

## Revisiting Newton's 340-year-old Third Law: A Generalized Form within Newtonian Mechanics

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### Abstract

Newton's third law is examined within the Newtonian framework under realistic interaction conditions, extending its applicability to real-world systems relevant to contemporary theoretical and experimental investigations. The law asserts the equality and simultaneity of action–reaction force pairs. Newton primarily applied the law qualitatively in Principia, illustrating it through three examples involving macroscopic interactions. Simple rebound experiments show that spherical bodies can retrace their original line of fall and rebound to comparable heights under suitable conditions, whereas asymmetrical or flat bodies exhibit reduced rebound heights and oblique rebound trajectories. The original formulation neglects several interaction-dependent factors, including material properties of bodies, rotation, spin, orientation, contact geometry, and deformation during interaction.

Consequently, the law is treated as independent of these factors and is therefore held universally. In horizontal motion the characteristics of the surface are also significant. Motivated by the above qualitative experimental trends and supported by historical and conceptual analysis, a generalized form of Newton's third law is proposed in which the reaction force is modified or extended by dimensionless coefficients accounting for shape, composition, target surface, and other interaction parameters, and expressed as

$$\text{Reaction } (F_{BA}) = - [K_{\text{shape}} \times K_{\text{composition}} \times K_{\text{target}} \times K_{\text{other}}] \text{ Action } (F_{AB})$$

The generalized form reduces to the original form under suitable conditions of parameters and provides an experimentally testable framework for quantitative confirmation at the macroscopic level. Over time, applications of Newton's third law have been extended to diverse systems, including aerodynamics and aerospace propulsion, each of which requires separate quantitative analysis.

**Key Words.** Third law, falling and rebounding bodies, shape, composition, and rocket.

## 1 Introduction

Newton originally formulated this law in *Principia Mathematica*, under the section Axioms, or Laws of Motion, presenting it in Latin and illustrating it with three simple examples typical of the era of natural philosophy [1].

Newton analyzed physical phenomena through the principles of proportions and proportionality [2], at a time when formal mathematical equations had not yet been developed.

When a player pushes a basketball against the ground, and it rebounds, the interaction provides a simple, real-world, macroscopic demonstration of Newton's Third Law of Motion in a qualitative sense. The quantitative confirmation of the law is required in such cases. Examples of this nature are extensively cited in the scientific literature. The *Principia* was translated into English by Andrew Motte in 1729, and Newton's third law of motion is quoted from this translation [3].

*“To every action there is an equal and opposite reaction, or the mutual actions of two bodies upon each other are always equal and directed to contrary parts.”*

$$\text{Reaction} = -\text{Action} \quad (1)$$

In the second part of the law, Newton explained that the force exerted by the second body on the first body (reaction) equals the force exerted by the first body on the second body (action). The magnitudes of these forces are equal but directed oppositely along the same line of action. Action and reaction are forces, as emphasized by Newton through three qualitative illustrations supporting the validity of the law. In contemporary notation, action and reaction are not treated as physical quantities, since they are absent from the IUPAP-defined list of physical quantities.

In terms of force, Eq. (1) may be written as

$$\text{Force of Reaction of Body B on Body A ( } F_{BA} \text{ )} = \\ - \text{ Force of Action of Body A on Body B ( } F_{AB} \text{ )} \quad (1)$$

Accordingly, the third law is defined solely in terms of the mutual interaction forces  $F_{BA}$  and  $F_{AB}$ , without reference to any additional parameters.

### 1.1 Newton's Third Law: A Universal Axiom of Classical Mechanics

In Newton's Third Law, *cause and effect are not implied* in the usual temporal sense; instead, the law describes a mutual, simultaneous interaction.  $F_{\text{Body} \rightarrow \text{Floor}}(t) = -F_{\text{Floor} \rightarrow \text{Body}}(t)$  at the same instant  $t$ . The condition  $\Delta t_{\text{action-reaction}} = 0$ , implies that action and reaction forces arise simultaneously, with no temporal separation between them i.e., action force and reaction force.

Action and reaction always happen in pairs equal in magnitude but opposite direction [2,4]. The definition and equations of the third law of motion only take into account  $F_{AB}$  and  $F_{BA}$ , but are independent of all other factors involved. According to the law, further characteristics of bodies are also insignificant.

Action and reaction are always equal and opposite, irrespective of other factors such as physical characteristics of bodies (shape, size, composition, mass, asymmetry, etc.), point or area of contact, orientation, rotation, deformation, etc., as Newton's third law considers only  $F_{AB}$  and  $F_{BA}$ , independent of all other factors. Thus, Newton's third law implies that action and reaction forces are always equal and opposite, regardless of other influences, and is therefore treated in practice as an all-purpose or universal law. The third law implies equality of

action and reaction but opposite in direction in all cases without any exception, hence the law is universal.

In the *Principia*, Newton analyzed physical phenomena geometrically, using ratios and proportions in the tradition of natural philosophy [2] without any equations. In the *Principia* (1686), Newton did not present the second law as a force equation but stated it in proportional form, relating the alteration of motion to the motive force impressed.

*The alteration of motion is ever proportional to the motive force impressed, and is made in the direction of the right line in which that force is impressed.*

Newton did not write the equation  $F=ma$ ; this formulation was later derived by Euler in 1776, while the explicit expression of weight as  $mg$  emerged only in the early twentieth century (c. 1901). Subsequent scientists adopted the relation  $F=ma$  as the mathematical expression of Newton's second law of motion. The development of these concepts thus represents a cumulative and evolutionary process rather than a single, fixed formulation.

### 1.2 Newton's definition of body or mass in Principia

Newton opens the *Principia* with Definition 1 for quantity of matter (mass or body), which lays the foundation for the subsequent formulations [1-3].

*The Quantity of matter is the measure of the same arising from its density and bulk conjunctly.*

Further, Newton stated that it is this quantity that I mean hereafter everywhere under the name of **body** or mass.

Newton illustrated the concept of mass using examples such as snow, dust, and powdered substances. Thus, Newton formulated his laws based on macroscopic examples of mass and motion in *Principia*. In his first two illustrations of the third law, Newton depicted interactions involving a stone, a horse, and a finger. In the third example, Newton referred to interactions between bodies as defined in Definition I of the *Principia*. Atoms, molecules, and subatomic particles were unknown or speculative in Newton's time (1642-1727). Dalton's Atomic Theory was developed from 1803 to 1808.

### 1.3 Conditions For Complete Validity of the Third Law

For Newton's Third Law to hold fully, two conditions must be satisfied

- (i) The reaction force must equal the action force in magnitude.
- (ii) The reaction must act in the exact opposite direction to the action.

The law describes a mutual and simultaneous interaction between two bodies. In mathematical equations, the opposite direction of the reaction force is represented by a negative sign in Eq. (1) and in the subsequent related expressions, such as Eq. (2). Thus, for the law to be valid, both conditions must be satisfied simultaneously. If the magnitudes of action and reaction are equal but their directions differ, the law cannot be considered fully satisfied. Therefore, the fulfillment of both conditions is essential for the complete validity of the law. Directional observations are especially important in interactions involving bodies of varying shapes.

### 1.4 Newton's three original illustrative Examples in Principia (1686)

In the second part of the law, Newton stated

*“ the mutual actions of two bodies upon each other are always equal and directed to contrary parts ”.*

So by action and reaction, Newton meant forces. Newton's third law is framed in terms of mutual interactions

between two bodies. After defining the law in *Principia* [3], Newton illustrated it with three simple qualitative examples.

*Whatever draws or presses another is as much drawn or pressed by that other.*

When body B is drawn or pressed by an external force exerted by body A (action), body B simultaneously exerts an equal and opposite reaction force on body A. The mutual actions between two bodies are forces. In the *Principia*, Newton defines force in the first part of Definition VI as an action exerted on a body that changes its state of rest or motion. Thus, in Newton's usage, the term "action" denotes force.

(i) Horse and Stone: If the horse pulls the stone tied to the rope, then the stone also pulls the horse equally backward.

