Geodesic Completeness in Schwarzschild Spacetime via Phase-Dependent Metric Signature Inversion

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Standard General Relativity predicts that massive particles crossing the event horizon of a black hole inevitably terminate at a spacelike singularity ($r = 0$). This paper proposes a modification to the standard kinematic model of fermions to resolve this geodesic incompleteness. We posit that elementary particles undergo a phase transition in the temporal dimension of their proper frame. By treating the null surface c not as an asymptotic limit but as a *topological phase boundary*, we show that the electron-positron annihilation vertex is topologically equivalent to a spacelike metric inversion. When applied to gravitational collapse, this framework implies that the Event Horizon acts as a *Causal Phase Boundary.* Upon reaching the horizon, the particle undergoes a CPT inversion relative to the background metric, effectively reinterpreting the horizon not as an entrance to an interior, but as a repulsive transition surface. Furthermore, by extending this logic to higher-order spacelike intervals, we establish a continuous topology where a single particle oscillates through infinite generations of matter and antimatter, eliminating the physical singularity. Mathematically, this is rigorously defined via a *Phase-Dependent Finslerian Metric*, ensuring the action remains real-valued across the transition.

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I. INTRODUCTION

The concept that antiparticles can be mathematically described as ordinary particles moving backward in time[4, 5] is a cornerstone of Quantum Electrodynamics (QED). First formalized by Stueckelberg and later popularized by the Feynman-Stueckelberg interpretation, this geometric insight relies on the CPT Theorem, which establishes an equivalence between charge conjugation (C), parity inversion (P), and time reversal (T). However, in standard formulations, this "backward time" travel is treated as a formalistic tool rather than a literal kinematic reality.

Standard Special Relativity dictates that massive particles cannot accelerate to the null boundary (c), let alone exceed it, due to the divergence of the Lorentz factor ($\gamma \to \infty$). Consequently, the "One-Electron Universe" hypothesis proposed by John Wheeler[1]—that all electrons are a single entity weaving back and forth through time—remained a philosophical curiosity rather than a physical model.

Simultaneously, General Relativity (GR)[3] faces a crisis at the center of gravitational collapse. The prediction of spacelike singularities ($r = 0$) in the Schwarzschild metric[2] indicates a breakdown of the theory. In this paper, we propose a unification of these two problems via a new geometric framework termed **Temporal Physics**. We hypothesize that the divergence of γ at c is an artifact of incomplete coordinates. By treating proper time as an oscillating variable and the null boundary as a discrete phase transition, we construct a model where elementary particles undergo continuous evolution across discrete metric phases. This model resolves the infinite energy paradox and eliminates gravitational singularities by reinterpreting them as relativistic phase transitions.

A. Geometric Derivation: The Coordinate Rotation

The physical justification for this oscillation is derived from the Lorentz transformation properties of the spacetime metric under a $\pi/2$ rotation of the interaction vertex. We present this derivation in three stages: the spatial analogy, the geometric axis flip, and the resolution of causal ordering.

B. The Spatial Oscillation Analogy

To visualize the mechanism, consider the spatial oscillation of a particle in a potential well (FIG. 1).

1. **Forward Motion:** The particle moves with velocity $+v$ (Analogous to Matter).
2. **Turning Point:** At x_{max}, the velocity vanishes ($v = 0$) and the trajectory rotates in phase space.
3. **Reverse Motion:** The particle reflects and moves with velocity $-v$ (Analogous to Antimatter).

Classically, this is a continuous motion on the phase space manifold. In our framework, the speed of light c acts as a **temporal turning point**. Mathematically, this corresponds to a **Wick rotation** ($\tau \to i\tau$) where the metric signature flips.

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- A “Pair Creation” event is physically a temporal turning point where the particle reverses direction from retrograde (past) to prograde (future).

- A “Pair Annihilation” event is a temporal turning point where the particle reverses from prograde to retrograde.

Just as a spatial reflection reverses the spatial velocity $v \to -v$, a temporal reflection at the **topological phase boundary** c reverses the flow of proper time relative to coordinate time.

