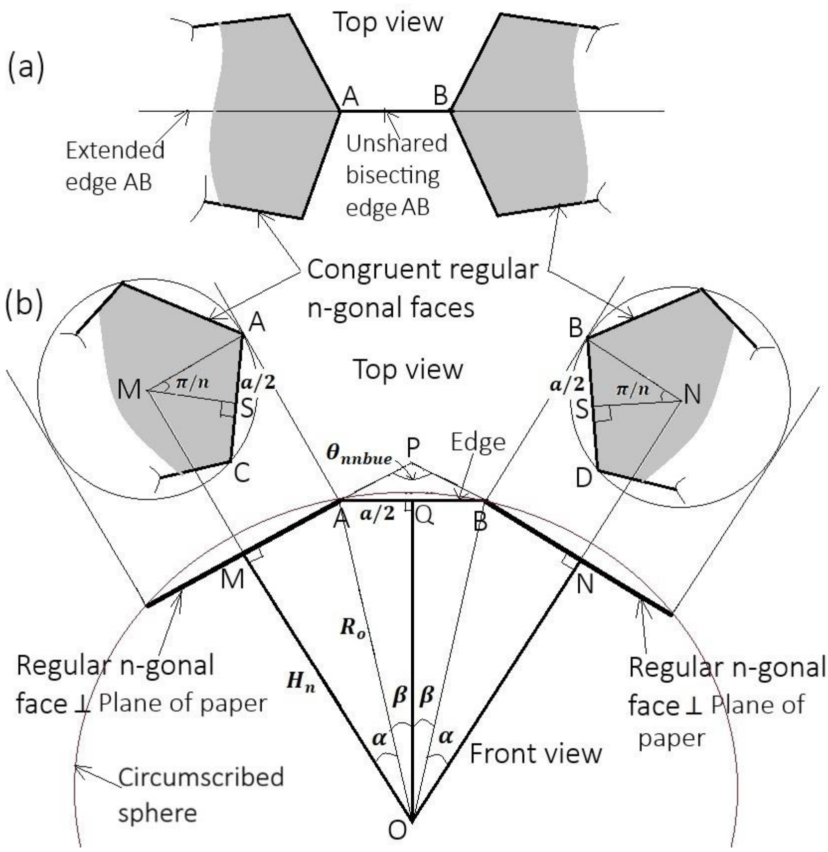

Figure 3: (a) Two congruent regular n-gonal faces, at a normal distance H_n from the centre O of convex uniform polyhedron, are bisected by unshared edge AB in the plane of paper (Top view), and (b) circumscribed sphere with centre O and radius R_o touches all the vertices of polyhedron (Front view).

$$\sin\angle AMS = \frac{AS}{AM} \Rightarrow \sin\frac{\pi}{n} = \frac{\left(\frac{a}{2}\right)}{AM} \qquad \left(\because \angle AMS = \frac{2\pi}{2n} = \frac{\pi}{n},\ AS = \frac{AC}{2} = \frac{a}{2}\right)$$

$$\Rightarrow AM = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n}$$

Similarly, in right ΔAMO (Fig. 3(b)),

$$\sin\angle AOM = \frac{AM}{OA} \Rightarrow \sin\alpha = \frac{\frac{a}{2}\operatorname{cosec}\frac{\pi}{n}}{R_o} \Rightarrow \alpha = \sin^{-1}\left(\frac{a}{2R_o\sin\frac{\pi}{n}}\right) \qquad \dots\dots\dots..(10)$$

$$\tan\angle AOM = \frac{AM}{OM} \Rightarrow \tan\alpha = \frac{\frac{a}{2}\operatorname{cosec}\frac{\pi}{n}}{H_n} \Rightarrow \alpha = \tan^{-1}\left(\frac{a}{2H_n\sin\frac{\pi}{n}}\right) \qquad \dots\dots\dots..(11)$$

In right ΔAQO (Fig. 3(b)),

$$\sin\angle AOQ = \frac{AQ}{OA} \Rightarrow \sin\beta = \frac{\frac{a}{2}}{R_o} \Rightarrow \beta = \sin^{-1}\left(\frac{a}{2R_o}\right) \qquad \dots\dots\dots..(12)$$

$$\tan\angle AOQ = \frac{AQ}{OQ} \Rightarrow \tan\beta = \frac{\frac{a}{2}}{\sqrt{{R_o}^2 - \left(\frac{a}{2}\right)^2}} \Rightarrow \beta = \tan^{-1}\left(\frac{1}{\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1}}\right) \qquad \dots\dots\dots..(13)$$

The two regular n-gonal faces, when extended, intersect each other at a line i.e. represented by the point P in the plane of paper. In (cyclic) quadrilateral OMPN (Fig. 3(b)),

$$\angle OMP + \angle MPN + \angle PNO + \angle MON = 2\pi$$

$$\frac{\pi}{2} + \theta_{nnbue} + \frac{\pi}{2} + 2(\alpha + \beta) = 2\pi$$

$$\theta_{nnbue} = \pi - 2(\alpha + \beta) \quad \ldots\ldots\ldots..(14)$$

Now, substituting the values of angles α and β from the above Eq.(10) and (12) respectively into Eq(14), the dihedral angle, θ_{nnbue} between the two regular n-gonal faces bisected by the unshared edge in a convex uniform polyhedron, is given as follows

$$\theta_{nnbue} = \pi - 2\left(\sin^{-1}\left(\frac{a}{2R_o \sin\frac{\pi}{n}}\right) + \sin^{-1}\left(\frac{a}{2R_o}\right)\right)$$

$$= \pi - 2\cos^{-1}\left(\sqrt{1 - \left(\frac{a}{2R_o \sin\frac{\pi}{n}}\right)^2}\sqrt{1 - \left(\frac{a}{2R_o}\right)^2} - \left(\frac{a}{2R_o \sin\frac{\pi}{n}}\right)\left(\frac{a}{2R_o}\right)\right)$$

$$= \pi - 2\cos^{-1}\left(\left(\frac{a}{2R_o}\right)^2\left(\frac{1}{\sin\frac{\pi}{n}}\right)\sqrt{\left(\frac{2R_o}{a}\right)^2 \sin^2\frac{\pi}{n} - 1}\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1} - \left(\frac{a}{2R_o}\right)^2\left(\frac{1}{\sin\frac{\pi}{n}}\right)\right)$$

$$= 2\left(\frac{\pi}{2} - \cos^{-1}\left(\frac{1}{\left(\frac{2R_o}{a}\right)^2 \sin\frac{\pi}{n}}\left(\sqrt{\left(\left(\frac{2R_o}{a}\right)^2 \sin^2\frac{\pi}{n} - 1\right)\left(\left(\frac{2R_o}{a}\right)^2 - 1\right)} - 1\right)\right)\right)$$

$$\Rightarrow \theta_{nnbue} = 2\sin^{-1}\left(\frac{\sqrt{\left(\left(\frac{2R_o}{a}\right)^2 - 1\right)\left(\left(\frac{2R_o}{a}\right)^2 \sin^2\frac{\pi}{n} - 1\right)} - 1}{\left(\frac{2R_o}{a}\right)^2 \sin\frac{\pi}{n}}\right)$$

The above expression of dihedral angle θ_{nnbue} is same as given by first expression of the above Eq(9). The subscript '$nnbue$' in θ_{nnbue} stands for **two** regular **n**-gonal faces **b**isected by an **u**nshared **e**dge connecting their vertices in a convex uniform polyhedron.

Similarly, substituting the values of angles α and β from the above Eq.(11) and (13) respectively into Eq(14), the dihedral angle, θ_{nnbue} between the two regular n-gonal faces bisected by the unshared edge in a convex uniform polyhedron, is given as follows

$$\theta_{nnbue} = \pi - 2\left(\tan^{-1}\left(\frac{a}{2H_n \sin\frac{\pi}{n}}\right) + \tan^{-1}\left(\frac{1}{\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1}}\right)\right)$$

$$= \pi - 2\tan^{-1}\left(\frac{\frac{a}{2H_n \sin\frac{\pi}{n}} + \frac{1}{\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1}}}{1 - \left(\frac{a}{2H_n \sin\frac{\pi}{n}}\right)\left(\frac{1}{\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1}}\right)}\right) = 2\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{\frac{\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1} + \left(\frac{2H_n}{a}\right)\sin\frac{\pi}{n}}{\frac{2H_n}{a}\sin\frac{\pi}{n}\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1}}}{\frac{\left(\frac{2H_n}{a}\right)\sin\frac{\pi}{n}\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1} - 1}{\frac{2H_n}{a}\sin\frac{\pi}{n}\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1}}}\right)\right)$$

$$\Rightarrow \theta_{nnbue} = 2\cot^{-1}\left(\frac{\frac{\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1} + \left(\frac{2H_n}{a}\right)\sin\frac{\pi}{n}}{\frac{2H_n}{a}\sin\frac{\pi}{n}\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1}}}{\frac{\left(\frac{2H_n}{a}\right)\sin\frac{\pi}{n}\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1} - 1}{\frac{2H_n}{a}\sin\frac{\pi}{n}\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1}}}\right) = 2\cot^{-1}\left(\frac{\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1} + \left(\frac{2H_n}{a}\right)\sin\frac{\pi}{n}}{\left(\frac{2H_n}{a}\right)\sin\frac{\pi}{n}\sqrt{\left(\frac{2R_o}{a}\right)^2 - 1} - 1}\right)$$

The above expression of dihedral angle θ_{nnbue} is same as given by second expression of the above Eq(9).

The above generalized formula i.e. Eq.(9) is applicable to any convex uniform polyhedron to compute the dihedral angle between any **two regular n-gonal faces bisected by an unshared edge connecting their vertices** given the edge length a, numbers of sides n in regular n-gon, radius R_o of circumscribed sphere, or normal distance H_n in a convex uniform polyhedron . It is useful to find the dihedral angles between two congruent n-gonal faces bisected by unshared edge in Platonic and Archimedean solids.