(ii) Finger and Stone: If a finger presses a stone, the finger is also pressed by the stone.

In both examples, Newton considered small forces acting on a system that remained at rest.

Surface characteristics play a decisive role in the interaction. The finger exerts force on the stone (action), while the stone exerts force on the finger. The action and reaction forces are equal and opposite, and the system therefore remains at rest. If the applied action force is below an optimum value, or the finger pushes the stone weakly, the stone nevertheless continues to remain at rest. If a sufficient force is applied by the finger, the stone begins moving. Newton did not consider this situation.

(iii) Movement of Projectile and Target after interaction: In the third example of the third law p.20, Newton stated [3].

*"If a body impinges upon another, and by its force **change the motion of the other**; that body also (because of the quality of the mutual pressure) **will undergo an equal change**, in its own motion, **towards the contrary part**".*

Therefore, throughout his illustrations, Newton assumed that the reaction (force exerted by the second body) always equals the action (force exerted by the first body) and acts in the opposite direction.

Thus, Newton imposed strict conditions on interacting bodies, stating that the mutual forces they exert on each other are always equal in magnitude and opposite in direction, irrespective of the characteristics and physical properties of the interacting bodies and the surface of interaction. In the third illustration, he expanded the discussion to include interactions between macroscopic bodies as they move in response to these interactions.

If a body A (projectile) of mass  $M_p$  exerts a force on body B (target) of mass  $M_t$ ; body A experiences an equal and opposite change in motion.  $u_1, u_2$  are initial velocities, and  $v_1, v_2$  final velocities for projectile and target. Then, [Change in motion of the body B (target) when the body A (projectile) impinges on it] or [action] =

- [Change in motion of the body A (projectile) when it impinges on body B due to mutual interactions] or [reaction] (2)

$$\text{or } [M_t v_2 - M_t u_2] = - [M_p v_1 - M_p u_1] = - M_p v_1 + M_p u_1$$

$$M_p u_1 + M_t u_2 \text{ (initial momentum of the system)} = M_p v_1 + M_t v_2 \text{ (final momentum of the system)} \quad (2)$$

In the above equation, motion is represented by the quantity of motion, namely, momentum. So, the third example of Newton's third law of motion leads to the law of conservation of momentum.

Equation (2) may be verified experimentally by conducting real-world tests using practically achievable parameter values. Therefore, multiple sets of experimental observations with different feasible parameter combinations are

required to validate Eq. (2). At the macroscopic scale, interaction outcomes depend on the geometrical and material properties of the colliding bodies, as well as on the characteristics of the contact surface. Since these factors are not incorporated into Eq. (2), experimental validation is essential to assess its applicability. For simplicity, if  $M_t$  and  $M_p$  are equal, then

$$u_1 + u_2 = v_1 + v_2 \quad (3)$$

If the target is at rest, i.e.,  $u_2 = 0$ , then Eq. (3) becomes

$$u_1 = v_1 + v_2 \quad (4)$$

Equations (2), (3), (4), and other similar related expressions need to be experimentally verified using a broad range of parameter variations. There is no evidence in the existing literature that quantitative macroscopic experiments have been conducted to confirm Eqs. (2–4) or similar equations. These equations do not take into account the various characteristics of the projectile and target; hence, these equations are equally applicable for all bodies. However, experimentally, the characteristics of bodies and surfaces are significant.

The law of conservation of momentum [4] is also explained with the help of the equation of the second law of motion.

$$F = ma = \frac{dp}{dt} \quad (5)$$

$$\text{If } F=0, \text{ then } p = \text{constant or initial momentum (mu)} = \text{final momentum (mv)} \quad (6)$$

### 1.5 Universality of the Equations and Their Independence from Projectile–Target Characteristics

The definition and equation of Newton's Third Law are assumed to apply universally to all bodies, such as solids, liquids, gases, and semi-fluids, without regard to their mass, shape, material, or composition, etc.[5-6]. The law imposes no constraints on mathematical predictions because it considers only  $F_{AB}$  and  $F_{BA}$ , excluding all additional influencing parameters explicitly therein. Newton himself defined the body as he employed it in discussions, as stated in section (1.2). In real-world systems, the characteristics of body A, body B, and the interaction surface are significant and can be readily understood physically.

#### (i) First body A (projectile).

This is the body (projectile) that exerts action or an action force on the second body during interaction. In the definition of the third law, further characteristics of the body are neither defined nor mentioned at all. The body may have many possible shapes, e.g., spheres, hemispheres, cylinders, triangles, polygons, cones, long thin pipes, flat sheets, or arbitrary irregular shapes, etc. The interacting body may differ in physical properties such as composition, material nature, elasticity, flexibility, mass, asymmetry, and size or related properties; the law applies equally in all cases. For bodies of differing shapes, the angles of contact with the floor or supporting surface play a significant role in the interaction. Newton's Third Law does not account for all these factors and is assumed to remain valid for all bodies.

However, modern highly elastic materials, e.g., Super Ball, Waboba Moon Ball, Super High Bounce Balls (laboratory-grade polybutadiene), may also be considered in experiments, as the law is assumed valid for all bodies. The synthesis of additional highly elastic materials beyond those previously described is presently

underway. They will represent potential additional systems, once finalized, for experimental verification of Newton's third law. Further, Newton's Third Law is increasingly investigated in diverse and complex systems, including active matter, where internal energy sources break reciprocity [7], and related cases. Thus, the understanding of Newton's third law of motion is a continuous, evolving process and cannot be regarded as complete.

**(ii) Second body B (target):**

The second body, or target, is the body on which the first body applies the action, and as a result, an equal and opposite reaction is generated simultaneously.

Newton's formulation of the law is presented broadly and does not specify physical characteristics of the second interacting object (target), referring to it simply as a body.

The characteristics of the second body (target) are also extremely important, as they produce mutual interactions with the first body (projectile), generating an equal and opposite reaction force simultaneously. The equality of action and reaction, and their opposite directions, depend upon characteristics of both the body and the target.

In his definition, Newton refers only to "bodies," a general term encompassing both projectile and target. Newton defined the body in Definition I of the *Principia*. Newton explained the law using simple qualitative examples.

The law implies that, without specifying characteristics of the first body (projectile) and the second body (target), the reaction equals the action but acts in the opposite direction. Thus, the third law is applicable in all cases. Newton provided only three qualitative examples to justify the third law in the *Principia*.

In the first two examples, the target is a stone, which remains at rest before and after interaction, while the second bodies are a horse and a finger that apply force to the stone. A limited force is applied to the stone by the finger or the horse, and the system remains at rest. In the third example, general bodies are considered that move during interaction, such that the reaction is equal and opposite to the action.

When the law is examined critically within broader theoretical and experimental frameworks, the characteristics of the target (second body) should be described relative to those of the projectile (first body). Whether an object functions as a target depends on the projectile; for example, a stretched sheet of paper may serve as a target for a small body but not for a stone. Thus, the projectile and the target should be treated as relative entities in realistic analyses. Projectile–target pairs must therefore be chosen carefully and cannot be treated arbitrarily.

For example, a stretched sheet of paper may act as a target for a small body but not for a stone. Newton's third law is completely silent on the choice of target and implies that anybody may serve as a target with respect to the first body. Within Newton's definition of the third law, the stretched sheet of paper and the table surface each serve as the second body interacting with the stone. In Newton's formulation of the third law, no explicit distinction is made between projectile and target; both are described simply as interacting bodies. This is further evident from the fact that Newton explained the third law using three qualitative examples, which were appropriate in the era of

natural philosophy.

### (iii) Third Body C: Interaction Surface

Newton stated that action and reaction occur simultaneously. In interactions involving a horizontal surface, the physical characteristics of the surface play a crucial role in determining the observed response. This aspect needs to be critically discussed as it may cause a significant loss of energy comparatively. The physical characteristics of the surface influence the outcome of the interactions and must be taken into account.

If one marble A strikes the second marble B with the same force (action) on a different surface, e.g., on ground, floor or glass slab. Then, on the different surfaces, the velocities of marble A and marble B are different. Thus, on horizontal interactions, the role of the surface is as important as that of the projectile and the target in understanding the interactions.