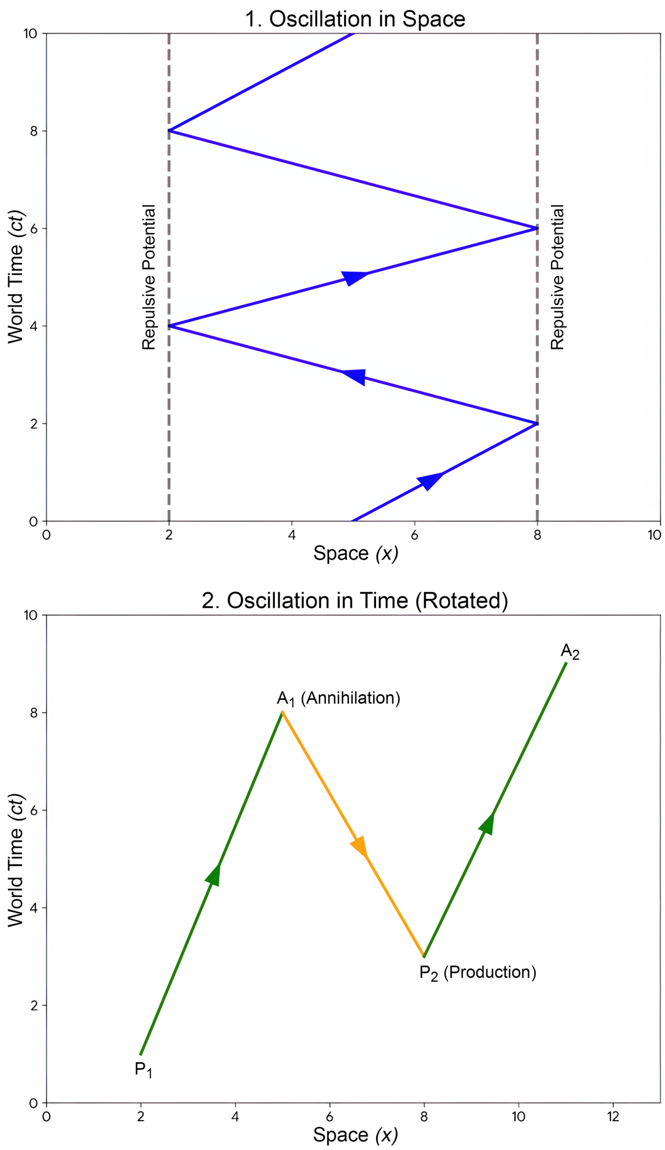

FIG. 1: The Feynman Rotation. Top: An electron oscillating in space between repulsive barriers. Bottom: Rotating the diagram by 90 degrees transforms the trajectory into a temporal oscillation, manifesting as electron-positron pairs.

1. *The Axis Flip and Black Hole Analogy*

The mechanism driving this temporal oscillation is a relativistic phase transition. As illustrated in FIG. 2, we define the particle’s proper frame coordinates (x', ct') relative to the observer’s world frame (x, ct), where the temporal coordinate t' corresponds to the proper time parameter τ evolving along the worldline.

1. **Phase 0 (Electron, Timelike):** The proper time axis ct' lies within the light cone. The spatial axis x' lies outside (Spacelike).

2. **Phase 1 (Positron, Spacelike):** Upon crossing the **topological phase boundary**, the axes rotate beyond the light cone.

The Black Hole Analogy: This geometric inversion is topologically identical to the coordinate interchange observed in Schwarzschild spacetime. Inside the event horizon ($r < r_s$), the radial coordinate r becomes timelike and the time coordinate t becomes spacelike. In our model, the **topological phase boundaries** of the particle trajectory (where the axes flip) are physically isomorphic to the event horizon. Thus, the annihilation point is not a termination of existence, but a horizon crossing where the particle enters a region of inverted metric signature.