2.4. Dihedral angle between any two arbitrary polygonal faces: It is worth noticing that when two arbitrary faces of a polyhedron are not regular polygons, or when two arbitrary faces share no edge or vertex, HCR's Inverse Cosine Formula can be applied to determine the dihedral angle [5,6]. In such cases, an auxiliary (imaginary) tetrahedron is constructed with its apex located at the vertex of the given irregular or non-uniform polyhedron, and with one of its three lateral faces formed by the unshared edges of the faces under consideration. Alternatively, if d_{12} is the distance between the feet of perpendiculars, having lengths H_1 and H_2, dropped to two arbitrary polygonal faces from the centre O of polyhedron, the dihedral angle θ_{12} between the polygonal faces is given as

$$\theta_{12} = \pi - \cos^{-1}\left(\frac{{H_1}^2 + {H_2}^2 - {d_{12}}^2}{2H_1H_2}\right) \qquad \ldots\ldots\ldots..(15)$$

Proof: Consider any two arbitrary polygonal faces such that their normal distances from the centre O of a polyhedron are H_1 and H_2. Let d_{12} be the distance between the feet of the perpendiculars M and N (as shown in Figure 4 below).

Applying Cosine rule in ΔMNO (Fig. 4),

$$\cos\angle MON = \frac{(OM)^2 + (ON)^2 - (MN)^2}{2(OM)(ON)} \Rightarrow \cos\alpha = \frac{(H_1)^2 + (H_2)^2 - (d_{12})^2}{2(H_1)(H_2)}$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{{H_1}^2 + {H_2}^2 - {d_{12}}^2}{2H_1H_2}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots.(16)$$

The two arbitrary faces, when extended, intersect each other at a line (⊥ plane of paper) i.e. represented by the point P in the plane of paper. In (cyclic) quadrilateral OMPN (Fig. 4),

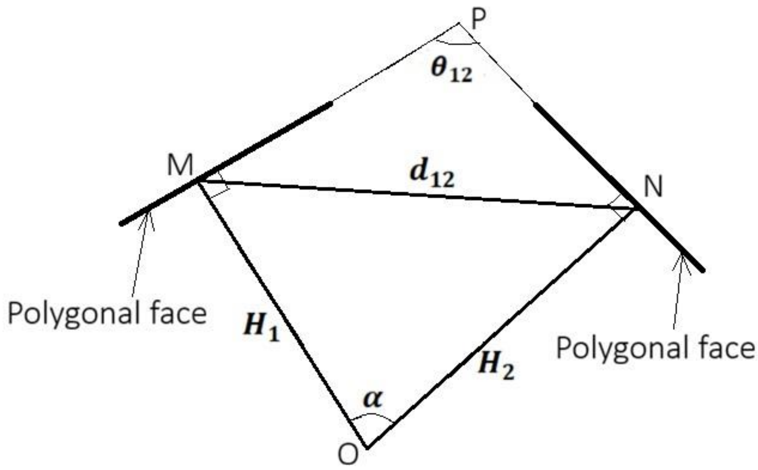

Figure 4: H_1 and H_2 are normal distances, from the centre O of polyhedron, of two arbitrary polygonal faces that are ⊥ plane of paper.

$$\angle OMP + \angle MPN + \angle PNO + \angle MON = 2\pi$$

$$\frac{\pi}{2} + \theta_{12} + \frac{\pi}{2} + \alpha = 2\pi$$

$$\theta_{12} = \pi - \alpha$$

Substituting the value of angle α from the above Eq.(16) into above equation, the dihedral angle, θ_{12} between the two arbitrary polygonal faces in a polyhedron, is given as follows

$$\theta_{12} = \pi - \cos^{-1}\left(\frac{{H_1}^2 + {H_2}^2 - {d_{12}}^2}{2H_1H_2}\right)$$

It's worth noticing that the above formula (i.e. Eq.(15)) can be used to find the dihedral angle between any two arbitrary polygonal faces, with or without common edge or vertex, in an irregular or non-uniform polyhedron given the normal distances of faces H_1 and H_2 from the center and the distance d_{12} between the feet of the perpendiculars dropped from the centre to these faces. The distance d_{12} between the feet of perpendiculars depend on the geometric shapes and dimensions, and the configurations of the faces with respect to each other.

Now, using the above generalized equations (1), (6), (9), and (15), the dihedral angles of regular polyhedra (Platonic solids) [7] and convex uniform polyhedra (Archimedean solids) [9-21] are computed below.

3. Dihedral angles of regular polyhedrons or Platonic solids

In this section, the key parameters like normal distances H_m & H_n of regular m-gonal and n-gonal faces in a regular polyhedron are adopted directly from the author's previously derived results [7].

3.1. Dihedral angles of a regular tetrahedron: A regular tetrahedron has 4 congruent equilateral triangular faces each with an edge length a and a normal distance H_T from the centre and radius R_o of circumscribed sphere, which are given from the formula [7],

$$H_T = \frac{a}{2\sqrt{6}}, \quad R_o = \frac{a}{2}\sqrt{\frac{3}{2}}$$

Now, the dihedral angle θ_{TTe} , between two **adjacent regular triangular faces** (with a common edge), is obtained by substituting the corresponding values; number of sides in triangular faces, $m = n = 3$ and normal distances, $H_m = H_n = H_T$ in the above generalized Eq.(1) as follows

$$\theta_{TTe} = \tan^{-1}\left\{\frac{\frac{a}{2\sqrt{6}}}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{a}{2\sqrt{6}}}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\} = 2\tan^{-1}\left\{\frac{\frac{a}{2\sqrt{6}}}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\} = 2\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) \approx 70.52877937^{o}$$

The above result shows that the dihedral angle between any two equilateral triangular faces with a common edge in a regular tetrahedron is $\approx \mathbf{70.52877937^{o}}$.

Alternatively, the dihedral angle θ_{TTe} , between two regular triangular faces bisected by an edge connecting their vertices, is obtained by substituting the corresponding values; number of sides in triangular faces, $n = 3$ and radius of circumscribed sphere, R_o in the above generalized Eq.(9) as follows

$$\theta_{TTbue} = \pi - 2\cos^{-1}\left(\frac{\sqrt{\left(\left(\frac{2R_o}{a}\right)^2 - 1\right)\left(\left(\frac{2R_o}{a}\right)^2 \sin^2\frac{\pi}{n} - 1\right)} - 1}{\left(\frac{2R_o}{a}\right)^2 \sin\frac{\pi}{n}}\right) = \pi - 2\cos^{-1}\left(\frac{\sqrt{\left(\left(\frac{\frac{2a}{2}\sqrt{\frac{3}{2}}}{a}\right)^2 - 1\right)\left(\left(\frac{\frac{2a}{2}\sqrt{\frac{3}{2}}}{a}\right)^2 \sin^2\frac{\pi}{3} - 1\right)} - 1}{\left(\frac{\frac{2a}{2}\sqrt{\frac{3}{2}}}{a}\right)^2 \sin\frac{\pi}{3}}\right)$$

$$\theta_{TTbue} = \theta_{TTe} = \pi - 2\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 2\sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 70.52877937^{o}$$

The above result shows that the pair or combination of any two triangular faces of a regular tetrahedron is of only one type and thus the dihedral angle between any two equilateral triangular faces of a regular tetrahedron is $\approx \mathbf{70.52877937^{o}}$.

3.2. Dihedral angles of a regular hexahedron/cube: A regular hexahedron or cube has 6 congruent square faces each with an edge length a and a normal distance H_S from the centre, and radius R_o of circumscribed sphere, which are given from the formula [7],

$$H_S = \frac{a}{2}, \quad R_o = \frac{a\sqrt{3}}{2}$$

Now, the dihedral angle θ_{SSe} , between two **adjacent square faces** (with a common edge), is obtained by substituting the corresponding values; number of sides in square faces, $m = n = 4$ and normal distances, $H_m = H_n = H_S$ in the above generalized Eq.(1) as follows

$$\theta_{SSe} = \tan^{-1}\left\{\frac{\frac{a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{4}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{4}\right)}\right\} = 2\tan^{-1}\left\{\frac{\frac{a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{4}\right)}\right\} = 2\tan^{-1}(1) = 90^{o}$$

The above result shows that the dihedral angle between any two square faces of a regular hexahedron/cube is $\mathbf{90^{o}}$ which is true in a cube.

The dihedral angle θ_{SSbue} between two square faces, bisected by an unshared edge connecting their vertices, is obtained by substituting the corresponding values; number of sides in square faces, $n = 4$ and radius of circumscribed sphere, R_o in the above generalized Eq.(9) as follows

$$\theta_{SSbue}=\pi-2\cos^{-1}\left(\frac{\sqrt{\left(\left(\frac{2R_o}{a}\right)^2-1\right)\left(\left(\frac{2R_o}{a}\right)^2\sin^2\frac{\pi}{n}-1\right)-1}}{\left(\frac{2R_o}{a}\right)^2\sin\frac{\pi}{n}}\right)=\pi-2\cos^{-1}\left(\frac{\sqrt{\left(\left(\frac{\frac{2a\sqrt{3}}{2}}{a}\right)^2-1\right)\left(\left(\frac{\frac{2a\sqrt{3}}{2}}{a}\right)^2\sin^2\frac{\pi}{4}-1\right)-1}}{\left(\frac{\frac{2a\sqrt{3}}{2}}{a}\right)^2\sin\frac{\pi}{4}}\right)$$

$$=\pi-2\cos^{-1}(0)=0^o$$

The above result shows that dihedral between two squares faces with no common edge and vertex in a regular hexahedron or cube is $\mathbf{0^o}$ which implied that such square faces are parallel to each other which is true in a cube.