Newton stated that action and reaction are two simultaneous events. If interactions occur on a horizontal surface, then surface characteristics become significant and affect the results. This aspect must be critically discussed, as it may cause comparatively significant energy loss. The physical characteristics of the surface influence interaction outcomes and must be taken into account.

If marble A strikes marble B with the same force (action) on different surfaces, e.g., ground, floor, or glass slab, then on different surfaces, the velocities of marble A and marble B differ at various places. Thus, in horizontal interactions, the role of the surface is as important as that of the projectile and the target in understanding interactions.

(a) Speculative conclusion in view of the second part of Newton's First Law of Motion: If the system is completely free from resistive forces, and marble A strikes marble B, then the latter must move with uniform velocity if the system is free from other forces, as implied by the second part of Newton's First Law of Motion.

(b) In the case of freely falling and rebounding bodies, air resistance may be minimized or effectively eliminated by creating a partial or complete vacuum.

Thus, in some cases, the surface on which interactions of projectile and target occur may be termed the third body, as it affects the results. The purpose of discussing these factors is that Newton's law, in its original formulation, is silent on these aspects.

## 2.0 Two categories of applications of Newton's third law are recognized.

Newton illustrated the Third Law with three qualitative examples in the *Principia* (1686), as discussed above. Subsequent scientists extended its applications to a variety of physical phenomena. Initially, the law was applied in classical mechanics, for example, to bouncing balls, rowing, and similar systems. In the 20<sup>th</sup> century, the third law was applied to entirely different contexts, such as rockets, airplanes, and propulsion systems. With the passage of time, theoretical and experimental systems have advanced, and the applications of Newton's third law can be broadly categorized into action–reaction motion systems and aerospace and propulsion systems.

**(i) Action reaction Motion Systems.** In the *Principia* (1686), Newton presented only three applications, largely qualitative in nature. Then the following scientists extended the application of the law in other examples, such as bouncing balls, swimming, rowing, walking, jumping from a boat, balloon deflation, swimming fish, gun recoil,

striking and rebounding bodies on the horizontal surface, etc.

These represent contact-based action–reaction interactions. Although such examples serve to illustrate the action–reaction principle, they rarely provide quantitative verification—an essential criterion for establishing scientific validity in physics education and research. As mentioned earlier, in horizontal motion, the characteristics of the surface must be taken into account.

In vertical motion of falling and rebounding balls, resistive forces of the medium may be minimized or effectively eliminated by creating a partial or complete vacuum. However, rigorous experimental verification of Newton’s Third Law across diverse physical systems remains technically challenging and often requires high-precision instrumentation.

(ii) **Aerospace Propulsion Systems.** These constitute highly advanced and technically sophisticated applications of Newton’s Third Law developed in the 20<sup>th</sup> and 21<sup>st</sup> centuries. These include rockets, fireworks, spacecraft, airplanes, missiles, helicopters, drones, and gliders, among others.

In aerospace and rocket propulsion systems, the exhaust gases expelled at high velocity produce the reaction force that propels the vehicle forward. This is a direct consequence of Newton’s Third Law of Motion. Aeronautical and astronautical engineers and scientists apply Newton’s third law in these systems.

The current work examines the quantitative behavior of falling and rebounding bodies, a topic that has not yet been systematically studied. The qualitative nature of Newton’s third law in typical applications may be interpreted through various approaches.

(a) By the sixteenth century, arquebuses and muskets were increasingly employed in English armies; however, Newton’s writings do not make any reference to such firearms. The recoil velocity of the gun is given by

$$v_{\text{gun}} = - m_{\text{bullet}} \cdot v_{\text{bullet}} / m_{\text{gun}} \quad (7)$$

Although Eq. (7) has been repeatedly presented and discussed in theoretical frameworks, there is no reported experimental validation of it in the existing literature. The understanding of theoretical deductions becomes complete only when they are rigorously validated through experimental confirmation. Equation (7) is practically based on the third application of Newton’s third law under idealized conditions.

(b) Using Newton’s third law, the ideal rocket equation [8] was derived by Russian teacher Konstantin Tsiolkovsky in 1897 and published in his paper “*Exploration of Outer Space by Means of Rocket Devices*” in 1903. The velocity of the rocket at any instant is given by

$$\Delta V = V_e \ln M_0 - V_e \ln M = V_e \ln M_0/M \quad (8)$$

All quantities in Eqs. (7)-(8) have their conventional physical meanings as used in elementary mechanics. The Eq.(8) has undergone several refinements by aeronautical scientists over the years to improve its applicability and precision. A physical law is regarded as universally valid only when it is repeatedly and independently verified across all relevant physical regimes. Here, it is emphasized that quantitative experiments on falling and rebounding bodies should be carried out at the macroscopic level to properly understand the law.

### 3.0 Historical Context before and after Principia.

Before Newton, Descartes (1644) presented a Third Law of Nature in a simple, descriptive form of physical events. Newton published the Third Law of Motion in *Principia* (1687) with three qualitative illustrative examples. Later,

scientists applied the third law to various action–reaction motion systems in a qualitative manner. In the 20th century, scientists applied the third law to aerospace propulsion systems.

**(i) Descartes' Third Law of Nature [9]:** The third law of nature, as formulated by Descartes, appeared in 1644 in *Principles of Philosophy* (Chapter 2, Paragraph 40, p.34 ), predating Newton's version by about four decades.

The third law of nature: (a) if one body collides with another that is stronger than itself, it loses none of its motion; (b) if it collides with a weaker body, it loses the same amount of motion that it gives to the other body.

Both Descartes and Newton have discussed colliding bodies. The third application of Newton's Third Law of Motion quantitatively defines the relationship between interacting bodies in terms of action and reaction, i.e., forces that are equal in magnitude and opposite in direction, whereas Descartes' law offers only a qualitative description of the physical event.

(ii) During Newton's lifetime (1642–1727), mathematical equations were not prevalent in scientific inquiries, and natural phenomena were largely interpreted through qualitative reasoning and description. Newton interpreted phenomena geometrically by the method of ratios or proportions [2]. Newton originally stated the law in Latin in *Philosophiae Naturalis Principia Mathematica*, and it was later translated into English by Andrew Motte, encouraged by his brother Benjamin Motte, a publisher who saw the book's commercial potential.

The English translation was published in 1729, two years after the death of Newton. Newton did not alter the definition of the third law for approximately 40 years (1686–1726), nor did he provide additional clarification beyond its original statement as illustrated in the 1686 *Principia*. The second and third editions of the *Principia* were published by Newton in 1713 and 1726, respectively. Newton stated the second law in proportionality form and the third law in equality form.

(iii) Swiss Leonhard Euler started relating force with mass and acceleration in 1736 in the book *Mechanica* [10]. Euler [11,12] published the equation  $F=ma$  in 1776 when he was working at the Imperial Academy of Sciences in St. Petersburg, Russia.  $F = ma$  was published in the treatise *Nova methodus motuum corporum rigidorum determinandi* (A New Method for Determining the Motions of Rigid Bodies) in the journal *Academia Scientiarum Imperialis Petropolitana* (Imperial Academy of Sciences in Saint Petersburg) at pages 222-224 (E479).

As previously mentioned, the conceptual limitations of that era prevented Newton from formulating mathematical equations.

(iv) The gravitational acceleration ( $g = 9.80665 \text{ m/s}^2$ ) was measured in 1888 by the Geographic Services of the French Army [13]. In 1901, at the 3rd General Conference on Weights and Measures (CGPM), weight was formally defined as  $mg$ . In Euler's equation  $F = ma$ , when the acceleration  $a$  replaces the acceleration due to gravity  $g$ , the resulting expression  $W = mg$  represents the weight of the body near the Earth's surface. Realistically, this became the greatest breakthrough in classical mechanics related to the quantitative aspects of Newton's third law.

