Calculation of the hyperfine splitting in the ground-state of atomic hydrogen

Hans Peter Good
Sargans, Switzerland
e-mail: hp.good at catv.rol.ch

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Abstract

In terms of the 1S–2S transition, defined as the 15-digit frequency 2 466 061 413.187 01 MHz, the hyperfine splitting in the ground-state of the neutral atom hydrogen (H) is predicted to be 1420.405 751 768 78 MHz, which should undergo experimental testing with sub-mHz accuracy for consistency. The conclusion is founded on the premise that the proton magnetic moment in Bohr magneton in hydrogen is the product of two simple algebraic expressions that include integers or fractions of integers, linking them through a basic building block to the subatomic structure of the vacuum space. The use of an alternative fine structure constant in the modeling necessitates a mass correction factor to the mass of the electron recommended by CODATA, which is established through both experimental data and a theoretical derivation.

Keywords: Relativistic quantum mechanics, hyperfine splitting, proton magnetic moment, atomic hydrogen, proton structure.

Introduction

In [1], the author outlined a technique for modeling transition energies among nS states in atomic hydrogen. The study, using the method of least squares, utilized optically measured data from two extraordinarily well known 1S–2S transitions and three not so accurate 1S–3S transitions for the analysis. Below is a summary of the result.

The electron binding energy in hydrogen associated with a metastable nS state is [2: eq.1]

$$E(nS1/2) = E(n,0,1/2) = E_D(n,1/2) \cdot \gamma_{zero} \cdot \{1 + B/n\} \quad (1)$$

E_D is the relativistic Dirac binding energy and B illustrates the dimensionless Lamb shift of the 1S state in hydrogen adjusted by [1: eq.1, 2, 3]

$$B \equiv \frac{\mu}{\mu_B} \cdot \left(-\frac{1}{2}\right) \cdot \gamma_{diamag\_shielding} \cdot (2\alpha)^2$$

with

$$\frac{\mu}{\mu_B} \equiv \frac{\mathbf{5}}{\left(\frac{\lambda_{e\_bar}}{L_2(L)} + \mathbf{3}\right)} \approx 0.023\,025 \quad ; \quad \frac{\lambda_{e\_bar}}{L_2(L)} \equiv 2^{16}\pi^{-5}$$

and

$$\gamma_{diamag\_shielding} \equiv \frac{1}{3}\left(1 + 2\sqrt{1 - (\mathbf{3}\alpha)^2}\right) \approx 1 - \frac{(3\alpha)^2}{3}$$

The speculative magnetic monopole μ/μ_B and the screening factor $\gamma_{diamag\_shielding}$ are dimensionless fundamental constants associated with the proton, while α represents the fine structure constant given within the geometrized framework by the axiomatic number constant [6: p.42]

$\alpha \equiv\ 2^{-6}\pi^{-\frac{2}{3}} \approx 0.00\,72\,843\ (\alpha^{-1} \approx 137.28)$

The magnitude of the scaling factor γ_{zero} changes depending on the atom due to the recoil effect and the unit of energy used for the measurement of the transitions. It can be linked with the precisely known 1S–2S transition $\Delta E_{1S-2S}^{meas} \equiv 2\,466\,061\,413.187\,01$ MHz, applying

$$\gamma_{zero} = \frac{\Delta E_{1S-2S}^{meas}}{X + B \cdot Y} \quad with \quad X \equiv E_D(2,1/2) - E_D(1,1/2) \quad and \quad Y \equiv \frac{E_D(2,1/2)}{2} - E_D(1,1/2) \qquad (2)$$

The experimental mass correction to the electron mass derived from the measured 1S1/2 hyperfine interval

In analogy to [2: eq.6], the hyperfine splitting of the 1S1/2 level $(\Delta E)_{1S1/2}^{hfs}$ in atomic hydrogen (H) can be calculated using

$$(\Delta E)_{1S1/2}^{hfs} = -E(1,0,1/2) \cdot \frac{16}{3} \cdot \tilde{B} \qquad (3)$$

where the energy $E(1,0,1/2)$ is defined by formula (1) and the parameter $\tilde{B}$ by [2: eq.7]

$$\tilde{B} \equiv \alpha^2 \cdot \frac{\mu_p(H)}{\mu_B} \cdot \gamma_{corr} = \alpha^2 \cdot \frac{\mu_p(H)}{\mu_e(H)} \cdot \frac{\mu_e(H)}{\mu_B} \cdot \gamma_{corr} \approx \alpha^2 \cdot \frac{\mu_p(H)}{\mu_e(H)} \cdot (1 + a_e) \cdot \gamma_{corr} \qquad (4)$$