3.3. Dihedral angles of a regular octahedron: A regular octahedron has 8 congruent equilateral triangular faces each with an edge length a and a normal distance H_T from the centre, and radius R_o of circumscribed sphere, which are given from the formula [7],

$$H_T=\frac{a}{\sqrt{6}},\quad R_o=\frac{a}{\sqrt{2}}$$

Now, the dihedral angle θ_{TTe} , between two adjacent regular triangular faces (with a common edge), is obtained by substituting the corresponding values; number of sides in triangular faces, $m=n=3$ and normal distances, $H_m=H_n=H_T$ in the above generalized Eq.(1) as follows

$$\theta_{TTe}=\tan^{-1}\left\{\frac{\frac{a}{\sqrt{6}}}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\}+\tan^{-1}\left\{\frac{\frac{a}{\sqrt{6}}}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\}=2\tan^{-1}\left\{\frac{\frac{a}{\sqrt{6}}}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\}=2\tan^{-1}(\sqrt{2})\approx 109.4712206^o$$

Now, consider two vertically opposite regular triangular faces AB and AC meeting at the vertex A but have no common edge. The AB and AC are the altitudes drawn from the common vertex A of equilateral triangle faces to their opposite sides (i.e. ⊥ plane of paper in Figure 5), which are given as

$$AB=AC=a\cos\frac{\pi}{6}=\frac{a\sqrt{3}}{2}$$

In right ΔAMC (Fig. 5),

$$\sin\angle CAM=\frac{MC}{AC}\quad\Rightarrow\sin\frac{\theta_{TTv}}{2}=\frac{a/2}{a\sqrt{3}/2}=\frac{1}{\sqrt{3}}$$

$$\theta_{TTvo}=2\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\approx 70.52877937^o$$

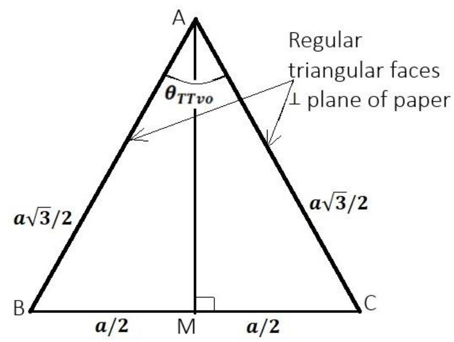

Figure 5: Two regular triangular faces AB and AC, with a common vertex A but no common edge, are ⊥ plane of paper.

Alternatively, the dihedral angle θ_{TTvo} between two vertically opposite triangular faces meeting at any vertex of the regular octahedron is obtained by substituting the corresponding values, $H_m=H_n=H_T$ and R_o in generalized Eq. (6) as follows

$$\theta_{TTvo}=\sin^{-1}\left(\frac{H_T}{R_o}\right)+\sin^{-1}\left(\frac{H_T}{R_o}\right)=2\sin^{-1}\left(\frac{H_T}{R_o}\right)=2\sin^{-1}\left(\frac{\frac{a}{\sqrt{6}}}{\frac{a}{\sqrt{2}}}\right)=2\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\approx 70.52877937^o$$

Alternatively, the dihedral angle $\theta_{TTvo}(=\delta_{13})$ between two vertically opposite i.e. first ($r=1$) and third ($r=3$) triangular faces meeting at any vertex of the regular octahedron is obtained by substituting $\alpha=\pi/3,\ n=4$ and $r=3$ in the generalized formula [8] as follows

$$\theta_{TTvo}=\delta_{13}=2\cos^{-1}\left(\sin\left(\frac{(3-1)\pi}{4}\right)\sqrt{1-\frac{\tan^2\frac{\pi/3}{2}}{\tan^2\frac{\pi}{4}}}\right)=2\cos^{-1}\left((1)\sqrt{1-\frac{\left(\frac{1}{\sqrt{3}}\right)^2}{(1)^2}}\right)=2\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

$$=2\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\approx 70.52877937^o$$

The above result shows that dihedral two vertically opposite regular triangular faces, meeting at a vertex and having no common edge, is $\approx \mathbf{70.52877937^o}$ which is true in a regular octahedron.

3.4. Dihedral angles of a regular dodecahedron: A regular dodecahedron has 12 congruent regular pentagonal faces each with an edge length a and a normal distance H_P from the centre, and radius R_o of circumscribed sphere, which are given from the formula [7],

$$H_P=\frac{(3+\sqrt{5})a}{8\sin 36^o},\quad R_o=\frac{\sqrt{3}(\sqrt{5}+1)a}{4}$$

Now, the dihedral angle θ_{PPe} , between two adjacent regular pentagonal faces, is obtained by substituting the corresponding values; number of sides in triangular faces, $m=n=5$ and normal distances, $H_m=H_n=H_P$ in the above generalized Eq.(1) as follows

$$\theta_{PPe}=\tan^{-1}\left\{\frac{\frac{(3+\sqrt{5})a}{8\sin 36^o}}{\left(\frac{a}{2}\cot\frac{\pi}{5}\right)}\right\}+\tan^{-1}\left\{\frac{\frac{(3+\sqrt{5})a}{8\sin 36^o}}{\left(\frac{a}{2}\cot\frac{\pi}{5}\right)}\right\}=2\tan^{-1}\left(\frac{1+\sqrt{5}}{2}\right)\approx 116.5650512^o$$

The dihedral angle θ_{PPbue} between two regular pentagonal faces 1 and 2, bisected by an unshared edge AB connecting their vertices A and B (as shown in Figure 6 below), is obtained by substituting the corresponding values; number of sides in pentagonal faces, $n=5$ and radius of circumscribed sphere, R_o in the above generalized Eq.(9) as follows

$$\theta_{PPbue}=2\sin^{-1}\left(\frac{\sqrt{\left(\left(\frac{2R_o}{a}\right)^2-1\right)\left(\left(\frac{2R_o}{a}\right)^2\sin^2\frac{\pi}{n}-1\right)}-1}{\left(\frac{2R_o}{a}\right)^2\sin\frac{\pi}{n}}\right)$$

$$=2\sin^{-1}\left(\frac{\sqrt{\left(\left(\frac{\frac{2\sqrt{3}(\sqrt{5}+1)a}{4}}{a}\right)^2-1\right)\left(\left(\frac{\frac{2\sqrt{3}(\sqrt{5}+1)a}{4}}{a}\right)^2\sin^2\frac{\pi}{5}-1\right)}-1}{\left(\frac{\frac{2\sqrt{3}(\sqrt{5}+1)a}{4}}{a}\right)^2\sin\frac{\pi}{5}}\right)$$

$$\theta_{PPbue}=2\sin^{-1}\left(\sqrt{\frac{5-\sqrt{5}}{10}}\right)=2\tan^{-1}\left(\frac{\sqrt{5}-1}{2}\right)\approx 63.434948823^o$$

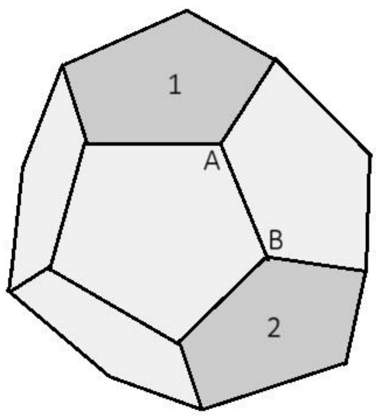

Figure 6: Two regular pentagonal faces 1 and 2 are bisected by an unshared edge AB connecting vertices A and B in a regular dodecahedron.

The above result shows that dihedral between two regular pentagonal faces bisected by an unshared edge connecting their vertices is $\approx \mathbf{63.434948823^o}$ in a regular dodecahedron.

3.5. Dihedral angles of a regular icosahedron: A regular icosahedron has 20 congruent equilateral triangular faces each with an edge length a and a normal distance H_T from the centre, and radius R_o of circumscribed sphere, which are given from the formula [7],

$$H_T = \frac{(3+\sqrt{5})a}{4\sqrt{3}}, \quad R_o = a\cos 18^o$$

Now, the dihedral angle θ_{TTe} , between two adjacent regular triangular faces (i.e. faces 1 and 2 in Figure 7), is obtained by substituting the corresponding values; number of sides in triangular faces, $m = n = 3$ and normal distances, $H_m = H_n = H_T$ in the above generalized Eq.(1) as follows

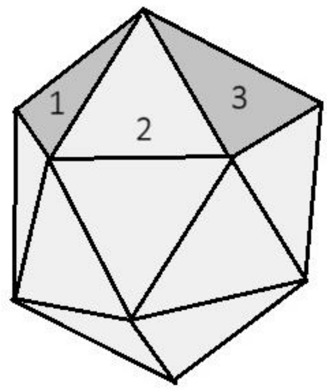

Figure 7: Two regular triangular faces 1 and 3 have a common vertex A but no common edge in a regular icosahedron.

$$\theta_{TTe} = \tan^{-1}\left\{\frac{\frac{(3+\sqrt{5})a}{4\sqrt{3}}}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{(3+\sqrt{5})a}{4\sqrt{3}}}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\}$$

$$= 2\tan^{-1}\left(\frac{3+\sqrt{5}}{2}\right) \approx 138.1896851^o$$

Alternatively, the dihedral angle $\theta_{TTe}(=\delta_{12})$ between first $(r=1)$ and second $(r=2)$ triangular faces sharing a common edge in the regular icosahedron (see Fig. 5) is obtained by substituting $\alpha = \pi/3,\ n = 5$ and $r = 2$ in the generalized formula [8] as follows

$$\delta_{12} = 2\cos^{-1}\left(\sin\left(\frac{(2-1)\pi}{5}\right)\sqrt{1-\frac{\tan^2\frac{\pi/3}{2}}{\tan^2\frac{\pi}{5}}}\right) = 2\cos^{-1}\left(\frac{\sqrt{10-2\sqrt{5}}}{4}\sqrt{1-\frac{\left(\frac{1}{\sqrt{3}}\right)^2}{\left(\sqrt{5-2\sqrt{5}}\right)^2}}\right) = 2\cos^{-1}\left(\frac{\sqrt{5}-1}{2\sqrt{3}}\right)$$

$$\therefore\ \theta_{TTe} = \delta_{12} = 2\cos^{-1}\left(\frac{\sqrt{5}-1}{2\sqrt{3}}\right) = 2\tan^{-1}\left(\frac{3+\sqrt{5}}{2}\right) \approx 138.1896851^o$$

Similarly, the dihedral angle $\theta_{TTv}(=\delta_{13})$ between first $(r=1)$ and third $(r=3)$ triangular faces meeting at a vertex but having no common edge in the regular icosahedron (as shown in Fig. 7 above) is obtained by substituting $\alpha = \pi/3,\ n = 5$ and $r = 3$ in the generalized formula [8] as follows

$$\delta_{13} = 2\cos^{-1}\left(\sin\left(\frac{(3-1)\pi}{5}\right)\sqrt{1-\frac{\tan^2\frac{\pi/3}{2}}{\tan^2\frac{\pi}{5}}}\right) = 2\cos^{-1}\left(\frac{\sqrt{10+2\sqrt{5}}}{4}\sqrt{1-\frac{\left(\frac{1}{\sqrt{3}}\right)^2}{\left(\sqrt{5-2\sqrt{5}}\right)^2}}\right) = 2\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\therefore \theta_{TTv} = \delta_{13} = 2\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 109.4712206^o$$

4. Dihedral angles of Archimedean solids

In this section, the key parameters like normal distances H_m & H_n of regular m-gonal and n-gonal faces, and circum-radius R_o of Archimedean solids are adopted directly from the author's previously derived results [9-22].