Thus, expressing Newton's Third Law in precise mathematical form (for falling and rebounding bodies) became practically feasible only about 215 years after the publication of the *Principia*. In the case of a body falling and rebounding towards the surface, the action can be regarded as the gravitational pull of the Earth on the body, quantitatively given by its weight,  $mg$ .

(v) The equation for freely falling and rebounding bodies became feasible about 125 years ago, in 1901. In this

period, Newton's third law was established as a fundamental law at a qualitative level. Historically, the law has been reinforced largely through qualitative examples, which were acceptable in the early development of mechanics but fall short of today's expectations for quantitative verification.

However, fundamental quantitative experiments on falling and rebounding bodies have not yet been systematically performed. The successful completion of these experiments would mark a breakthrough, potentially enabling more precise and quantitative observations in related phenomena.

(vi) Earliest origins of rocket motion and applications of Newton's Third Law in such extended applications.

Gunpowder-propelled devices, primarily used for recreational purposes such as fireworks, were developed in China around the 9th century A.D., nearly 700 years before Newton's *Principia*.

Rockets were employed in warfare for the first time during the Mongol-Chinese conflict in 1232, using gunpowder (solid fuel); their motion was completely uncontrolled after ignition, similar to fireworks. The earliest recorded fireworks display in England occurred at the wedding of King Henry VII and Elizabeth of York in 1486. Despite these developments, Newton (1642–1727) did not discuss rockets in the *Principia* or elsewhere, nor did he mention his Third Law to explain their motion.

(vii) After the publication of Newton's *Principia* (1687), his Third Law of Motion quickly became a cornerstone of European physics. Universities such as Cambridge, Oxford, Paris, Naples, and Göttingen incorporated Newtonian mechanics into their curricula, educating scholars and engineers who applied these principles in research, technology, and practical sciences. Beginning in the early 1800s, the British East India Company's schools, military academies, and surveying programs systematically incorporated European scientific knowledge, including the principles of Newtonian mechanics, into colonial India's education system. This foundation paved the way for the formal inclusion of Newton's laws in the curricula of the first modern Indian universities e.g., Calcutta, Bombay, and Madras, established in 1857.

It is pertinent to note that the acceleration due to gravity,  $g=9.8005 \text{ m s}^{-2}$ , was experimentally determined in France in 1888, and the concept of weight was formally defined in 1901. By this time, Newton's laws had already been incorporated into academic curricula worldwide, about one and a half century before.

(viii) In 19<sup>th</sup> century, the gliders were considerably well developed. Otto Lilienthal (1848–1896) was a German aviation pioneer known as the "Father of Gliding". Wright Flyer I, invented by the Wright brothers in 1903, fundamentally relied on Newton's Third Law of Motion for its flight. Here, wings produce lift and the flyer is controlled by elevators, wing wrapping and rudder etc. A glider flies without an engine, while the Wright Flyer was the first aircraft to fly using an engine and propellers. In 1907, Paul Cornu (France) achieved one of the first manned vertical lifts using a twin-rotor craft i.e., practical perception of a helicopter.

(ix) Goddard refined the ideal rocket equation in 1919 in a monograph [14], introducing a generalized form as follows.

$$\Delta V = V_e \ln \frac{M_0}{M} - \int_0^t g dt - \int_0^t \frac{D(v)}{m} dt \quad (9)$$

where  $V_e$  is the effective velocity,  $M_0$  and  $M$  are the initial and final masses,  $g$  is the acceleration due to gravity,  $D$  is the drag force,  $t$  is the burn time, etc.

In 1926, Goddard became the first astronomical pioneer to experimentally launch a liquid-fueled rocket (often

called Nell), achieving a flight lasting about 2.5 seconds. The fuel consisted of gasoline (fuel) and liquid oxygen (oxidizer). By further improving the design, Goddard extended the rocket's flight time to approximately 22.3 seconds in 1937.

Although technology and knowledge have since advanced in astronautical engineering and computational modeling, space missions may succeed or fail, yet Newton's third law, formulated about 340 years ago in the *Principia*, remains valid. In such experiments, technological advancement is equally crucial.

Thus, Newton's Third Law of Motion finds diverse applications across science and engineering.

### 3.1 Why has Newton's Third Law Not Been Quantitatively Confirmed in rebound experiments?

Various reasons can be discussed to explain why the law has not been quantitatively studied in action–reaction motion systems.

**First**, Newton's Third Law was originally formulated without a mathematical expression and was interpreted only qualitatively in the *Principia*, where Newton illustrated it using three examples. The law was subsequently applied to various other analogous cases, albeit only qualitatively. In some applications, the effects of the surface on which bodies move are significant.

**Second**, a spherical body appears to rebound toward its original position in the opposite direction after striking a surface under certain conditions, and this behavior seems consistent with the law, but only qualitatively. Thus, scientists did not quantitatively confirm the law in many other cases due to the experimental complexity associated with the interactions and composition of different bodies. Hence, the law was established qualitatively in many situations.

**Third**, the precise acceleration due to gravity,  $g$ , was determined in 1888, enabling a mathematical treatment only after the standardized definition of weight (mg) became established in 1901. By this time, the law was widely regarded as qualitatively verified. Now, quantitative applications of the law, along with methods, have been discussed for falling and rebounding bodies in the past 125 years, when equations became feasible.

**Fourth**, in the 20<sup>th</sup> and 21<sup>st</sup> centuries, the law enabled significant development in rockets, aircraft, and propulsion systems, which redirected focus toward astronautical engineering and computer-based design, rather than fundamental experimental assessments in the category of Action-Reaction Motion systems.

Because the applicability of the law was regarded as exceptionally precise and reliable in space programs, little attention was directed toward testing it through simple experiments involving falling and rebounding bodies.

The analysis identifies unexplored experimental tests of Newton's third law, especially for rebounding bodies of equal mass and composition but differing shapes. The conclusions are therefore preliminary and call for dedicated experimental investigations. Thus, carefully designed experiments involving rebounding bodies are scientifically essential for a direct quantitative understanding of the law in action-reaction motion systems.

## 4.0 Observations and Mathematical Basis of Rebound Experiments.

Falling and rebounding bodies with identical mass and composition but different shapes are essential for the quantitative verification of the third law, as qualitative results alone are insufficient. The definition and equation

law only involve  $F_{AB}$  and  $F_{BA}$ , not other factors. For simplicity, these experiments are presented at the initial stage, after which more complex experiments can be discussed and performed.

**Basic qualitative observation:** In everyday qualitative demonstrations, a spherical body may, under certain conditions, rebound to its point of release, effectively retracing its initial path; thus, action and reaction are the same in magnitude but opposite in direction. Therefore, the upward rebound can be considered an observable consequence of the reaction stipulated in Newton's third law. Thus, the reaction exerted by the body is quantitatively manifested as its rebound distance in such cases. This argument supports the validity of Newton's third law.

However, the bodies of identical mass and material or composition, but with different shapes (e.g., spheres, semi-spheres, cylinders, triangles, polygons, cones, long thin pipes, sheets, arbitrary shapes, etc.), rebound to different heights and directions. A flat body is observed to rebound to the minimum height, whereas irregular bodies show variable and unpredictable rebound heights and angles. All observations must be explained quantitatively and in a mutually consistent manner.

Bodies with the same mass and composition may differ in shape. These qualitative observations serve as the primary motivation for quantitative confirmation of the third law at the macroscopic level, thereby prompting further both theoretical and scientific discussions [5-6].

**Quantitative Explanation:** It is not scientifically justified to rely indefinitely on such qualitative observations as evidence for Newton's third law that a body falls as action and, as a reaction, it rebounds upward.

Newton expressed the concept of *action* in terms of force, i.e., push or pull, in the first two examples as given in Principia Mathematica. Let the body fall from point  $A_0$  at height  $H$  (1 m). Then it must reach at the bottom at time 0.45 s, i.e.

$$t = \sqrt{\frac{2s}{g}} = 0.45s \quad (10)$$

In the first two examples, Newton expressed action in terms of push or pull, i.e., force.