The precise measurements of

$$(\Delta E)_{1S1/2}^{hfs\_meas} = 1420.405\,751\,768(2)\ MHz \qquad [3]$$

$$\frac{\mu_e(H)}{\mu_p(H)} = 658.210\,7063(66) \qquad [4]$$

$$\frac{\mu_e(geonium)}{\mu_B} \equiv 1 + a_e = 1.001\,159\,652\,1884(43) \qquad [5]$$

enable to estimate ${\gamma_{corr}}^{hfs}$ using formula (1), (2), (3) and (4). In terms of the 1S–2S reference transition, defined as the 15-digit frequency 2 466 061 413.187 01 MHz , the calculation straightforwardly yields ${\gamma_{corr}}^{hfs} \approx 1.003\,594\,936$.

Winkler and colleagues determined the electron-proton magnetic moment ratio $\mu_e(H)/\mu_p(H)$ by analyzing the spectrum of an atomic hydrogen MASER in a magnetic field and observing both the proton and the electron spin flip frequencies. The ratio is the most fragile aspect of formula (4), as only one literature precision value [4] is available, and it heavily relies on the theoretical framework used to interpret the energy levels of atomic hydrogen in an external magnetic field. The modeling process is difficult, and the technique is complex, requiring careful management of the experimental setting. The parameter a_e represents the unbound electron's anomalous magnetic moment, which the Dyck-Schwinberg-Dehmelt group at the University of Washington [5] measured with great precision.[1]

[1] The calculation of a_e within the geometrized framework is available in [6: p.130]. Three simple algebraic interaction terms are adequate to match the precise experimental value of a_e.

The theoretical mass correction to the electron mass obtained within the geometrized framework

The values of the natural constants h and c are precisely established by committee decision with absolute certainty due to the redefinition of the kilogram, in contrast to the electron mass m_e, which is derived from theoretical interpretations utilizing experimental data and whose true origin is still unclear. CODATA presently connects it to the Rydberg constant and the fine-structure constant [7: eq. 22]. In Penning trap experiments, the mass of the electron is determined by calculating the g-factor of the electron by bound-state QED. In every instance, it is impossible to determine the mass of the electron without a theoretical foundation. At present, the theoretical framework is free or bound-state QED and CODATA regularly supplies an adapted value with ever-higher precision for m_e that is consistent with the dominating QED, despite the theory's complexity (renormalization techniques to eliminate infinities), lack of physical clarity, and insensible mathematics. The upcoming content makes use of the CODATA values for h, c, and m_e from [8] without restating it again.

In [6], the author examined simple axioms to shed light on the origin of fundamental constants and suggested that the mass of the electron within the geometrized framework is inter-related to h and c according to [6: p.56]

$$m_{e\_geom} \equiv 2^{\frac{15}{4}} \cdot \pi^{\frac{26}{3}} \cdot h^{\frac{3}{4}} \cdot c^{-\frac{5}{4}}$$

Comparing m_e with m_{e_geom}, along with the value ${\gamma_{corr}}^{hfs}$ experimentally obtained earlier, prompts the hypothesis

$${\gamma_{corr}}^{theo} \equiv \left(\frac{m_e}{m_{e\_geom}}\right)^{\frac{3}{4}} \approx 1.003\,595\,267 \qquad (5)$$

on the theoretical mass correction factor to the electron mass, which raises the question whether the alignment of ${\gamma_{corr}}^{theo}$ with ${\gamma_{corr}}^{hfs}$ is coincidental, or if formula (5) is applicable for any unit set $\{m_e, h, c\}$. The relative difference between ${\gamma_{corr}}^{theo}$ and ${\gamma_{corr}}^{hfs}$ amounts to 3.3 parts per 10^7.

Prediction of the 1S1/2 hyperfine interval and the bound-state proton magnetic moment ratio $\mu_p(H)/\mu_B$

The Standard Model has failed to provide a reliable explanation for the magnetic moment of the proton. By relying on the theoretically predicted factor ${\gamma_{corr}}^{theo}$ and ensuring that $(\Delta E)^{hfs}_{1S1/2}$ corresponds with experimental data, the magnetic moment of the proton in Bohr magneton $\mu_p(H)/\mu_B$ can be established through numerical testing using

$$\tilde{B} \equiv \alpha^2 \cdot \frac{\mu_p(H)}{\mu_B} \cdot {\gamma_{corr}}^{theo} \qquad (6)$$