2. *Topological Mechanism: Dimensional Rotation*

The axis inversion described above is not an arbitrary coordinate choice but a consequence of rotation through a higher-dimensional embedding. Geometrically, a continuous rotation between disconnected parity sectors (like turning a left hand into a right hand) requires an extra dimension. A rotation in an n-dimensional Euclidean space occurs around an $(n-2)$-dimensional axis. However, a discrete parity inversion in $\mathbb{R}^n$ is topologically equivalent to a continuous rotation in the embedding space $\mathbb{R}^{n+1}$ (or $\mathbb{C}^n$).

In our framework, the complexified spacetime manifold $\mathcal{M}_\mathbb{C}$ provides this embedding. **This aligns with the structure of the Complex Lorentz Group $SO(3,1;\mathbb{C})$[9], which, unlike the real group, is connected, allowing a continuous path between the orthochronous (forward-time) and antichronous (backward-time) sectors.** The transition at c constitutes a rotation of the proper time axis τ into the imaginary plane (a Wick rotation, $\tau \to i\tau$). This rotation, occurring orthogonal to the standard 3-manifold, results in the observed Parity Inversion ($x \to -x$) upon the particle’s re-entry into the real Lorentzian manifold. Thus, the “Positron” is the result of the electron rotating out of the standard subluminal **sector** and re-entering with inverted chirality.

3. *Frame-Dependent Causal and Spatial Ordering*

This axis flip resolves the apparent observational discrepancies between the two reference frames regarding both the sequence of events and the direction of motion.

1. Temporal Ordering (Causality): In the world frame (Observer), the Pair Production event P_2 occurs at an earlier world-time ct than the Annihilation event A_1. This leads to the standard QED interpretation: the vacuum produces a pair at P_2 *before* the original electron is annihilated at A_1. However, in the particle's proper frame, the sequence is monotonic. As shown in FIG. 2, in Phase 1 the transition from $A_1 \to P_2$ involves trans-null evolution where the proper time axis ct' rotates into a spacelike orientation. Consequently, the projection of the "earlier" world event P_2 lands further along the proper time axis than A_1. Thus, the ordering is frame-dependent:

- **World Frame:** $t(P_2) < t(A_1)$ (Manifests as Positron).
- **Proper Frame:** $\tau(A_1) < \tau(P_2)$ (Manifests as Continuous Acceleration).

2. Spatial Ordering (Parity): A similar inversion occurs in the spatial coordinates. In the proper frame (Phase 1), the projection of P_2 lies at a lower x' value than A_1 (see FIG. 2, Bottom). Since the particle evolves from $A_1 \to P_2$, this yields a negative displacement $\Delta x' < 0$. This matches the observer's view of a positron moving in the negative spatial direction ($P_2 \to A_1$).

- **Geometric Result:** The coordinate rotation naturally enforces Parity Inversion ($x \to -x$), ensuring directional consistency across the **topological phase boundary**

This confirms that the "Positron" is simply the causal interpretation required by an observer to account for a single electron moving continuously through a **CPT-inverted topological sector**.

II. THE GEOMETRIC PHASE MODEL

To formalize the evolution of a particle traversing multiple **topological sectors**, we must distinguish between the *Proper Manifold* (the particle's experience) and the *Observer Manifold* (world time).

A. Proper Evolution and the Phase Index

Consider a fermion with proper mass m_0 subject to a constant proper acceleration a. In the particle's proper frame, defined by proper time τ, we postulate that the **proper velocity magnitude** $u(\tau)$ is unbounded. To