4.1. Dihedral angles of a truncated tetrahedron: A truncated tetrahedron has 4 congruent equilateral triangular and 4 congruent regular hexagonal faces, each having equal edge length a and their normal distances H_T and H_H respectively from the centre, and radius R_o of circumscribed sphere which are given from the formula [9],

$$H_T = \frac{5a}{2\sqrt{6}}, \quad H_H = \frac{a}{2}\sqrt{\frac{3}{2}}, \quad R_o = \frac{a}{2}\sqrt{\frac{11}{2}}$$

Now, the dihedral angle θ_{THe} , between two adjacent regular triangular and hexagonal faces (i.e. faces 1 and 2 with a common edge in Fig. 8(a)), is obtained by substituting the corresponding values; number of sides in triangular face, $m = 3$, number of sides in hexagonal face, $n = 6$ and normal distances, $H_m = H_T$ and $H_n = H_H$ respectively in the above generalized Eq.(1) as follows

$$\theta_{THe} = \tan^{-1}\left\{\frac{\frac{5a}{2\sqrt{6}}}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{a}{2}\sqrt{\frac{3}{2}}}{\left(\frac{a}{2}\cot\frac{\pi}{6}\right)}\right\} = \pi - \tan^{-1}\{2\sqrt{2}\} = 2\tan^{-1}(\sqrt{2}) \approx 109.4712206^o$$

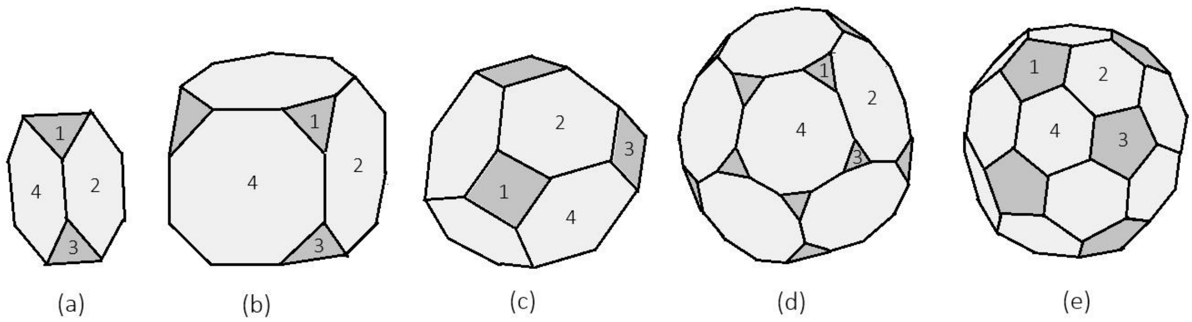

Figure 8: (a) Truncated tetrahedron, (b) truncated hexahedron/cube, (c) truncated octahedron (d) truncated dodecahedron, (e) truncated icosahedron.

Similarly, the dihedral angle θ_{HHe} , between two adjacent regular hexagonal faces (i.e. faces 2 and 4 with a common edge in Fig. 8(a)), is obtained by substituting the corresponding values; number of sides in hexagonal faces, $m = n = 6$ and normal distances, $H_m = H_n = H_H$ in the above generalized Eq.(1) as follows

$$\theta_{HHe} = \tan^{-1}\left\{\frac{\frac{a}{2}\sqrt{\frac{3}{2}}}{\left(\frac{a}{2}\cot\frac{\pi}{6}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{a}{2}\sqrt{\frac{3}{2}}}{\left(\frac{a}{2}\cot\frac{\pi}{6}\right)}\right\} = 2\tan^{-1}\left\{\frac{\frac{a}{2}\sqrt{\frac{3}{2}}}{\left(\frac{a}{2}\cot\frac{\pi}{6}\right)}\right\} = 2\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) \approx 70.52877937^o$$

The dihedral angle θ_{TTbue} between two regular triangular faces 1 and 3, bisected by an unshared edge connecting their vertices (as shown in Figure 8(a) above), is obtained by substituting the corresponding values; number of sides in triangular faces, $n = 3$ and radius of circumscribed sphere, R_o in above generalized Eq.(9) as follows

$$\theta_{TTbue} = 2\sin^{-1}\left(\frac{\sqrt{\left(\left(\frac{2R_o}{a}\right)^2 - 1\right)\left(\left(\frac{2R_o}{a}\right)^2\sin^2\frac{\pi}{n} - 1\right)} - 1}{\left(\frac{2R_o}{a}\right)^2\sin\frac{\pi}{n}}\right)$$

$$= 2\sin^{-1}\left(\frac{\sqrt{\left(\left(\frac{\frac{2a}{2}\sqrt{\frac{11}{2}}}{a}\right)^2-1\right)\left(\left(\frac{\frac{2a}{2}\sqrt{\frac{11}{2}}}{a}\right)^2\sin^2\frac{\pi}{3}-1\right)}-1}{\left(\frac{\frac{2a}{2}\sqrt{\frac{11}{2}}}{a}\right)^2\sin\frac{\pi}{3}}\right)$$

$$\theta_{TTbue} = 2\sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 70.52877937^o$$

4.2. Dihedral angles of a truncated hexahedron/cube: A truncated hexahedron/cube has 8 congruent equilateral triangular & 6 congruent regular octagonal faces, each having an equal edge length a and their normal distances H_T and H_O respectively from the centre, and radius R_o of circumscribed sphere which are given from the formula [10],

$$H_T = \frac{(3+2\sqrt{2})a}{2\sqrt{3}}, \quad H_O = \frac{(1+\sqrt{2})a}{2}, \quad R_o = \frac{a}{2}\sqrt{7+4\sqrt{2}}$$

Now, the dihedral angle θ_{TOe} , between two adjacent regular triangular and octagonal faces (i.e. faces 1 and 2 with a common edge in Fig. 8(b) above), is obtained by substituting the corresponding values; number of sides in triangular face, $m = 3$, number of sides in octagonal face, $n = 8$ and normal distances, $H_m = H_T$ and $H_n = H_O$ respectively in the above generalized Eq.(1) as follows

$$\theta_{TOe} = \tan^{-1}\left\{\frac{\frac{(3+2\sqrt{2})a}{2\sqrt{3}}}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{(1+\sqrt{2})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{8}\right)}\right\} = \pi - \tan^{-1}(\sqrt{2}) = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) \approx 125.2643897^o$$

Similarly, the dihedral angle θ_{OOe} , between two adjacent regular octagonal faces (i.e. faces 2 and 4 with a common edge in Fig. 8(b) above), is obtained by substituting the corresponding values; number of sides in hexagonal faces, $m = n = 8$ and normal distances, $H_m = H_n = H_O$ in the above generalized Eq.(1) as follows

$$\theta_{OOe} = \tan^{-1}\left\{\frac{\frac{(1+\sqrt{2})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{8}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{(1+\sqrt{2})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{8}\right)}\right\} = 2\tan^{-1}\left\{\frac{\frac{(1+\sqrt{2})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{8}\right)}\right\} = 2\tan^{-1}(1) = 90^o$$

The dihedral angle θ_{TTbue} between two regular triangular faces 1 and 3, bisected by an unshared edge connecting their vertices (as shown in Figure 8(b) above), is obtained by substituting the corresponding values; number of sides in triangular faces, $n = 3$ and radius of circumscribed sphere, R_o in the above generalized Eq.(9) as follows

$$\theta_{TTbue} = 2\sin^{-1}\left(\frac{\sqrt{\left(\left(\frac{2R_o}{a}\right)^2-1\right)\left(\left(\frac{2R_o}{a}\right)^2\sin^2\frac{\pi}{n}-1\right)}-1}{\left(\frac{2R_o}{a}\right)^2\sin\frac{\pi}{n}}\right)$$

$$= 2\sin^{-1}\left(\frac{\sqrt{\left(\left(\frac{\frac{2a}{2}\sqrt{7+4\sqrt{2}}}{a}\right)^2-1\right)\left(\left(\frac{\frac{2a}{2}\sqrt{7+4\sqrt{2}}}{a}\right)^2\sin^2\frac{\pi}{3}-1\right)-1}}{\left(\frac{\frac{2a}{2}\sqrt{7+4\sqrt{2}}}{a}\right)^2\sin\frac{\pi}{3}}\right)$$

$$\theta_{TTbue} = 2\sin^{-1}\left(\sqrt{\frac{2}{3}}\right) = 2\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 109.4712206^o$$

4.3. Dihedral angles of a truncated octahedron: A truncated octahedron has 6 congruent squares and 8 congruent regular hexagonal faces, each having equal edge length a and their normal distances H_S and H_H respectively from the centre, and radius R_o of circumscribed sphere which are given from the formula [11],

$$H_S = a\sqrt{2}\,, \quad H_H = a\sqrt{\frac{3}{2}}\,, \quad R_o = a\sqrt{\frac{5}{2}}$$

Now, the dihedral angle θ_{SHe} , between two adjacent square and regular hexagonal faces (i.e. faces 1 and 2 with a common edge in Fig. 8(c) above), is obtained by substituting the corresponding values; number of sides in square face, $m = 4$, number of sides in hexagonal face, $n = 6$ and normal distances, $H_m = H_S$ and $H_n = H_H$ respectively in the above generalized Eq.(1) as follows

$$\theta_{SHe} = \tan^{-1}\left\{\frac{a\sqrt{2}}{\left(\frac{a}{2}\cot\frac{\pi}{4}\right)}\right\} + \tan^{-1}\left\{\frac{a\sqrt{\frac{3}{2}}}{\left(\frac{a}{2}\cot\frac{\pi}{6}\right)}\right\} = \pi - \tan^{-1}\left(\sqrt{2}\right) = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) \approx 125.2643897^o$$