For better understanding of the complex mechanisms involved, the magnetic moment in atomic hydrogen $\mu_p(H)/\mu_B$ has been decomposed into the product $\mu_p/\mu_B(free) \times \gamma_{binding}$ consisting of two algebraic expressions. The factor $\gamma_{binding}$ serves as a convenient adjustment that conceals our lack of understanding regarding how the magnetic moment in hydrogen is genuinely generated due to the presence of the proton and the electron. Numerical modeling yields

$$\frac{\mu_p(H)^{theo}}{\mu_B} \equiv \frac{\mu_p}{\mu_B}(free) \times \gamma = \frac{\mathbf{\frac{1}{3}}}{\left(\frac{\lambda_{e_{bar}}}{L_2(L)} + \mathbf{5}\right)} \times \gamma \approx 0.001\,521\,031\,659 \quad ; \quad \frac{\lambda_{e\_bar}}{L_2(L)} \equiv 2^{16}\pi^{-5} \tag{7}$$

$$\gamma \equiv \gamma_{binding} = 1 + \frac{\mathbf{4}}{\mathbf{7}}\alpha^2 + (\mathbf{3} \cdot \mathbf{5} + \frac{\mathbf{5}}{\mathbf{9}})\alpha^4$$

Using the relations (1), (2), (3), (5), (6), and (7), the theoretical value of $(\boldsymbol{\Delta E})_{\boldsymbol{1S1/2}}^{\boldsymbol{hfs\_theo}}$ is determined to be **1420.405 751 768 78 MHz**. This falls within the acceptable error margins (2 mHz) and should undergo experimental testing with sub-mHz accuracy for consistency to fulfill the criteria of the philosopher Popper. Just as in the case of the Lamb shift [1], the energy building block $1/L_2(L)$ in the energy unit $1/\lambda_{e\_bar}$, where λ_{e_bar} corresponds to the reduced Compton wavelength of the electron, is connected to the saturation density of matter established through elastic electron scattering experiments. Likewise, the conjectural factors such as 3, 5, 1/3, 4/7, 5/9 in formula (7) are consistent with the concept that symmetries in quantum theory result in simple mathematical equations that include integers or fractions of integers. In [6: p.77, p.125], supplementary examples are provided that emphasize the databased significance of the numbers 4/7 or 5/9.

The bound-state electron-proton magnetic moment ratio $\mu_e(H)/\mu_p(H)$

Employing $\mu_e(H)/\mu_B \approx 1 + a_e$ to compute ${\gamma_{corr}}^{hfs}$ is inconsistent with the assumption that the hydrogen electron represents a Dirac particle[2] with $g_e = 2$, meaning that $\mu_e(H) = \mu_B$ [2: eq.6]. The multiplicative factor $\mu_e(H)/\mu_B$ to convert the experimental value $\mu_p(H)/\mu_e(H)$ to $\mu_p(H)/\mu_B$ likely carries a different significance, considering that the electron-proton magnetic-moment ratio in hydrogen

$$\frac{\mu_e(H)}{\mu_p(H)} = \frac{\mu_B \cdot [1 + a_e]}{\mu_p(H)^{theo}} \approx 658.210\,923 \tag{8}$$

is beyond the error limits of the experimental value $\mu_e(H)/\mu_p(H) = 658.210\,706(7)$ [4: Table III] measured in an applied magnetic field by Winkler et al. (1972) at MIT.[3] Interestingly,

$$\frac{\boldsymbol{\mu_e(H)}}{\boldsymbol{\mu_p(H)}}(\boldsymbol{theo}) = \frac{\mu_B \cdot [1 + \alpha/2\pi]}{\mu_p(H)^{theo}} \approx \mathbf{658.210\,710\,905} \tag{9}$$

fits within the permitted deviation limits, raising the question of whether formula (9) is an accidental finding or it indicates that the electron in the atomic hydrogen MASER is coupled to the applied magnetic field. Relations (8) and (9) are not the result of hypothesis (7), but rather arise

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[2] The existence of two spin states in Dirac's quantum relativistic wave equation is a key result of merging special relativity and quantum mechanics. Assumptions on the electron spin are irrelevant.