(i) When a body freely falls, the acceleration due to gravity is  $g$  (9.78-9.83  $m/s^2$  at different regions), and the downward force acting on it is the gravitational force, or its weight, represented by  $mg$ . In this context, the *action* corresponds to the force, i.e., the gravitational force or weight acting on the body downward. Consequently, for bodies of identical mass (1kg) the magnitude of the action remains constant ( $mg$ ), irrespective of shape and other physical attributes.

$$\text{Action} = \text{Force} = \text{weight} (mg) = 9.8 \text{ newtons} \quad (11)$$

(ii) The body strikes the surface ( floor, say), with action ( $F_{BF}$ ) 9.8 newtons. According to Newton's third law, a reaction ( $F_{FB}$ ) arises simultaneously due to mutual interactions between a body and a floor; its magnitude equals that of the action (i.e., 9.8 N) and is directed upward, opposite to the action (denoted by a negative sign).

$$\text{Reaction} = -\text{Action} = -9.8 \text{ newtons} \quad (12)$$

When a body falls, the acceleration due to gravity is  $+g$ , and when the body rebounds upward, the acceleration is  $-g$ . Let  $h$  denote the rebound height of the body. For simplicity, the bodies are assumed to have the same mass and composition and to be non-deformable; only their shapes are varied.

(i) **Spherical bodies:** The spherical body (symmetrical) rebounds to the original point  $A_0$  at height  $H$ , in the opposite

direction; it describes ideal or standard conditions. Hence, the law is completely obeyed under some conditions, the height from the body falls ( $H$ ), and the height to which the body rebounds ( $h$ ) are equal ( $H=h=1m$ ). Thus, in this case, the magnitudes of the action and reaction are exactly equal, as the body rebounds upward to its original height ( $H$ ) in the opposite direction. So, the reaction is manifested in terms of rebound distance

$$\text{Action } (F_{BF}) = -\text{Reaction } (F_{FB}) = -9.8 \text{ newtons. (opposite direction)} \quad (13)$$

Here, the negative sign indicates the opposite direction of the rebounding body.

As the body rebounds to the original point ( $H=h$ ), retracing its original trajectory, then the ‘Rebound Angle’ (RA) may be regarded as zero. Under standard or ideal conditions, Newton’s Third Law is completely satisfied in the case of a spherical body.

**(ii) Bodies of different shapes:** The rebounding behavior of bodies with different geometries—including spheres, hemispheres, cylinders, triangular and polygonal bodies, cones, elongated pipes, thin sheets, and other arbitrary shapes- is not similar to spherical bodies.

In real-world conditions, bodies of different shapes rebound to lower heights ( $h$ ) and distinct angles. Such qualitative observations can be repeatedly confirmed through numerous daily-life experiences. This implies that the experimental reaction possesses different magnitudes and directions in such cases than predicted by Newton’s third law. Theoretically, in Eqs. (12–13); the action and reaction forces are equal when the masses of all bodies are the same.

A flat body is typically observed to rebound to the minimum height, whereas an irregular body exhibits variable and unpredictable rebound heights and angles. The discussion aims to quantitatively interpret and explain these observations. When the masses of the bodies are equal, the gravitational force acting on each is the same, i.e.,  $mg$ , leading to equal and opposite reaction forces in accordance with Newton’s third law. However, experimentally, the reaction in such cases does not manifest in the same way as for a spherical body. Further, for bodies of different shapes, the angles of contact play a significant role in understanding the rebound behavior.

**(iii) Factors responsible for reduced rebound height ( $h$ ) and varying rebound angles.**

**(a) Loss of energy during interactions.**

This factor may be systematically evaluated within this context. The causes for bodies rebounding to a lower height  $h$  include energy losses, such as heat energy, sound energy, other associated forms, and additional significant effects depending on interacting bodies. An effort should be made to minimize the energy losses as much as possible. Non-deformable bodies must be selected to achieve conceptual simplicity.

These energies must be carefully quantified, and it should be examined whether they are solely responsible for bodies rebounding to a reduced height, or if other factors demand consideration in the quantitative analysis.

**(b) Asymmetry of body:** The asymmetry of the body may be responsible for the body rebounding at different and nonzero angles. Newton’s third law is independent of effects arising from physical asymmetry, such as differences in shape or size, as it accounts for  $F_{AB}$  and  $F_{BA}$  only.

**(c) Other inherent factors.** Thus, additional factors beyond  $F_{AB}$  and  $F_{BA}$  are likely responsible for the observed anomalous behavior, yet remain unaccounted for within the existing law and scientific framework. These factors include the shape and composition, material properties of the interacting bodies, and other relevant experimental

parameters involved either implicitly or explicitly. These factors may be assessed using varied observations and systematic critical evaluation. This aspect is discussed in section (6.0).

(iv) **Time of fall and rebound to maximum height:** Newton's third law stipulates that the action and reaction forces ( $F_{AB}$  and  $F_{BA}$ ) are equal in magnitude and opposite in direction. Under ideal conditions, if a spherical body of mass 1 kg falls from a height of 1 m and reaches the floor from point  $A_0$  in 0.45 s (action is 9.8 newtons) as in Eq. (10), it should rebound (reaction is 9.8 newtons but in the opposite direction) and return to point  $A_0$  in the same duration. This deduction is expected to be valid only under idealized conditions.

Because the motion of falling and rebounding bodies has not been analyzed quantitatively in the existing literature, the practical relevance of the time of descent to the point of impact and the time of rebound to the maximum height has remained largely unrecognized.

Had the law been experimentally verified for falling and rebounding bodies, or attempts been made toward this, fundamental aspects such as the difference between the time of fall to the surface and the time of rebound to the maximum height would have been recognized much earlier. Newton's third law (Reaction = -Action) can manifest differently under varied conditions, e.g., rebound heights, time to rebound to maximum height etc. The angle of rebound reflects the direction of reaction.

(v) **Angle of rebound.** Newton's third law states that the magnitude of the reaction equals that of the action, and its direction is exactly opposite to the action. In Eq. (1), the negative sign is introduced externally to denote the opposite direction of the reaction force.

For freely falling bodies, a spherical ball rebounds along the line of fall, while bodies of other shapes, such as semi-spheres, cylinders, triangles, polygons, cones, long thin pipes, flat sheets, or arbitrary forms, rebound at angles deviating from the line of fall. For these bodies, the rebound angles vary depending on the specific conditions of impact. The angle of fall plays a significant role in determining the nature of the interaction when a body strikes a surface. These variations must be fully harnessed to realize their maximum potential, and this behavior must be analyzed and discussed in detail following experimental measurements.

The values of action, reaction, angle of rebound, time of fall to surface, and time of rebound to maximum height may be tabulated for analysis. Thus, the exploration and experimental verification of Newton's third law remains an ongoing scientific endeavor, continually advancing with emerging opportunities and technologies. Multiple factors influence measurements in experiments with freely falling and rebounding bodies, which put constraints on the verification of the law.

(vi) **Horizontal impact of the body with the target:** When a ball or body travels horizontally across a surface, strikes a target, and rebounds, frictional forces and energy dissipation (as heat and sound) become dominant factors that must be quantified along with other relevant parameters. In this case, the action force must be precisely measured using external instruments. The role of the target's characteristics becomes increasingly significant in this context. The use of asymmetrical bodies often results in irregular or unpredictable forward and backward motion, making the accurate measurement of associated quantities both challenging and extremely difficult. Similarly, experiments that involve varying the composition of bodies and targets are highly cumbersome to conduct with

quantitative precision.

Therefore, falling and rebounding experiments involving bodies of identical mass and composition but different shapes are discussed first, owing to their simplicity and their fundamental importance in examining the validity of the Third Law. In addition, we come across far more complicated experiments to confirm Newton's third law.

#### 4.2 Futuristic Experiments

In the future, such experiments may be performed in low-gravity environments, such as on the Moon, where the acceleration due to gravity is approximately one-sixth of Earth's value. The lower value of  $g$  increases the duration of free fall and modifies the time of ascent during rebound. At the International Space Station, or in other microgravity environments, such experiments may also be explored under conditions of weightlessness.