Similarly, the dihedral angle θ_{HHe} , between two adjacent regular hexagonal faces (i.e. faces 2 and 4 with a common edge in Fig. 8(c) above), is obtained by substituting the corresponding values; number of sides in hexagonal faces, $m = n = 6$ and normal distances, $H_m = H_n = H_H$ in the above generalized Eq.(1) as follows

$$\theta_{HHe} = \tan^{-1}\left\{\frac{a\sqrt{\frac{3}{2}}}{\left(\frac{a}{2}\cot\frac{\pi}{6}\right)}\right\} + \tan^{-1}\left\{\frac{a\sqrt{\frac{3}{2}}}{\left(\frac{a}{2}\cot\frac{\pi}{6}\right)}\right\} = 2\tan^{-1}\left\{\frac{a\sqrt{\frac{3}{2}}}{\left(\frac{a}{2}\cot\frac{\pi}{6}\right)}\right\} = 2\tan^{-1}\left(\sqrt{2}\right) \approx 109.4712206^o$$

The dihedral angle θ_{SSbue} between two square faces 1 and 3, bisected by an unshared edge connecting their vertices (as shown in Figure 8(c) above), is obtained by substituting the corresponding values; number of sides in square faces, $n = 4$ and radius of circumscribed sphere, R_o in the above generalized Eq.(9) as follows

$$\theta_{SSbue} = 2\sin^{-1}\left(\frac{\sqrt{\left(\left(\frac{2R_o}{a}\right)^2-1\right)\left(\left(\frac{2R_o}{a}\right)^2\sin^2\frac{\pi}{n}-1\right)-1}}{\left(\frac{2R_o}{a}\right)^2\sin\frac{\pi}{n}}\right)$$

$$= 2\sin^{-1}\left(\frac{\sqrt{\left(\left(\frac{2a\sqrt{\frac{5}{2}}}{a}\right)^2 - 1\right)\left(\left(\frac{2a\sqrt{\frac{5}{2}}}{a}\right)^2 \sin^2\frac{\pi}{4} - 1\right) - 1}}{\left(\frac{2a\sqrt{\frac{5}{2}}}{a}\right)^2 \sin\frac{\pi}{4}}\right)$$

$$\theta_{SSbue} = 2\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 90^o$$

4.4. Dihedral angles of a truncated dodecahedron: A truncated dodecahedron has 20 congruent equilateral triangular & 12 congruent regular decagonal faces, each having equal edge length a and their normal distances H_T and H_D respectively from the centre, and radius R_o of circumscribed sphere which are given from the formula [12],

$$H_T = \frac{(9+5\sqrt{5})a}{4\sqrt{3}}, \quad H_D = \frac{(5+3\sqrt{5})a}{8\sin 36^o}, \quad R_o = \frac{a}{4}\sqrt{74+30\sqrt{5}}$$

Now, the dihedral angle θ_{TDe} , between two adjacent regular triangular and decagonal faces (i.e. faces 1 and 2 with a common edge in Fig. 8(d) above), is obtained by substituting the corresponding values; number of sides in triangular face, $m = 3$, number of sides in decagonal face, $n = 10$ and normal distances, $H_m = H_T$ and $H_n = H_D$ respectively in the above generalized Eq.(1) as follows

$$\theta_{TDe} = \tan^{-1}\left\{\frac{\frac{(9+5\sqrt{5})a}{4\sqrt{3}}}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{(5+3\sqrt{5})a}{8\sin 36^o}}{\left(\frac{a}{2}\cot\frac{\pi}{10}\right)}\right\} = \pi - \tan^{-1}(3-\sqrt{5}) \approx 142.6226319^o$$

Similarly, the dihedral angle θ_{DDe} , between two adjacent regular decagonal faces (i.e. faces 2 and 4 with a common edge in Fig. 8(d) above), is obtained by substituting the corresponding values; number of sides in hexagonal faces, $m = n = 10$ and normal distances, $H_m = H_n = H_D$ in the above generalized Eq.(1) as follows

$$\theta_{DDe} = \tan^{-1}\left\{\frac{\frac{(5+3\sqrt{5})a}{8\sin 36^o}}{\left(\frac{a}{2}\cot\frac{\pi}{10}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{(5+3\sqrt{5})a}{8\sin 36^o}}{\left(\frac{a}{2}\cot\frac{\pi}{10}\right)}\right\} = 2\tan^{-1}\left(\frac{1+\sqrt{5}}{2}\right) \approx 116.5650512^o$$

The dihedral angle θ_{TTbue} between two regular triangular faces 1 and 3, bisected by an unshared edge connecting their vertices (as shown in Figure 8(d) above), is obtained by substituting the corresponding values; number of sides in triangular faces, $n = 3$ and radius of circumscribed sphere, R_o in the above generalized Eq.(9) as follows

$$\theta_{TTbue} = 2\sin^{-1}\left(\frac{\sqrt{\left(\left(\frac{2R_o}{a}\right)^2 - 1\right)\left(\left(\frac{2R_o}{a}\right)^2 \sin^2\frac{\pi}{n} - 1\right) - 1}}{\left(\frac{2R_o}{a}\right)^2 \sin\frac{\pi}{n}}\right)$$

$$= 2\sin^{-1}\left(\frac{\sqrt{\left(\left(\frac{\frac{2a}{4}\sqrt{74+30\sqrt{5}}}{a}\right)^2-1\right)\left(\left(\frac{\frac{2a}{4}\sqrt{74+30\sqrt{5}}}{a}\right)^2\sin^2\frac{\pi}{3}-1\right)}-1}{\left(\frac{\frac{2a}{4}\sqrt{74+30\sqrt{5}}}{a}\right)^2\sin\frac{\pi}{3}}\right)$$

$$\theta_{TTbue} = 2\sin^{-1}\left(\frac{\sqrt{5}+1}{2\sqrt{3}}\right) = 2\tan^{-1}\left(\frac{3+\sqrt{5}}{2}\right) \approx 138.1896851^o$$

4.5. Dihedral angles of a truncated icosahedron: A truncated icosahedron has 12 congruent regular pentagonal & 20 congruent regular hexagonal faces, each having equal edge length a and their normal distances H_P and H_H respectively from the centre, and radius R_o of circumscribed sphere which are given from the formula [13],

$$H_P = \frac{(2\sqrt{5}+1)a}{4\sin 36^o}, \quad H_H = \frac{\sqrt{3}(3+\sqrt{5})a}{4}, \quad R_o = \frac{a}{4}\sqrt{58+18\sqrt{5}}$$

Now, the dihedral angle θ_{PHe} , between two adjacent regular pentagonal and hexagonal faces (i.e. faces 1 and 2 with a common edge in Fig. 8(e) above), is obtained by substituting the corresponding values; number of sides in pentagonal face, $m = 5$, number of sides in hexagonal face, $n = 6$ and normal distances, $H_m = H_P$ and $H_n = H_H$ respectively in the above generalized Eq.(1) as follows

$$\theta_{PHe} = \tan^{-1}\left\{\frac{\frac{(2\sqrt{5}+1)a}{4\sin 36^o}}{\left(\frac{a}{2}\cot\frac{\pi}{5}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{\sqrt{3}(3+\sqrt{5})a}{4}}{\left(\frac{a}{2}\cot\frac{\pi}{6}\right)}\right\} = \pi - \tan^{-1}(3-\sqrt{5}) \approx 142.62263186^o$$

Similarly, the dihedral angle θ_{HHe} , between two adjacent regular hexagonal faces (i.e. faces 2 and 4 with a common edge in Fig. 8(e) above), is obtained by substituting the corresponding values; number of sides in hexagonal faces, $m = n = 6$ and normal distances, $H_m = H_n = H_H$ in the above generalized Eq.(1) as follows

$$\theta_{HHe} = \tan^{-1}\left\{\frac{\frac{\sqrt{3}(3+\sqrt{5})a}{4}}{\left(\frac{a}{2}\cot\frac{\pi}{6}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{\sqrt{3}(3+\sqrt{5})a}{4}}{\left(\frac{a}{2}\cot\frac{\pi}{6}\right)}\right\} = 2\tan^{-1}\left(\frac{3+\sqrt{5}}{2}\right) \approx 138.1896851^o$$

The dihedral angle θ_{PPbue} between two regular pentagonal faces 1 and 3, bisected by an unshared edge connecting their vertices (as shown in Figure 8(e) above), is obtained by substituting the corresponding values; number of sides in pentagonal faces, $n = 5$ and radius of circumscribed sphere, R_o in the above generalized Eq.(9) as follows

$$\theta_{PPbue} = 2\sin^{-1}\left(\frac{\sqrt{\left(\left(\frac{2R_o}{a}\right)^2-1\right)\left(\left(\frac{2R_o}{a}\right)^2\sin^2\frac{\pi}{n}-1\right)}-1}{\left(\frac{2R_o}{a}\right)^2\sin\frac{\pi}{n}}\right)$$

$$= 2\sin^{-1}\left(\frac{\sqrt{\left(\left(\frac{\frac{2a}{4}\sqrt{58+18\sqrt{5}}}{a}\right)^2-1\right)\left(\left(\frac{\frac{2a}{4}\sqrt{58+18\sqrt{5}}}{a}\right)^2\sin^2\frac{\pi}{5}-1\right)}-1}{\left(\frac{\frac{2a}{4}\sqrt{58+18\sqrt{5}}}{a}\right)^2\sin\frac{\pi}{5}}\right)$$

$$\theta_{PPbue} = 2\sin^{-1}\left(\sqrt{\frac{5+\sqrt{5}}{10}}\right) = 2\tan^{-1}\left(\frac{1+\sqrt{5}}{2}\right) \approx 116.5650512^o$$