[3] The authors modified the measured value $\mu_e(H)/\mu_p(H)$, taking into account relativistic (Breit term) and radiative interactions of the proton and the electron, using the Grotch-Hegstrom theory, to derive the free space value μ_e/μ_p [4: eq.97]. As the correction terms for the proton and the electron are nearly equal, they largely cancel each other out in the calculation [8: Table XXXI, D41] of the ratio. By employing $\mu_e/\mu_B \equiv (1+a_e)$ the authors indirectly obtained the magnetic moment of the free proton in terms of the Bohr magneton $\mu_p/\mu_B = 0.001\,521\,032\,181(15)$ [4: eq.100]. Since the publication by Winkler et al. CODATA involves the same approach, taking $\mu_e(H)/\mu_p(H)$ as an input datum [8: Table XXV, D41]. In summary, establishing μ_p/μ_B from $\mu_e(H)/\mu_p(H)$ requires substantial theoretical input, that is, untested bound-state QED. The CODATA 2022 value for μ_p/μ_B is 0.001 521 032 202 30(45).

from hypothesis (5) through the application of formulae (6) and (3), as $(\Delta E)_{1S1/2}^{hfs\_meas}$ is a very accurately measured quantity. In essence, relation (9) serves as a theoretical framework for interpreting the measurement conducted by Winkler et al., a clarification that CODATA cannot offer due to circular reasoning, given that the measured value $\mu_e(H)/\mu_p(H)$ is an input datum for calculating the free space (vacuum space) magnetic moment of the proton by means of two theoretical correction factors regarded as exact. To strengthen hypothesis (9), further measurement data for $\mu_e(H)/\mu_p(H)$ is necessary.

Direct Measurement of the Proton Magnetic Moment ratio μ_p/μ_N

The weighted mean of μ_p/μ_N (=g_p/2) recorded by CODATA in 1973, 1986, 1998, 2002, 2006, and 2010, all derived from the indirect measurement by Winkler et al., amounts to 2.792 847 352 (12). The value 2.792 847 3446(8) obtained from recent direct measurements by Schneider et al. [9] shows a minor absolute deviation from the indirect measurement, yet it possesses significantly improved accuracy. These authors directly measured the unshielded (free space) magnetic moment in nuclear magneton of a single proton suspended in a Penning trap needing no theoretical corrections due to atomic binding. This suggests that the two theoretical bound-state QED correction factors[4] employed by CODATA and Winkler et al. in deriving the free space value μ_p/μ_N must be highly precise, or the measurements are aligned within the deviation limits of the weighted mean of μ_p/μ_N recommended by CODATA in the years 1973 until 2010.[5]

Based on relation (7), that is, incorporating the definition

$$\frac{\mu_p(free)}{\mu_B} \equiv \frac{\mathbf{\frac{1}{3}}}{\left(\frac{\lambda_{e_{bar}}}{L_2(L)} + \mathbf{5}\right)} \approx \mathbf{0.001\,520\,985\,475} \tag{10}$$

along with m_p/m_e [8], the free proton magnetic moment ratio $\mu_p(free)/\mu_N$ is ≈2.792 762.

By using m_p/m_e [8] and the shielding constant σ_{proton} ≈ 17.7354 ppm for the diamagnetic effect of the orbital electron [8: Table XXIV, 10: Table II] to calculate the free space value μ_p/μ_N based on $\mu_p(H)^{theo}/\mu_B$, the outcome is ≈2.792 896.

Both approaches reflect an enormous discrepancy (≈−31 or ≈17 ppm) in relation to the findings of Schneider and colleagues. As a result, it is impossible to ascertain whether $\mu_p(free)/\mu_B$ or $\mu_p(H)^{theo}/\mu_B/(1-\alpha^2/3)$ ≈ **0.001 521 058 562** indicates the magnetic moment of the unbound proton in terms of the Bohr magneton.[6]

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4 The relevant factors are: $\frac{\mu_p(H)}{\mu_p}(theo) \equiv 1 - \sigma_p$, $\frac{\mu_e(H)}{\mu_e}(theo)$, $\frac{\mu_e}{\mu_B}(exp) \equiv 1 + a_e$, $\frac{m_p}{m_e}(exp)$

5 The author found no substantial explanations or compelling arguments in the scientific literature to help understand why the indirect method, which includes two theoretical shielding constants around 18 ppm [8: Table XXIV] and two experimental ratios, is unequivocally comparable to the direct method, apart from issues related to accuracy. This implies that CODATA is defining the proton magnetic moment ratio μ_p/μ_N with ever-higher precision without solid scientific evidence.

6 The term $\alpha^2/3$ (≈ 17.6869 ppm) is the result of the analytical calculation of the diamagnetic shielding constant σ_p for the spherically symmetrical 1s ground-state of atomic hydrogen, achievable with the Lamb formula and the Schrödinger approximation.

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