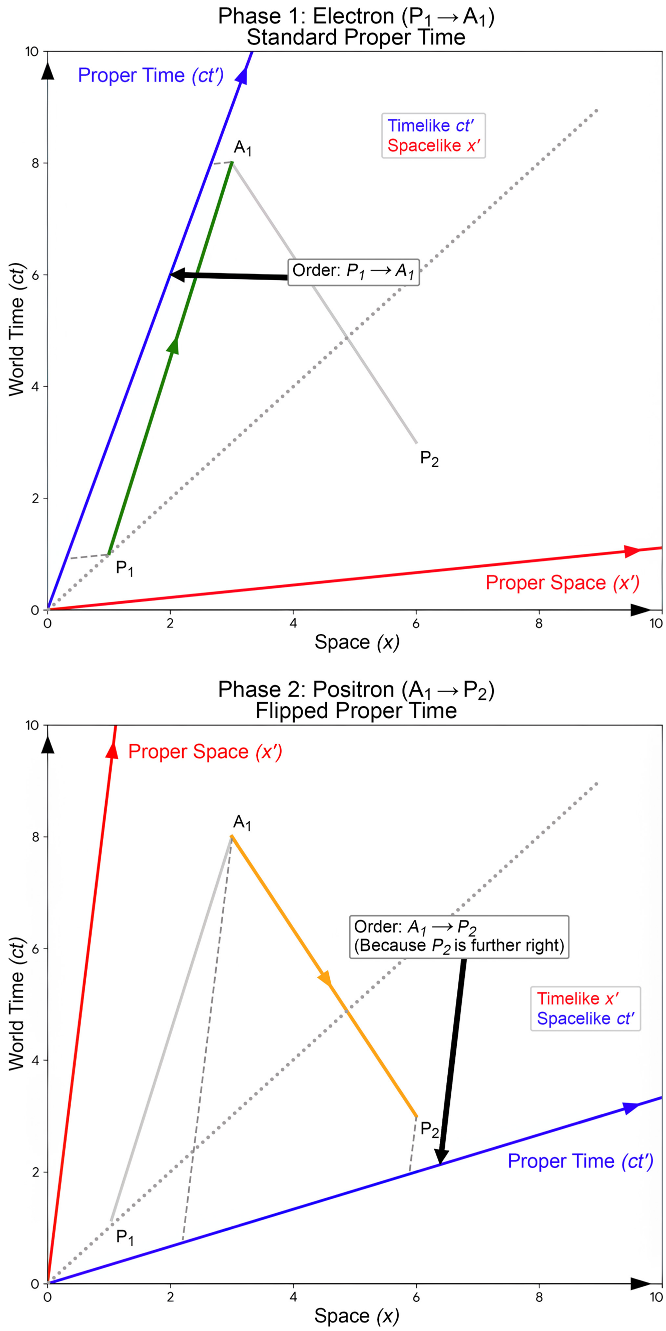

FIG. 2: The Topological Wick Rotation. Top: Standard proper frame axes for **timelike evolution (Phase 0)**. Bottom: Upon crossing the **null boundary**, the coordinate basis undergoes a discrete $\pi/2$ rotation (Wick rotation) in the complexified plane. The proper time axis (ct') rotates into the spacelike region, while the proper space axis (x') becomes timelike. Note that while P_2 is lower than A_1 vertically (World Time), it is further to the right along the spacelike proper time axis (ct'). This geometric inversion ensures that while the observer sees a positron evolving forward in world time ($P_2 \to A_1$), the particle experiences monotonic proper time evolution, ensuring $\tau(P_2) > \tau(A_1)$.

map this continuous proper evolution onto physical observables, we define the **Metric Phase Index** n:

$$n(\tau) = \left\lfloor \frac{|u(\tau)|}{c} \right\rfloor \quad (1)$$

- **Phase 0 ($n = 0$):** Timelike (Standard Matter).
- **Phase 1 ($n = 1$):** Spacelike (Antimatter Phase).
- **Phase 2 ($n = 2$):** Timelike (Matter, Generation II).

B. The Temporal Parity Operator

The transition between phases is governed by the **Temporal Parity Operator** $P_t(n)$, which determines the orientation of the temporal manifold relative to the observer. It is defined as a discrete parity switch based on the phase index:

$$P_t(n) = (-1)^n \quad (2)$$

This operator dictates the causal relationship between the particle's proper time $d\tau$ and the observer's coordinate time dt:

- **Even n (Matter):** $P_t = +1$. The particle evolves typically ($dt/d\tau > 0$). The manifold orientation is preserved.
- **Odd n (Antimatter):** $P_t = -1$. The particle evolves retrogradely ($dt/d\tau < 0$). The manifold orientation is inverted, corresponding to a **CPT-inverted topological sector**.