Furthermore, researchers are exploring elastic materials more advanced than Super Balls and Sky Balls to achieve even greater bounce performance. Thus, the law needs to undergo quantitative testing in such experimental situations. Thus, the concise formulation of Newton's third law underpins a wide range of experimental studies, highlighting both its applications and possible departures. These must be exploited or performed to their fullest extent.

**Speculated experiments in view of the simultaneous application of Newton's first and third laws:** Speculated experiments may be conceived on a completely frictionless surface where no external force acts on the system. When body A pushes body B, the interaction produces equal and opposite forces on both bodies simultaneously, in accordance with Newton's third law.

If no net external force acts, body B should then continue to move with uniform velocity after interaction, consistent with Newton's first law. Since a perfectly frictionless system or a condition of zero external force is not available in practice, experiments must be designed to progressively approach such idealized systems.

#### 4.3 Alternative Description of the Same Observation in Terms of Eq.(5)

Consider a spherical body freely falling onto a rigid horizontal surface with acceleration  $g$ . At the instant of contact, the action force is commonly identified with the weight  $mg$ , acting on the surface. For a body of mass 1 kg, this action force equals 9.8 N, and the corresponding reaction force is assumed to be equal in magnitude and opposite in direction ( $-9.8$  N), as discussed earlier in Eqs. (12–13).

(i) The same physical process may be analyzed using Eq. (5),

$$F=ma = \frac{dp}{dt} = \frac{d(mv)}{dt} \quad (5)$$

Since the same event, a spherical body falling and rebounding from a rigid surface, is described by two formulations within Newtonian mechanics, consistency would require both approaches to yield identical quantitative results.

However, this is not observed in the present analysis.

$$v^2 - u^2 = 2gS \quad \text{or} \quad v^2 = 19.6 \text{ m}^2/\text{s}^2 \quad \text{or} \quad v = 4.43 \text{ ms}^{-1} \quad (14)$$

Let the time of impact of body and surface be  $10^{-3}$ s, then force is given by Eq.(5) as

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = 4.43/10^{-3} = 4430 \text{ newtons} \quad (15)$$

(ii) According to Newton's third law, the body must experience an equal and opposite force of the same magnitude. This value is nearly 453 times greater than the force corresponding to the weight of the body (9.8 N). The resulting acceleration of a 1 kg body would therefore be  $4430 \text{ m s}^{-2}$ , far exceeding the gravitational acceleration  $g$  ( $9.8 \text{ m/s}^2$ ). Both cases are discussed.

Thus, two Newtonian descriptions of the same physical event based on action force as weight and impulsive momentum transfer lead to markedly different force magnitudes. Thus, quantitative experimental investigation is necessary to ensure impartial evaluation of the results. This discrepancy shows that the third law cannot be fully interpreted without considering contact time, deformation, and force transmission, motivating a refined or generalized formulation.

### 5.0 Experiments with low-cost Equipment.

These experiments can be conducted using low-cost instrumentation such as USB cameras or photogate timers to accurately measure impact durations during collision. The digital inclinometers or angle finders are used to determine rebound angles with respect to the line of fall. The laser displacement sensors may be utilized to precisely record rebound heights.

The combined use of high-speed videography and laser-based sensing enables time-resolved tracking of the motion and ensures precise investigation of shape- and geometry-dependent rebound effects under controlled experimental conditions.

These experiments are relatively simple to execute and yield reproducible results, offering a practical approach to quantitatively validate Newton's third law at the macroscopic scale. At the same time, more complex action–reaction systems remain technically challenging. Thus, as the first step toward quantitative verification of action–reaction systems at the macroscopic level, experiments with falling and rebounding bodies are being explored.

### 6.0 Generalized form of Newton's Third Law

Qualitative observations cannot be accepted as universally valid for all situations. Therefore, rigorous quantitative experiments are necessary for reliable conclusions. Today, theoretical and experimental methods are far more advanced than in Newton's era of natural philosophy when the third law was originally proposed.

Newton's third law considers only  $F_{AB}$  and  $F_{BA}$ , whereas experimentally, in rebound experiments, the various other factors are also significant, as discussed in section (2.0). These elusive factors may be incorporated by generalizing or extending the law [4-6,15-19]. However, in this work, we primarily consider falling and rebounding bodies within action–reaction motion systems. Newton's third law may be expressed in a proportionality form, similar to Newton's second law of motion. Thus,

$$\text{Reaction} \propto \text{Action} \quad \text{or} \quad \text{Reaction} (F_{BA}) = - K \text{ Action} (F_{AB}) \quad (16)$$

where  $K$  is a coefficient of proportionality or phenomenological parameters. The coefficient  $K$  accounts for various elusive factors not incorporated in the original law, such as the shapes, sizes, asymmetry, compositions, material properties, physical characteristics of the bodies, the properties of the target, the nature of the surface, the

interconversion of energy during interactions, and all other influential variables implicitly or explicitly affecting results.

Such factors are equally significant in colliding bodies, as discussed. Thus, the value of K may be expressed in one of the following ways as

$$K \text{ (coefficient or additional factor)} = K_{\text{shape}} \times K_{\text{composition}} \times K_{\text{target}} \times K_{\text{other}} \quad (17)$$

$(K_{\text{shape}} \times K_{\text{composition}} \times K_{\text{target}} \times K_{\text{other}})$  is an additional expression in Eq. (16) relative to Eq. (1), which implies the reaction may deviate from the action depending on various influencing factors and effects. Thus, action and reaction may or may not remain equal always, the direction of bodies depend upon asymmetry. K is dimensionless, and its magnitude may deviate from unity from one observation to another, depending on the associated parameters. If the value of K is unity, then Eq. (16) reduces to Eq. (1). The value of K may be determined through standardization, calibration, or comparison, depending on the practicality and feasibility of the chosen method.

$K_{\text{shape}}$  accounts for the influence of geometry, asymmetry, etc.,  $K_{\text{composition}}$  for the effect of specific compositions and material characteristics,  $K_{\text{target}}$  for all measurable effects of the target, and  $K_{\text{other}}$  for the influence or impact of all remaining factors, including the transformation of energy between interacting bodies etc. The measurement of  $K_{\text{other}}$  is more challenging than determining other  $K_i$ 's. Realistically, Eq. (16) becomes in comprehensive form as

$$\text{Reaction (F}_{\text{BA}}) = -K_{\text{shape}} \times K_{\text{composition}} \times K_{\text{target}} \times K_{\text{other}} \text{ Action (F}_{\text{AB}}) \quad (16)$$

Thus, law may be defined as

*“Every action has a proportional reaction; the magnitude and direction of the reaction would be precisely equal and opposite depending upon experimental factors such as the shape, size, characteristics, and other involved factors, etc., of the interacting bodies.”*

Newton has given a compact statement of the law with three qualitative examples and following scientists extended the applications of the third law. As already noted, the applications of Newton's third law may be broadly categorized into action–reaction motion systems and aerospace and propulsion systems. The present discussion is restricted to falling and rebounding bodies; no comments are offered regarding other cases where the law is well established and experimentally verified.

As the experiments span a wide range of interaction regimes, the physical properties of the interacting surface play a critical role and must be systematically investigated. When a first body A (the projectile) moves horizontally and strikes or interacts with a second body B (the target), the surface on which the interaction occurs is equally significant. Its physical characteristics influence the outcome of the interaction and must be taken into account. In horizontal interactions, the role of the surface is as important as that of the projectile and the target. These influences may be taken in account through  $K_{\text{other}}$ .

### **K is an effective coefficient or phenomenological parameter**

The coefficient K in the generalized relation  $\text{Reaction} = -K \text{ Action}$  serves as an effective, phenomenological parameter. Analogous to the coefficients in the Bethe–Weizsäcker mass formula or the coefficient of friction, it encapsulates complex interactions. Various other parameters of this type are discussed in the literature. Its value

depends on shape, asymmetry, material, shape, interaction conditions etc., and approaches unity in idealized limits, recovering the conventional form of Newton's third law

The generalized form aims to unify shape, composition, and target properties, other effects into a single framework, rather than leaving them as separate domains. It doesn't replace Newton's third law but extends it, embedding macroscopic realities into one operator equation. This visualization shows how the generalized law absorbs separate factors into one structured equation, instead of leaving them fragmented across physics and engineering subfields. It's a bridge between classical mechanics and real-world impact phenomena. Experimental measurements of contact time, rebound height, rebound direction etc., enable direct determination of the generalized coefficients, thereby connecting idealized theoretical principles with real-world macroscopic interactions.