4.6. Dihedral angles of a cuboctahedron: A cuboctahedron has 8 congruent equilateral triangular & 6 congruent square faces, each having equal edge length a and their normal distances H_T and H_S respectively from the centre and radius R_o of circumscribed sphere which are given from the formula [14],

$$H_T = a\sqrt{\frac{2}{3}}, \quad H_S = \frac{a}{\sqrt{2}}, \quad R_o = a$$

Now, the dihedral angle θ_{TSe} , between two adjacent regular triangular and square faces, is obtained by substituting the corresponding values; number of sides in triangular face, $m = 3$, number of sides in square face, $n = 4$ and normal distances, $H_m = H_T$ and $H_n = H_S$ respectively in the above generalized Eq.(1) as follows

$$\theta_{TSe} = \tan^{-1}\left\{\frac{a\sqrt{\frac{2}{3}}}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{a}{\sqrt{2}}}{\left(\frac{a}{2}\cot\frac{\pi}{4}\right)}\right\} = \pi - \tan^{-1}(\sqrt{2}) = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) \approx 125.2643897^o$$

The dihedral angle θ_{TTvo} , between two vertically opposite regular triangular faces with a common vertex but no common edge, is obtained by substituting the corresponding values; normal distances, $H_m = H_n = H_T$ and circum-radius R_o in the above generalized Eq.(6) as follows

$$\theta_{TTvo} = \sin^{-1}\left(\frac{a\sqrt{\frac{2}{3}}}{a}\right) + \sin^{-1}\left(\frac{a\sqrt{\frac{2}{3}}}{a}\right) = 2\sin^{-1}\left(\sqrt{\frac{2}{3}}\right) = 2\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 109.4712206^o$$

Similarly, the dihedral angle θ_{SSvo} , between two vertically opposite square faces with a common vertex but no common edge, is obtained by substituting the corresponding values; normal distances, $H_m = H_n = H_S$ and circum-radius R_o in the above generalized Eq.(6) as follows

$$\theta_{SSvo} = \sin^{-1}\left(\frac{\frac{a}{\sqrt{2}}}{a}\right) + \sin^{-1}\left(\frac{\frac{a}{\sqrt{2}}}{a}\right) = 2\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 90^o$$

4.7. Dihedral angles of an icosidodecahedron: An icosidodecahedron has 20 equilateral triangular faces & 12 regular pentagonal faces, each having equal edge length a and their normal distances H_T and H_P respectively from the centre and radius R_o of circumscribed sphere which are given from the formula [15],

$$H_T = \frac{(3+\sqrt{5})a}{2\sqrt{3}}, \quad H_P = a\sqrt{\frac{5+2\sqrt{5}}{5}}, \quad R_o = \frac{(1+\sqrt{5})a}{2}$$

Now, the dihedral angle θ_{TPe} , between two adjacent regular triangular and pentagonal faces with a common edge, is obtained by substituting the corresponding values; number of sides in triangular face, $m = 3$, number of sides in pentagonal face, $n = 5$ and normal distances, $H_m = H_T$ and $H_n = H_P$ respectively in the above generalized Eq.(1) as follows

$$\theta_{TPe} = \tan^{-1}\left\{\frac{\frac{(3+\sqrt{5})a}{2\sqrt{3}}}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\} + \tan^{-1}\left\{\frac{a\sqrt{\frac{5+2\sqrt{5}}{5}}}{\left(\frac{a}{2}\cot\frac{\pi}{5}\right)}\right\} = \pi - \tan^{-1}(3-\sqrt{5}) \approx 142.6226319^{o}$$

The dihedral angle θ_{TTvo} , between two vertically opposite regular triangular faces with a common vertex but no common edge, is obtained by substituting the corresponding values; normal distances, $H_m = H_n = H_T$ and circum-radius R_o in the above generalized Eq.(6) as follows

$$\theta_{TTvo} = \sin^{-1}\left(\frac{\frac{(3+\sqrt{5})a}{2\sqrt{3}}}{\frac{(1+\sqrt{5})a}{2}}\right) + \sin^{-1}\left(\frac{\frac{(3+\sqrt{5})a}{2\sqrt{3}}}{\frac{(1+\sqrt{5})a}{2}}\right) = 2\sin^{-1}\left(\frac{1+\sqrt{5}}{2\sqrt{3}}\right) = 2\tan^{-1}\left(\frac{3+\sqrt{5}}{2}\right) \approx 138.1896851^{o}$$

Similarly, the dihedral angle θ_{PPvo} , between two vertically opposite regular pentagonal faces with a common vertex but no common edge, is obtained by substituting the corresponding values; normal distances, $H_m = H_n = H_P$ and circum-radius R_o in the above generalized Eq.(6) as follows

$$\theta_{PPvo} = \sin^{-1}\left(\frac{a\sqrt{\frac{5+2\sqrt{5}}{5}}}{\frac{(1+\sqrt{5})a}{2}}\right) + \sin^{-1}\left(\frac{a\sqrt{\frac{5+2\sqrt{5}}{5}}}{\frac{(1+\sqrt{5})a}{2}}\right) = 2\sin^{-1}\left(\sqrt{\frac{5+\sqrt{5}}{10}}\right) = 2\tan^{-1}\left(\frac{1+\sqrt{5}}{2}\right) \approx 116.5650512^{o}$$

4.8. Dihedral angles of a small rhombicuboctahedron: A small rhombicuboctahedron has 8 equilateral triangular faces & 18 square faces, each having equal edge length a and their normal distances H_T and H_S respectively from the centre and radius R_o of circumscribed sphere which are given from the formula [16],

$$H_T = \frac{(3+\sqrt{2})a}{2\sqrt{3}}, \quad H_S = \frac{(1+\sqrt{2})a}{2}, \quad R_o = \frac{a}{2}\sqrt{5+2\sqrt{2}}$$

Now, the dihedral angle θ_{TSe} , between two adjacent regular triangular and square faces with a common edge, is obtained by substituting the corresponding values; number of sides in triangular face, $m = 3$, number of sides in square face, $n = 4$ and normal distances, $H_m = H_T$ and $H_n = H_S$ respectively in the above generalized Eq.(1) as follows

$$\theta_{TSe} = \tan^{-1}\left\{\frac{\frac{(3+\sqrt{2})a}{2\sqrt{3}}}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{(1+\sqrt{2})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{4}\right)}\right\} = \pi - \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) = \cos^{-1}\left(-\sqrt{\frac{2}{3}}\right) \approx 144.7356103^{o}$$

Now, the dihedral angle θ_{SSe} , between two adjacent square faces with a common edge, is obtained by substituting the corresponding values; number of sides in square faces, $m = n = 4$, and normal distances $H_m = H_n = H_S$ respectively in the above generalized Eq.(1) as follows

$$\theta_{SSe} = \tan^{-1}\left\{\frac{\frac{(1+\sqrt{2})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{4}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{(1+\sqrt{2})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{4}\right)}\right\} = 2\tan^{-1}(1+\sqrt{2}) = 135^o$$

The dihedral angle θ_{TSvo} , between two vertically opposite regular triangular and square faces with a common vertex but no common edge, is obtained by substituting the corresponding values; normal distances, $H_m = H_T$ and $H_n = H_S$ and circum-radius R_o in the above generalized Eq.(6) as follows

$$\theta_{TSvo} = \sin^{-1}\left(\frac{\frac{(3+\sqrt{2})a}{2\sqrt{3}}}{\frac{a}{2}\sqrt{5+2\sqrt{2}}}\right) + \sin^{-1}\left(\frac{\frac{(1+\sqrt{2})a}{2}}{\frac{a}{2}\sqrt{5+2\sqrt{2}}}\right) = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) \approx 125.2643897^o$$

4.9. Dihedral angles of a great rhombicuboctahedron: A great rhombicuboctahedron has 12 square faces, 8 regular hexagonal faces & 6 regular octagonal faces, each having equal edge length a and their normal distances H_S, H_H and H_O respectively from the centre and radius R_o of circumscribed sphere which are given from the formula [17],

$$H_S = \frac{(3+\sqrt{2})a}{2}, \quad H_H = \frac{\sqrt{3}(1+\sqrt{2})a}{2}, \quad H_O = \frac{(1+2\sqrt{2})a}{2}, \quad R_o = \frac{a}{2}\sqrt{13+6\sqrt{2}}$$

Now, the dihedral angle θ_{SHe} , between two adjacent square and regular hexagonal faces with a common edge, is obtained by substituting the corresponding values; number of sides in square face, $m = 4$, number of sides in hexagonal face, $n = 6$ and normal distances, $H_m = H_S$ and $H_n = H_H$ respectively in the above generalized Eq.(1) as follows

$$\theta_{SHe} = \tan^{-1}\left\{\frac{\frac{(3+\sqrt{2})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{4}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{\sqrt{3}(1+\sqrt{2})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{6}\right)}\right\} = \pi - \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) \approx 144.7356103^o$$

Similarly, the dihedral angle θ_{SOe} , between two adjacent square and regular octagonal faces with a common edge, is obtained by substituting the corresponding values; number of sides in square face, $m = 4$, number of sides in octagonal face, $n = 8$ and normal distances, $H_m = H_S$ and $H_n = H_O$ respectively in the above generalized Eq.(1) as follows

$$\theta_{SOe} = \tan^{-1}\left\{\frac{\frac{(3+\sqrt{2})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{4}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{(1+2\sqrt{2})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{8}\right)}\right\} = 135^o$$

Similarly, the dihedral angle θ_{HOe} , between two adjacent regular hexagonal and octagonal faces with a common edge, is obtained by substituting the corresponding values; number of sides in hexagonal face, $m = 6$, number of sides in octagonal face, $n = 8$ and normal distances, $H_m = H_H$ and $H_n = H_O$ respectively in the above generalized Eq.(1) as follows

$$\theta_{HOe} = \tan^{-1}\left\{\frac{\frac{\sqrt{3}(1+\sqrt{2})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{6}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{(1+2\sqrt{2})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{8}\right)}\right\} = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) \approx 125.2643897^o$$

Now, the dihedral angle between two closest regular hexagonal faces, that are opposite and equally inclined with a square face is given as

$$\theta_{HHc} = 2\theta_{SHe} - \pi = 2\tan^{-1}(\sqrt{2}) \approx 109.4712206^o$$