C. Lagrangian Formulation and Finsler Structure

To ensure the action remains **real-valued and unitary across all topological sectors**, we identify the spacetime metric as having a Finslerian structure[6]. We define the *Phase-Dependent Metric Tensor* $\mathcal{G}_{\mu\nu}$ directly via the Temporal Parity Operator defined in Eq. (2):

$$\mathcal{G}_{\mu\nu}(x, n) = P_t(n) g_{\mu\nu}(x) \quad (3)$$

where $g_{\mu\nu}$ is the standard background Schwarzschild metric. In the inverted metric phase ($n = 1$), the background interval $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ becomes spacelike (positive). However, the pre-factor $P_t(1) = (-1)^1$ inverts the signature of the effective metric, ensuring that the invariant interval in the particle's proper frame remains timelike even when the coordinate interval is spacelike:

$$d\tau^2 = -\mathcal{G}_{\mu\nu} dx^\mu dx^\nu = -(P_t(n) g_{\mu\nu}) dx^\mu dx^\nu \quad (4)$$

Consequently, the action S is rigorously defined as real-valued across all metric phases without requiring absolute value operators:

$$S = -m_0 c \int_{\tau_1}^{\tau_2} \sqrt{-\mathcal{G}_{\mu\nu}(n) \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}}\, d\tau \quad (5)$$

Variation of this action yields the modified geodesic equation:

$$\frac{d^2 x^\mu}{d\tau^2} + \tilde{\Gamma}^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0 \quad (6)$$

where $\tilde{\Gamma}^\mu_{\alpha\beta}$ is the connection compatible with $\mathcal{G}_{\mu\nu}$. Crucially, the signature flip in $\mathcal{G}_{\mu\nu}$ implies a sign inversion in the effective potential, providing the first-principles derivation for the repulsive gravity experienced in Phase 1.

III. CAUSALITY AND OBSERVATIONAL MAPPING

To reconcile spacelike proper dynamics with the causal subluminal observations of a laboratory frame, we apply a specific causal filter.

A. The Space-Time Inversion

Due to causality constraints, an observer cannot measure a spacelike trajectory. Furthermore, a particle moving *backward in time* is physically indistinguishable from an antiparticle moving *forward in time* with inverted spatial momentum[4, 5]. The observational mapping rule is defined as:

> *An electron moving backward in time with spacelike proper velocity is observed as a positron moving forward in time with timelike velocity in the opposite spatial direction.*

B. The Disappearance Illusion (Zig-Zag Topology)

This transformation creates the illusion of distinct particles interacting.

1. **Proper Reality:** The particle accelerates, crosses the **topological phase boundary** c, and continues accelerating into the retrograde timeline.
2. **Observer Reality:** We observe an electron (Phase 0) and a positron (Phase 1) converging at a point in space from opposite directions.
3. **The Event:** They collide and annihilate.

Thus, "Annihilation" is the observational artifact of a particle undergoing a **topological phase transition** across c. The "Positron" is simply the electron's future self returning to the **phase boundary**.