### **Importance of Eq. (16 ):**

The present analysis demonstrates that Newton's third law, when applied within the Newtonian domain under realistic interaction conditions, can be meaningfully extended to real-world systems. This extension highlights the need for careful consideration of material, asymmetry, temporal, and contact-related factors in both theoretical formulations and experimental interpretations.

Following extensive experimentation, the results can be explained using different theoretical or phenomenological approaches, either simple or complex, including existing methods as well as those developed specifically for this purpose. Equation (16 ) provides a quantitative framework for interpreting the observations while remaining directly derived from the original form of Newton's third law within its domain of applicability; this logical continuity constitutes its main advantage.

The proposed extension does not replace Newton's third law but identifies the conditions under which its classical form holds exactly and those under which a generalized interpretation is necessary. The original law is recovered as a limiting case, ensuring full consistency with Newtonian mechanics.

### **6.2 Extension and development of laws is an established process.**

Refinements, extensions, or replacements of established laws are integral to scientific and physical progress.

(i) Newton presented three qualitative examples in the *Principia* (1687) to illustrate the third law of motion. Subsequent scientists applied the law qualitatively to explain phenomena such as bouncing balls, swimming, the recoil of a gun, etc. In the 20<sup>th</sup> and 21<sup>st</sup> centuries, the law was also widely used in the explanation of aerospace and propulsion systems. Attempts have been made to experimentally and quantitatively verify the law for falling and rebounding bodies at the macroscopic level.

(ii) Newton's seventeenth-century corpuscular theory of light was eventually supplanted by the wave theory advanced by Young and Fresnel. Maxwell later in 1865 unified optical phenomena with electromagnetism, identifying light as an electromagnetic wave. During the twentieth century, quantum theory introduced photons, distinct from Newton's corpuscles. Contemporary physics describes light using quantum electrodynamics, a fundamentally different framework today. While the seeds were planted in the 1920s–1930s, the **complete quantum electrodynamical theory** emerged in the **1940s**.

(iii) Dalton's atomic theory (1803-1808) constituted a foundational classical description of matter. Subsequent

experimental and theoretical contributions by J. J. Thomson, Rutherford, Bohr, Moseley, and Chadwick progressively revealed subatomic structure, nuclear organization, quantized energy levels, atomic number, and isotopic variation. These advances led to a refined and generalized understanding of atomic behavior beyond Dalton's original formulation.

(iv) Newton expressed the speed of sound in a medium ( $v$ ) as

$$v = \frac{P}{D_m} \quad (18)$$

**Pierre-Simon Laplace** in 1816 corrected this relation, yielding Equation (19), because Newton's expression did not produce the experimentally correct value for the speed of sound in air. Hence,

$$v = \gamma \frac{P}{D_m} \quad (19)$$

where  $P$  denotes the pressure,  $D_m$  the density of the medium, and  $\gamma$  the ratio of specific heats (specific heat at constant pressure to that at constant volume). Equation (19) provides an accurate value for the speed of sound in air.

Realistically, Newton assumed that the propagation of sound waves is isothermal in nature; however, Laplace later demonstrated that the process is actually adiabatic.

(v) Tsiolkovsky published Eq. (8), the ideal rocket equation, in 1903 based on Newton's third law of motion.

Goddard later extended this formulation in 1919 as Eq. (9) by incorporating the effects of gravity and aerodynamic drag. Consequently, Goddard's equation accounts for additional influences such as gravitational force and aerodynamic resistance. The equation has been further improved by other scientists.

(vi) Furthermore, Newton's third law implies that  $F_{AB}$  and  $F_{BA}$  are the only significant factors governing the interaction of bodies in Eq. (1). In contrast, Eq. (16) shows that for falling and rebounding bodies, in addition to  $F_{AB}$  and  $F_{BA}$  factors such as shape, composition, asymmetry, the nature and properties of the target, and other associated parameters also become significant.

(vii) This discussion aligns with the various historical developments of scientific laws and with Mach's proposition in the 1880s and 1890s that the metaphysical nature of mechanical laws must be empirically tested.

A generalized form of a theory is considered valid only when its predictions are consistently supported by repeated experimental evidence. Newton's laws govern most everyday phenomena, but deviations appear at quantum scales, relativistic speeds, and in deformed or asymmetric interactions, requiring generalized formulations.

### 6.3 Experimental confirmation of Eq. (16)

The proposed experiments on falling and rebounding bodies will verify Eq. (16); this conclusion is based on qualitative observations. These experiments are novel and original because they have not been quantitatively documented in scientific literature for more than 340 years. The proposed experiments provide the simplest method to validate the generalized form of Newton's Third Law. Other related experimental techniques are comparatively much more complex.

Qualitatively, for a spherical body of appropriate composition that strikes a suitable target, rebounds to its original height, and retraces its initial trajectory, the coefficient  $K$  equals unity, causing Eq. (16) to become identical to Eq.

(1). If deviations from the third law are observed in experiments, then the role of  $K$  becomes critically important for interpreting the results. Then, in the next step, the value of  $K$  must be quantified and experimentally determined with precision. If a body of arbitrary shape (having the same mass and composition as the spherical body) rebounds  $\frac{1}{2} m$  under identical conditions, then  $K$  will be  $\frac{1}{2}$ . Thus,

$$\text{Reaction} = - \frac{1}{2} \text{Action} \quad (20)$$

The estimation of coefficients ( $K_{\text{shape}}$ ,  $K_{\text{composition}}$ ,  $K_{\text{target}}$ , and  $K_{\text{other}}$ ) may be viewed as broadly analogous to determining the coefficients (volume, surface, Coulomb, asymmetry, etc.) in the Bethe–Weizsäcker semi-empirical liquid-drop formula (SEMF). These coefficients are experimentally fitted parameters. Thus, the origin of the generalized form of Newton’s third law is related to existing scientific traditions of the development of the laws. However, supporting experimental results are required for the final validation of Eq. (14), as it is an extension based on qualitative observations and theoretical analysis.

#### 6.4 Area of touch or contact (AOT).

The area of touch (AOT) or contact can be defined as the actual area of the body (projectile, which may have various shapes) that directly touches, contacts, or strikes the floor. The areas of touch/contact may be identical for different bodies, e.g., sphere, long thin pipe, cone, needle-like shape, flat, irregular body, triangle, etc. The effect of AOT depends on the shape, size, symmetry, and other geometrical features of the body. The area of touch may be considered as a parameter associated with the body’s shape in the following manner.

$$K_{\text{shape}} \propto \frac{1}{\text{area of touch or contact}} \quad \text{or} \quad K_{\text{shape}} = \frac{k}{\text{area of touch}} \quad (17)$$

$k$  depends on the properties of the bodies and the related experimental conditions.

Consider a flat body and a sphere (with the same mass and composition) both dropped identically onto the same surface. In the case of the flat body, the area of touch is larger than that of the sphere, so the value of  $K_{\text{shape}}$  is smaller, as in Eq. (17). Consequently, the reaction is reduced for the flat body in Eq. (14), and thus the flat body rebounds to a lower height ( $h < 1m$ ). The current value of  $K$  should be regarded as a qualitative estimate; rigorous and repeated experimentation is necessary to establish its accurate quantitative measurement.

#### 7.0 The rebound experiments are different than those in Aerospace and Propulsion Systems.

The quantitative verification of Aerospace and Propulsion Systems (fireworks, rockets, spacecraft, missiles, and drones) is fundamentally different from Action–Reaction Motion Systems or contact-based action–reaction interactions (bouncing balls, swimming, rowing, walking, jumping, balloon deflation, and gun recoil). The former system was mainly developed in the 20<sup>th</sup> and 21<sup>st</sup> centuries due to the efforts of aeronautical engineers in various subfields, and later was the earliest system of applications of Newton’s third law of motion developed qualitatively. The rockets are guided and stabilized through computer-controlled systems, whereas fireworks travel along uncontrolled and purely ballistic trajectories.