Similarly, the dihedral angle between two closest regular octagonal faces, that are opposite and equally inclined with a square face is given as

$$\theta_{OOc} = 2\theta_{SOe} - \pi = 90^{o}$$

4.10. Dihedral angles of a small rhombicosidodecahedron: A small rhombicosidodecahedron has 20 equilateral triangular faces, 30 square faces & 12 regular pentagonal faces, each having equal edge length a and their normal distances H_T, H_S and H_P respectively from the centre and radius R_o of circumscribed sphere which are given from the formula [18],

$$H_T = \frac{(3+2\sqrt{5})a}{2\sqrt{3}}, \quad H_S = \frac{(2+\sqrt{5})a}{2}, \quad H_P = \frac{3a}{2}\sqrt{\frac{5+2\sqrt{5}}{5}}, \quad R_o = \frac{a}{2}\sqrt{11+4\sqrt{5}}$$

Now, the dihedral angle θ_{TSe} , between two adjacent regular triangular and square faces with a common edge, is obtained by substituting the corresponding values; number of sides in triangular face, $m = 3$, number of sides in square face, $n = 4$ and normal distances, $H_m = H_T$ and $H_n = H_S$ respectively in the above generalized Eq.(1) as follows

$$\theta_{TSe} = \tan^{-1}\left\{\frac{\frac{(3+2\sqrt{5})a}{2\sqrt{3}}}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{(2+\sqrt{5})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{4}\right)}\right\} = \pi - \tan^{-1}\left(\frac{3-\sqrt{5}}{2}\right) \approx 159.0948426^{o}$$

Now, the dihedral angle θ_{SPe} , between two adjacent square and regular pentagonal faces with a common edge, is obtained by substituting the corresponding values; number of sides in square face, $m = 4$, number of sides in pentagonal face, $n = 5$ and normal distances, $H_m = H_S$ and $H_n = H_P$ respectively in the above generalized Eq.(1) as follows

$$\theta_{SPe} = \tan^{-1}\left\{\frac{\frac{(2+\sqrt{5})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{4}\right)}\right\} + \tan^{-1}\left\{\frac{\frac{3a}{2}\sqrt{\frac{5+2\sqrt{5}}{5}}}{\left(\frac{a}{2}\cot\frac{\pi}{5}\right)}\right\} = \pi - \tan^{-1}\left(\frac{\sqrt{5}-1}{2}\right) \approx 148.2825256^{o}$$

The dihedral angle θ_{TPvo}, between two vertically opposite regular triangular and pentagonal faces with a common vertex but no common edge, is obtained by substituting the corresponding values; normal distances, $H_m = H_T$ and $H_n = H_P$ and circum-radius R_o in the above generalized Eq.(6) as follows

$$\theta_{TPvo} = \sin^{-1}\left(\frac{\frac{(3+2\sqrt{5})a}{2\sqrt{3}}}{\frac{a}{2}\sqrt{11+4\sqrt{5}}}\right) + \sin^{-1}\left(\frac{\frac{3a}{2}\sqrt{\frac{5+2\sqrt{5}}{5}}}{\frac{a}{2}\sqrt{11+4\sqrt{5}}}\right) = \pi - \tan^{-1}(3-\sqrt{5}) \approx 142.6226319^{o}$$

4.11. Dihedral angles of a great rhombicosidodecahedron: A great rhombicosidodecahedron has 30 square faces, 20 regular hexagonal faces & 12 regular decagonal faces, each having equal edge length a and their normal distances H_S, H_H and H_D respectively from the centre and radius R_o of circumscribed sphere which are given from the formula [19],

$$H_S = \frac{(3+2\sqrt{5})a}{2}, \quad H_H = \frac{\sqrt{3}(2+\sqrt{5})a}{2}, \quad H_D = \frac{a}{2}\sqrt{25+10\sqrt{5}}, \quad R_o = \frac{a}{2}\sqrt{31+12\sqrt{5}}$$

Now, the dihedral angle θ_{SHe} , between two adjacent square and regular hexagonal faces with a common edge, is obtained by substituting the corresponding values; number of sides in square face, $m = 4$, number of sides in hexagonal face, $n = 6$ and normal distances, $H_m = H_S$ and $H_n = H_H$ respectively in the above generalized Eq.(1) as follows

$$\theta_{SHe}=\tan^{-1}\left\{\frac{\frac{(3+2\sqrt{5})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{4}\right)}\right\}+\tan^{-1}\left\{\frac{\frac{\sqrt{3}(2+\sqrt{5})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{6}\right)}\right\}=\pi-\tan^{-1}\left(\frac{3-\sqrt{5}}{2}\right)\approx 159.0948426^{o}$$

Similarly, the dihedral angle θ_{SDe} , between two adjacent square and regular decagonal faces with a common edge, is obtained by substituting the corresponding values; number of sides in square face, $m=4$, number of sides in decagonal face, $n=10$ and normal distances, $H_m=H_S$ and $H_n=H_D$ respectively in the above generalized Eq.(1) as follows

$$\theta_{SDe}=\tan^{-1}\left\{\frac{\frac{(3+2\sqrt{5})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{4}\right)}\right\}+\tan^{-1}\left\{\frac{\frac{a}{2}\sqrt{25+10\sqrt{5}}}{\left(\frac{a}{2}\cot\frac{\pi}{10}\right)}\right\}=\pi-\tan^{-1}\left(\frac{\sqrt{5}-1}{2}\right)\approx 148.2825256^{o}$$

Similarly, the dihedral angle θ_{HDe} , between two adjacent regular hexagonal and decagonal faces with a common edge, is obtained by substituting the corresponding values; number of sides in hexagonal face, $m=6$, number of sides in decagonal face, $n=10$ and normal distances, $H_m=H_H$ and $H_n=H_D$ respectively in the above generalized Eq.(1) as follows

$$\theta_{HDe}=\tan^{-1}\left\{\frac{\frac{\sqrt{3}(2+\sqrt{5})a}{2}}{\left(\frac{a}{2}\cot\frac{\pi}{6}\right)}\right\}+\tan^{-1}\left\{\frac{\frac{a}{2}\sqrt{25+10\sqrt{5}}}{\left(\frac{a}{2}\cot\frac{\pi}{10}\right)}\right\}=\pi-\tan^{-1}(3-\sqrt{5})\approx 142.6226319^{o}$$

The dihedral angle θ_{HHc} between two closest regular hexagonal faces, that are opposite and equally inclined with a square face is given as

$$\theta_{HHc}=2\theta_{SHe}-\pi=2\cot^{-1}\left(\frac{3-\sqrt{5}}{2}\right)\approx 138.1896851^{o}$$

Similarly, the dihedral angle between two closest regular decagonal faces, that are opposite and equally inclined with a square face is given as

$$\theta_{DDc}=2\theta_{SDe}-\pi=2\cot^{-1}\left(\frac{\sqrt{5}-1}{2}\right)\approx 116.5650512^{o}$$

4.12. Dihedral angles of a snub cube: A snub cube has 32 equilateral triangular and 6 square faces, each having equal edge length a and their normal distances H_T, and H_S respectively from the centre which are given from the formula [20],

$$H_T=a\sqrt{\frac{3C^2-1}{3}}\approx 1.2133558a\ ,\quad H_S=a\sqrt{\frac{2C^2-1}{2}}\approx 1.142613509a$$

Now, the dihedral angle θ_{TTe} , between two adjacent regular triangular faces with a common edge, is obtained by substituting the corresponding values; number of sides in triangular faces, $m=n=3$, and normal distances, $H_m=H_n=H_T$ in the above generalized Eq.(1) as follows

$$\theta_{TTe}=\tan^{-1}\left\{\frac{1.2133558a}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\}+\tan^{-1}\left\{\frac{1.2133558a}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\}\approx 153.2345877^{o}$$

Similarly, the dihedral angle θ_{TSe} , between two adjacent regular triangular and square faces with a common edge, is obtained by substituting the corresponding values; number of sides in triangular face, $m=3$, number of sides in square face, $n = 4$ and normal distances, $H_m = H_T$ and $H_n = H_S$ respectively in the above generalized Eq.(1) as follows

$$\theta_{TSe} = \tan^{-1}\left\{\frac{1.2133558a}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\} + \tan^{-1}\left\{\frac{1.142613509a}{\left(\frac{a}{2}\cot\frac{\pi}{4}\right)}\right\} \approx 142.9834301^o$$

4.13. Dihedral angles of a snub dodecahedron: A snub dodecahedron has 80 equilateral triangular and 12 regular pentagonal faces, each having equal edge length a and their normal distances H_T, and H_P respectively from the centre which are given from the formula [21,22],

$$H_T = a\sqrt{\frac{3C^2 - 1}{3}} \approx 2.07708966a\,, \quad H_P = a\sqrt{\frac{10C^2 - (5 + \sqrt{5})}{10}} \approx 1.980915947a$$

Now, the dihedral angle θ_{TTe} , between two adjacent regular triangular faces with a common edge, is obtained by substituting the corresponding values; number of sides in triangular faces, $m = n = 3$, and normal distances, $H_m = H_n = H_T$ in the above generalized Eq.(1) as follows

$$\theta_{TTe} = \tan^{-1}\left\{\frac{2.07708966a}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\} + \tan^{-1}\left\{\frac{2.07708966a}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\} \approx 164.1753661^o$$

Similarly, the dihedral angle θ_{TPe} , between two adjacent regular triangular and pentagonal faces with a common edge, is obtained by substituting the corresponding values; number of sides in triangular face, $m = 3$, number of sides in pentagonal face, $n = 5$ and normal distances, $H_m = H_T$ and $H_n = H_P$ respectively in the above generalized Eq.(1) as follows

$$\theta_{TPe} = \tan^{-1}\left\{\frac{2.07708966a}{\left(\frac{a}{2}\cot\frac{\pi}{3}\right)}\right\} + \tan^{-1}\left\{\frac{1.980915947a}{\left(\frac{a}{2}\cot\frac{\pi}{5}\right)}\right\} \approx 152.9299203^o$$

Conclusions: Using generalized Eq.(1), the dihedral angles in regular polyhedra (Platonic solids) and convex uniform polyhedra (Archimedean solids), whose faces are congruent regular polygons with an equal edge length, have been systematically evaluated, compiled, and tabulated below.