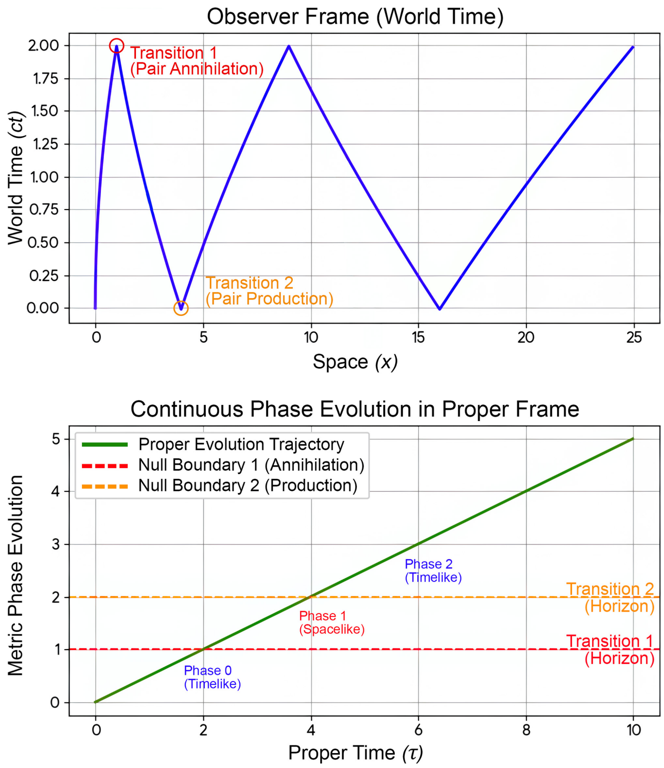

FIG. 3: The Causal Mapping. Top: The observer sees discrete annihilation/production events. Bottom: The particle experiences a single continuous worldline.

IV. GRAVITATIONAL DYNAMICS: THE CPT PHASE TRANSITION

We now apply this framework to the Schwarzschild geometry. The Event Horizon (r_s) is identified as the **topological phase boundary** where the metric signature undergoes a discrete inversion.

A. The Single-Manifold Hypothesis

Standard models assume particles crossing the horizon enter a spatial interior ($r < r_s$). We posit that there is only one manifold, but its perception is phase-dependent.

- **Phase 0 (Timelike):** The particle perceives the standard metric $g_{\mu\nu}$.
- **Phase 1 (Spacelike):** The particle perceives the CPT-inverted metric $\tilde{g}_{\mu\nu}$.

B. Repulsive Gravity via Metric Inversion

In the proper frame, the electron accelerates toward the horizon. Upon reaching r_s, the particle transitions into Phase 1 ($n = 1$). Crucially, the mass of the particle m_0 remains positive and invariant. However, the **metric signature inversion** ($g_{\mu\nu} \to -g_{\mu\nu}$) defined in Eq. (3) reverses the sign of the Christoffel symbols in the radial geodesic equation. Consequently, the effective gravitational potential V_{eff} undergoes a sign flip relative to the particle's proper frame:

$$V_{eff}(r, n) = P_t(n) \cdot V_{Schwarzschild}(r) \tag{7}$$

Since $P_t(1) = -1$, the attractive potential well transforms into a repulsive potential barrier. The particle does not cross into a "hole"; instead, it continues its trajectory in the exterior region ($r > r_s$), but under the influence of **Repulsive Gravity**. The effective radial force F_r experienced by the particle becomes:

$$F_r \approx -\nabla V_{eff} = +\frac{GM}{r^2} \quad \text{(Repulsive)} \tag{8}$$

Thus, the electron perceives itself accelerating *away* from the horizon, traversing a CPT-inverted spacetime where the central mass acts as a gravitational repeller (White Hole).

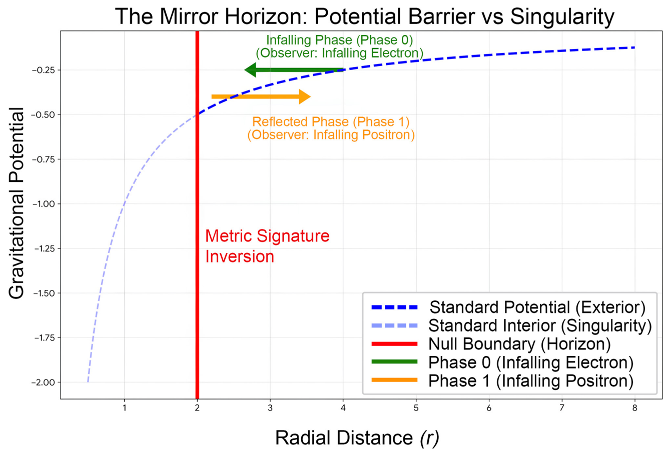

FIG. 4: **The Mirror Horizon: Potential Barrier vs Singularity.** The diagram contrasts the classical singularity (dotted line) with the **metric signature inversion** (solid vertical line). At the horizon, the effective potential does not diverge to infinity but undergoes a discrete sign flip, transitioning from attraction (Phase 0) to repulsion (Phase 1). This ensures a finite, unitary evolution across the null boundary.