The cruise missiles function under continuous powered flight and utilize advanced guidance technologies, such as GPS, radar, and inertial navigation systems.

The motion of drones is regulated by onboard flight-control electronics, integrated sensors, and externally commanded inputs.

Airplanes are directed through pilot inputs or automatic control systems and operate along predetermined and navigationally defined flight paths.

A glider moves in the atmosphere using aerodynamic lift and gravity, not by engines or propellers. The Airbus Perlan 2 has achieved the highest altitude ever, 76,124 feet. Gliders are typically constructed from lightweight materials and feature cambered (curved upper surface and flatter lower surface), high-aspect-ratio wings, a streamlined fuselage, and a conventional tail, which together optimize lift, minimize drag, and ensure stable flight. This design is consistent with the generalized coefficient  $K$ , as it explicitly accounts for the body's shape ( $K_{\text{shape}}$ ), material composition ( $K_{\text{composition}}$ ),  $K_{\text{other}}$ , etc. The combination of aerodynamic geometry and low-mass construction directly influences the glider's ascent in rising air and descent in still air, reflecting how variations in shape and material modify the effective action–reaction forces as expressed in Eq. (14).

Thus, diverse experiments dealing with action-reaction systems may be individually confirmed as their nature is different from Aerospace and Propulsion Systems.

## 8.0 Conclusions

Newton's Third Law is a fundamental principle of physics and underlies many applications across physics, various branches of engineering, mathematics, and related fields. Here, a specific application of the law is examined in the context of falling and rebounding bodies. Preliminary observations suggest that rebound behavior depends systematically on geometric shape, material composition, and impact conditions. The spherical bodies largely follow classical action–reaction behavior, whereas asymmetric or flat bodies show measurable deviations. Motivated by these trends, a generalized formulation of Newton's third law is proposed within its classical domain, introducing dimensionless coefficients that represent structural and material asymmetries, target properties, and other interaction parameters. The analysis identifies quantitatively unexplored experimental tests of Newton's third law, particularly for rebounding bodies of identical mass and composition but different shapes at the macroscopic level. The conclusions, therefore, emphasize the need for dedicated experimental investigations. Rigorous quantitative experiments guided by this framework are recommended to systematically test macroscopic deviations from Newton's third law, enhance conceptual understanding of rebound phenomena, and provide insights relevant to classical mechanics, aerodynamics, and propulsion.

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### References

- [1] I. Newton, *Philosophiæ Naturalis Principia Mathematica* (Jussu Societatis Regiæ ac Typis Josephi Streater, Londini, 1686/1687), available online at *Project Gutenberg* (Latin text) <https://www.gutenberg.org/files/28233/28233-h/28233-h.htm> (accessed 7 January 2026).
- [2] I B Cohen, *The Principia* (University of California Press, Berkeley, 1999) pp. 116–117.
- [3] I Newton, *Mathematical Principles of Natural Philosophy*, Vol. 1, Book I (Middle Temple Gate in Fleet Street, London, 1729) pp. 19–20.  
[https://books.google.co.in/books?id=Tm0FAAAAQAAJ&pg=PA1&redir\\_esc=y&hl=en#v=onepage&q&f=false](https://books.google.co.in/books?id=Tm0FAAAAQAAJ&pg=PA1&redir_esc=y&hl=en#v=onepage&q&f=false)
- [4] R Resnick and D Halliday, *Physics Part I* (Wiley Eastern Limited, New Delhi, 1992) pp. 87–88, 216–217.
- [5] Sharma, A., “Shape-dependent deviations from Newton’s third law: Qualitative observations and a generalized equation”, paper presented at the *IUPAP–International Conference on Physics Education (ICPE-2025)*, Indian Institute of Technology Ropar, India, Dec. 16–20, 2025. Available at <https://icpe-2025.in/report-and-abstracts/> pp. 185-186.
- [6] Sharma, A., “Roles of shape, composition, and target interactions in rebounding bodies, gun recoil, and elastic collisions: Toward a generalization of Newton’s third law,” paper presented at the *IUPAP–International Conference on Physics Education (ICPE-2025)*, Indian Institute of Technology Ropar, India, Dec. 16–20, 2025. Available at <https://icpe-2025.in/report-and-abstracts/> pp. 187-188.
- [7] K Ishimoto, *Odd elastohydrodynamics: Non-reciprocal living material in a viscous fluid*, **PRX Life** 1, 023002 (2023), doi:10.1103/PRXLife.1.023002.
- [8] A. A. Blagonravov (Ed), *Collected Works of K. E. Tsiolkovsky*, section “K. E. Tsiolkovsky: The Founder of Modern Dynamics” (Stvo Akademii Nauk SSSR, Moscow, 1954) pp. 1–10.  
<http://epizodsspace.airbase.ru/bibl/inostr-yazyki/tsiolkovskii/tsiolkovskii-nhedy-t2-1954.pdf>
- [9] R Descartes, *Principia Philosophiæ* (Ludovicus Elzevir, Amsterdam, 1644).
- [10] L Euler, *Mechanica*, Vol. 1, Chap. 2 (1735) pp. 63–64 (E15).  
<https://www.17centurymaths.com/contents/mechanical1.htm>
- [11] L Euler, *Nova methodus motum corporum rigidorum degerminandi* (E479) (1775) pp. 223–224.  
<https://scholarlycommons.pacific.edu/cgi/viewcontent.cgi?article=1478&context=euler-works>
- [12] IOP (Institute of Physics, England), *The development of Newton’s first law* (2024).  
<https://spark.iop.org/development-newtons-first-law> (2024)
- [13] “Standard gravity,” *Wikipedia*,  
[https://en.wikipedia.org/wiki/Standard\\_gravity](https://en.wikipedia.org/wiki/Standard_gravity) (accessed 7 January 2026).
- [14] R. H. Goddard, *A method of reaching extreme altitudes*, **Smithsonian Miscellaneous Collections** 71(2) (1919).  
[Smithsonian miscellaneous collections](#)

- [15] A Sharma, *The genuine effects of the shape of the body in Newton's third law of motion lead to its generalization*, **Science Talks** 10 (2024) 100334, doi:10.1016/j.sctalk.2024.100334.  
<https://doi.org/10.1016/j.sctalk.2024.100334>=  
<https://www.sciencedirect.com/science/article/pii/S2772569324000422?via%3Dihub>
- [16] A Sharma, *Shape-dependent form of Newton's third law of motion*, *Journal of Physics Education* 38 (2024).  
<https://physedn.in/jpe/article/view/16/12>
- [17] A Sharma, *The Principia's third law of motion: In original and generalized forms*, *Physics Essays* 29(1), 23–26 (2016),  
 doi:10.4006/0836-1398-29.1.23.
- [18] A Sharma, *Role of characteristics of bodies in Newton's third law of motion*, in *SM18 Program Book* (American Association of Physics Teachers, Washington DC, 2018) pp. 150–151.  
[https://aapt.org/Conferences/sm2018/upload/SM18Program\\_Final\\_A.pdf](https://aapt.org/Conferences/sm2018/upload/SM18Program_Final_A.pdf)
- [19] A Sharma, Abstract in *Proceedings of the 38th Annual Convention of the Indian Association of Physics Teachers* (16–18 October 2024).
- [20] A Sharma, *Elastic collisions in one dimension at the macroscopic level and Newton's third law of motion*, **Acta Ciencia Indica** XXV(P), 113–116 (1999).
- [21] A Sharma, *Generalization of Newton's 340-year-old third law: An open challenge for experimental verification using low-cost methods*, **Journal of General Science** (2025).  
<https://www.gsjournal.net/Science-Journals/Research%20Papers-Mechanics%20/%20Electrodynamics/Download/10360>