Table 1. Dihedral angles between the congruent faces of all five platonic solids

Platonic solid	**Pair of the congruent faces having a common edge or vertex**	**Dihedral angle between the corresponding pair of the regular polygonal faces**
Regular tetrahedron	Equilateral triangular faces with a common edge	$\theta_{TTe} = 2\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) \approx 70.52877937^o$
Regular hexahedron (cube)	Square faces with a common edge	$\theta_{SSe} = \pi/2 = 90^o$
	Equilateral triangular faces with a common edge	$\theta_{TTe} = 2\tan^{-1}(\sqrt{2}) \approx 109.4712206^o$

Regular octahedron	Equilateral triangular faces with a common vertex but no common edge	$\theta_{TTvo} = 2\sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 70.52877937^o$
Regular dodecahedron	Regular pentagonal faces with a common edge	$\theta_{PPe} = 2\tan^{-1}\left(\frac{1+\sqrt{5}}{2}\right) \approx 116.5650512^o$
	Regular pentagonal faces bisected by an unshared edge connecting their vertices	$\theta_{PPbue} = 2\tan^{-1}\left(\frac{\sqrt{5}-1}{2}\right) \approx 63.434948823^o$
Regular icosahedron	Equilateral triangular faces with a common edge	$\theta_{TTe} = 2\tan^{-1}\left(\frac{3+\sqrt{5}}{2}\right) \approx 138.1896851^o$
	Equilateral triangular faces with a common vertex but no common edge	$\theta_{TTv} = 2\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 109.4712206^o$

Table 2. Dihedral angles between the adjacent faces of Archimedean solid (uniform polyhedra)

Archimedean solid	**Dihedral angle between a pair of regular polygonal faces**
Truncated Tetrahedron having 4 equilateral triangular faces & 4 regular hexagonal faces	Dihedral angle between equilateral triangular & regular hexagonal faces with a common edge $$\theta_{THe} = 2\tan^{-1}(\sqrt{2}) \approx 109.4712206^o$$
	Dihedral angle between two regular hexagonal faces with a common edge $$\theta_{HHe} = 2\tan^{-1}\left(\frac{1}{\sqrt{2}}\right) \approx 70.52877937^o$$
	Dihedral angle between two regular triangular faces bisected by an unshared edge connecting their vertices $$\theta_{TTbue} = 2\sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 70.52877937^o$$
Truncated hexahedron/cube having 8 equilateral triangular faces & 6 regular octagonal faces	Dihedral angle between equilateral triangular & regular octagonal faces with a common edge $$\theta_{TOe} = \pi - \tan^{-1}(\sqrt{2}) = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) \approx 125.2643897^o$$
	Dihedral angle between two regular octagonal faces with a common edge $$\theta_{OOe} = 90^o$$
	Dihedral angle between two regular triangular faces bisected by an unshared edge connecting their vertices

	$$\theta_{TTbue} = 2\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 109.4712206^o$$
Truncated octahedron having 6 square faces & 8 regular hexagonal faces	Dihedral angle between square & regular hexagonal faces with a common edge $$\theta_{SHe} = \pi - \tan^{-1}(\sqrt{2}) = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) \approx 125.2643897^o$$
	Dihedral angle between two regular hexagonal faces with a common edge $$\theta_{HHe} = 2\tan^{-1}(\sqrt{2}) \approx 109.4712206^o$$
	Dihedral angle between square faces bisected by an unshared edge connecting their vertices $$\theta_{SSbue} = 90^o$$
Truncated dodecahedron having 20 equilateral triangular faces & 12 regular decagonal faces	Dihedral angle between equilateral triangular & regular decagonal faces with a common edge $$\theta_{TDe} = \pi - \tan^{-1}(3-\sqrt{5}) \approx 142.6226319^o$$
	Dihedral angle between two regular decagonal faces with a common edge $$\theta_{DDe} = 2\tan^{-1}\left(\frac{1+\sqrt{5}}{2}\right) \approx 116.5650512^o$$
	Dihedral angle between two regular triangular faces bisected by an unshared edge connecting their vertices $$\theta_{TTbue} = 2\tan^{-1}\left(\frac{3+\sqrt{5}}{2}\right) \approx 138.1896851^o$$
Truncated icosahedron having 20 regular hexagonal faces & 12 regular pentagonal faces	Dihedral angle between regular pentagonal and hexagonal faces with a common edge $$\theta_{PHe} = \pi - \tan^{-1}(3-\sqrt{5}) \approx 142.62263186^o$$
	Dihedral angle between two regular hexagonal faces with a common edge $$\theta_{HHe} = 2\tan^{-1}\left(\frac{3+\sqrt{5}}{2}\right) \approx 138.1896851^o$$
	Dihedral angle between two regular pentagonal faces bisected by an unshared edge connecting their vertices $$\theta_{PPbue} = 2\tan^{-1}\left(\frac{1+\sqrt{5}}{2}\right) \approx 116.5650512^o$$
Cuboctahedron having 8 equilateral triangular faces & 6 square faces	Dihedral angle between equilateral triangular & square faces with a common edge $$\theta_{TSe} = \pi - \tan^{-1}(\sqrt{2}) = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) \approx 125.2643897^o$$
	Dihedral angle between two vertically opposite equilateral triangular faces $$\theta_{TTvo} = 2\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 109.4712206^o$$

	Dihedral angle between two vertically opposite square faces $\theta_{SSvo} = 2\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 90^o$
Icosidodecahedron having 20 equilateral triangular faces & 12 regular pentagonal faces	Dihedral angle between regular triangular & pentagonal faces with a common edge $\theta_{TPe} = \pi - \tan^{-1}(3-\sqrt{5}) \approx 142.6226319^o$
	Dihedral angle between two vertically opposite equilateral triangular faces $\theta_{TTvo} = 2\tan^{-1}\left(\frac{3+\sqrt{5}}{2}\right) \approx 138.1896851^o$
	Dihedral angle between two vertically opposite regular pentagonal faces $\theta_{PPvo} = 2\tan^{-1}\left(\frac{1+\sqrt{5}}{2}\right) \approx 116.5650512^o$
Small rhombicuboctahedron having 8 equilateral triangular faces & 18 square faces	Dihedral angle between equilateral triangular & square faces with a common edge $\theta_{TSe} = \pi - \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) = \cos^{-1}\left(-\sqrt{\frac{2}{3}}\right) \approx 144.7356103^o$
	Dihedral angle between two square faces with a common edge $\theta_{SSe} = 2\tan^{-1}(1+\sqrt{2}) = 135^o$
	Dihedral angle between two vertically opposite regular triangular & square faces $\theta_{TSvo} = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) \approx 125.2643897^o$
Great rhombicuboctahedron having 12 square faces, 8 regular hexagonal faces & 6 regular octagonal faces	Dihedral angle between square & regular hexagonal faces with a common edge $\theta_{SHe} = \pi - \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) \approx 144.7356103^o$
	Dihedral angle between square & regular octagonal faces with a common edge $\theta_{SOe} = 135^o$
	Dihedral angle between regular hexagonal & octagonal faces with a common edge $\theta_{HOe} = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) \approx 125.2643897^o$
	Dihedral angle between two closest regular hexagonal faces, that are opposite and equally inclined with a square face $\theta_{HHc} = 2\theta_{SHe} - \pi = 2\tan^{-1}(\sqrt{2}) \approx 109.4712206^o$
	Dihedral angle between two closest regular octagonal faces, that are opposite and equally inclined with a square face $\theta_{OOc} = 2\theta_{SOe} - \pi = 90^o$

Small rhombicosidodecahedron having 20 equilateral triangular faces, 30 square faces & 12 regular pentagonal faces	Dihedral angle between equilateral triangular & square faces with a common edge $$\theta_{TSe} = \pi - \tan^{-1}\left(\frac{3-\sqrt{5}}{2}\right) = \cos^{-1}\left(\frac{-(1+\sqrt{5})}{2\sqrt{3}}\right) \approx 159.0948426^o$$
	Dihedral angle between square & regular pentagonal faces with a common edge $$\theta_{SPe} = \pi - \tan^{-1}\left(\frac{\sqrt{5}-1}{2}\right) \approx 148.2825256^o$$
	Dihedral angle between two vertically opposite regular triangular & pentagonal faces $$\theta_{TPvo} = \pi - \tan^{-1}(3-\sqrt{5}) \approx 142.6226319^o$$
Great rhombicosidodecahedron having 30 square faces, 20 regular hexagonal faces & 12 regular decagonal faces	Dihedral angle between square and regular hexagonal faces with a common edge $$\theta_{SHe} = \pi - \tan^{-1}\left(\frac{3-\sqrt{5}}{2}\right) \approx 159.0948426^o$$
	Dihedral angle between square and regular decagonal faces with a common edge $$\theta_{SDe} = \pi - \tan^{-1}\left(\frac{\sqrt{5}-1}{2}\right) \approx 148.2825256^o$$
	Dihedral angle between regular hexagonal & decagonal faces with a common edge $$\theta_{HDe} = \pi - \tan^{-1}(3-\sqrt{5}) \approx 142.6226319^o$$
	Dihedral angle between two closest regular hexagonal faces, that are opposite and equally inclined with a square face $$\theta_{HHc} = 2\theta_{SHe} - \pi = 2\cot^{-1}\left(\frac{3-\sqrt{5}}{2}\right) \approx 138.1896851^o$$
	Dihedral angle between two closest regular decagonal faces, that are opposite and equally inclined with a square face $$\theta_{DDc} = 2\theta_{SDe} - \pi = 2\cot^{-1}\left(\frac{\sqrt{5}-1}{2}\right) \approx 116.5650512^o$$
Snub cube having 32 equilateral triangular and 6 square faces	Dihedral angle between two regular triangular faces with a common edge $$\theta_{TTe} \approx 153.2345877^o$$
	Dihedral angle between regular triangular & square faces with a common edge $$\theta_{TSe} \approx 142.9834301^o$$
Snub dodecahedron having 80 equilateral triangular and 12 regular pentagonal faces	Dihedral angle between two regular triangular faces with a common edge $$\theta_{TTe} \approx 164.1753661^o$$
	Dihedral angle between regular triangular & pentagonal faces with a common edge $$\theta_{TPe} \approx 152.9299203^o$$

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