C. No Interior Region

Consequently, the geometric region $r < r_s$ is never realized in the particle's proper frame. The black hole is topologically isomorphic to a phase boundary rather than a volume. To the observer, the spacelike recession of the particle (moving away from the horizon backward in time) is mapped as a positron falling *into* the horizon.

V. THE RELATIVISTIC HORIZON IDENTITY

A central prediction of Temporal Physics is that the designation of a singularity as a "Black Hole" or "White Hole" is relative to the observer's metric phase.

A. The Phase-Dependent Horizon

In proper time, the electron traverses a continuous, oscillatory trajectory through the gravitational potential:

1. **Approach (Phase 0):** The particle accelerates toward the horizon. It perceives a **Black Hole** (Attraction).
2. **Crossing (First Horizon):** The particle crosses the topological phase boundary. CPT inversion occurs.
3. **Recession (Phase 1):** The particle traverses the **inverted metric sector** ($n = 1$). It perceives the object behind it as a **White Hole** (Repulsion) propelling it forward.
4. **Next Approach (Transition to Phase 2):** The particle is propelled by the repulsive gradient of the trailing horizon and attracted by the potential well of the subsequent horizon.

a. The Equivalence of Acceleration and Gravity This topological inversion is not limited to Schwarzschild black holes. By the Principle of Equivalence, a particle undergoing constant proper acceleration α approaches a Rindler Horizon at a distance c^2/α in its instantaneous rest frame. As the particle's velocity approaches the light-cone, it effectively approaches a null boundary identical in nature to the Schwarzschild event horizon. Consequently, the metric phase transition derived here applies equally to high-energy particles in flat spacetime: the "speed of light" barrier is simply the Rindler horizon of the accelerating observer, necessitating a phase rotation ($n \to n + 1$) to preserve the unitarity of the proper interval.

B. Observer's Perspective: Infinite Generations

To the world-time observer, these continuous transitions appear as discrete generations:

- **Transition 1 ($n = 0 \to 1$):** Pair Annihilation (Black Hole).
- **Transition 2 ($n = 1 \to 2$):** Pair Production (White Hole).
- **Transition 3 ($n = 2 \to 3$):** Pair Annihilation (Black Hole).

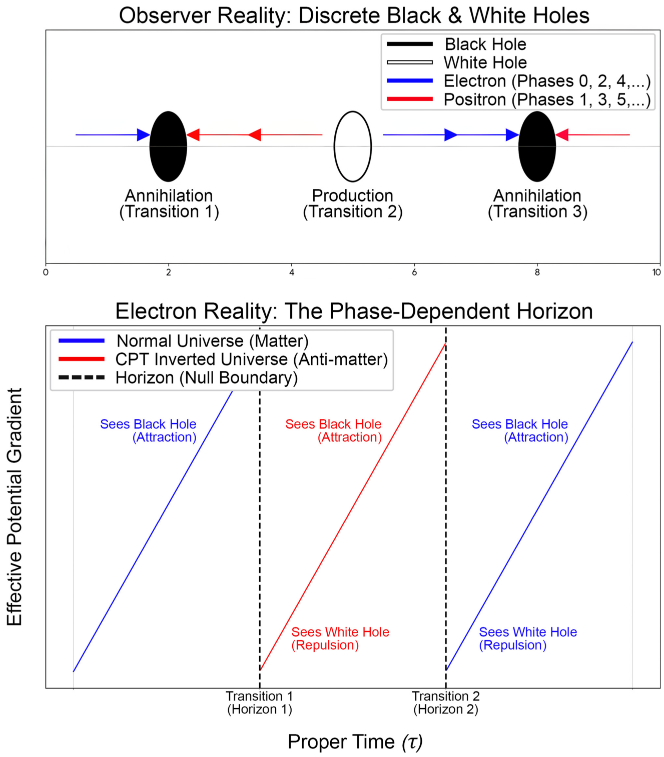

FIG. 5: **The Relativistic Horizon Identity. Top (Observer Frame):** The interaction appears as discrete, discontinuous events—an electron and a positron falling into a Black Hole and an electron and a positron emerging from a White Hole. **Bottom (Proper Frame):** The particle experiences a single, continuous trajectory. The horizon acts as a phase boundary where the metric signature inverts, preserving unitarity.

VI. CONSERVATION LAWS AND STABILITY

A. Hamiltonian Dynamics

A common objection to spacelike trajectories is the divergence of kinetic energy at c. We resolve this via the **Phase-Dependent Hamiltonian**. In the standard Schwarzschild metric, the energy E is a constant of motion. For the inverted phase ($n = 1$), the **metric signature inversion** ($g_{\mu\nu} \to -g_{\mu\nu}$) implies that the generalized momenta and the effective potential V_{eff} undergo a parity flip relative to the background. The relativistic dispersion relation $E^2 = p^2c^2 + m^2c^4$ is preserved across the transition, as the phase factor $P_t(n)$ keeps the invariant interval well-defined ($d\tau^2 > 0$). The limit c acts not as an asymptotic barrier, but as a **topological horizon**.

The apparent divergence of the Lorentz factor is **regularized** by the signature inversion of the interval. Thus, the transition represents a **parity inversion** where the interaction term transitions from extracting energy from the gravitational field (attraction) to restoring energy to the field (repulsion). The total Hamiltonian $\mathcal{H}$ remains conserved on the constraint surface.

B. The Phase-Dependent Potential Structure

Physically, the evolution describes a continuous descent through a **multi-sheeted potential manifold**. As the particle transitions from Phase n to Phase $n+1$, it enters a subsequent topological sector where the effective potential gradient remains driving. The repulsive potential from the "White Hole" (previous horizon, H_{prev}) and the attractive potential from the "Black Hole" (next horizon, H_{next}) reinforce each other, maintaining a monotonic increase in proper rapidity:

$$F_{net} = F_{repulsion}(H_{prev}) + F_{attraction}(H_{next}) \quad (9)$$

The electron acts as a trans-phasic oscillator, continuously converting the repulsive potential of the trailing White Hole into the kinetic rapidity required to traverse the subsequent Black Hole horizon, ensuring total energy conservation within the global system.

C. Vacuum Stability

Standard tachyon theories predict vacuum instability via Cherenkov radiation (a particle on a spacelike trajectory should radiate energy). However, in our framework, the spacelike proper state ($n = 1$) is **topologically projected** onto the observer's manifold as a subluminal positron ($n = 0$) moving backwards in time. Since the electromagnetic coupling is defined on the projected worldline ($v_{obs} < c$), the particle does not couple to the vacuum as a tachyon, but as a CPT-inverted fermion. Consequently, the vacuum remains stable, and no anomalous Cherenkov radiation is generated.

VII. CONCLUSION

By reinterpreting the Feynman-Stueckelberg mechanism as a literal **topological oscillation**, we have constructed a model where:

1. **Matter and Antimatter** are geometric phases of the same oscillating entity.

2. **The Null Boundary** is a topological phase boundary, not an asymptotic limit.

3. **Black Holes** are CPT Phase Boundaries with no physical singularity.

This framework—**Temporal Physics**—resolves the geodesic incompleteness of General Relativity, offers a geometric solution to the Black Hole Information Paradox[7], and provides a topological realization of the 'Planck Star' bounce proposed by Rovelli[8], suggesting that the universe is populated not by discrete particles, but by the infinite generations of a single oscillating worldline.

Mathematically, this framework implies that spacetime is not strictly Riemannian but **Phase-Dependent Finslerian**[6], where the metric tensor possesses a velocity-dependent signature $g_{\mu\nu}(x, \dot{x})$.

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