

# Making a Cayley Group Table With a TI84 Calculator

Timothy W. Jones

November 20, 2025

## Abstract

We quickly define permutation group  $S_3$  and then make a Cayley Table for it. We use a surprisingly simple program written on a TI84-CE calculator.

## Introduction

Birkoff and MacLane [1] introduces groups, Chapter 6, with symmetries of the square. In the problem set for this section 6.1, they ask readers to emulate the ideas using the symmetries of a triangle.

In both cases vertices can be labeled with numbers and the symmetries (same shapes after rotations and reflections) can be given by permutations of these vertex labels. The resulting groups are  $S_3$ , for the triangle, and  $S_4$  for the square.

One doesn't need to create a triangle to get the idea. The idea of permuting (123) to (231) is given by a rotation: 1 to 2; 2 to 3; and 3 to 1. All the 3! permutations correspond to the *symmetries* of the triangle.

One can immediately understand that the possible permutations of three books on a shelf are such that any permutation can be combined with any other permutation to yield a third permutation. From (123) I use (231) to arrive at (231), a rotation. I can also then subject this result to a flip, say (132) and arrive at (213); another flip this time (213) returns the original, identity order: (123). Such operations are then closed, have an identity, and each book arrangement has an inverse that returns the book order to the original, the identity. Such things are called groups. There are 36 such calculations in this  $S_3$  group.

We can use the matrix and programming features of the TI84 calculator to make a *multiplication* table of the  $6 \times 6$  products. Such a table in the context of abstract algebra are called Cayley Tables.

## A Cayley Table for $S_3$

As the permutations are three digit numbers, we can use the trick of multiplying a number by 100, then 10, and then 1 and then adding them up. So if the permutation is (123), we can generate this with  $100*1+10*2+3 = 123$ . First store all six permutations in matrix [A]; see Line 003 in Figure 1, Left. Then, its a bit of trick, use the I and J variables in for loops to index into this matrix and form the product table, Lines 004 - 009, Figure 1, Right.

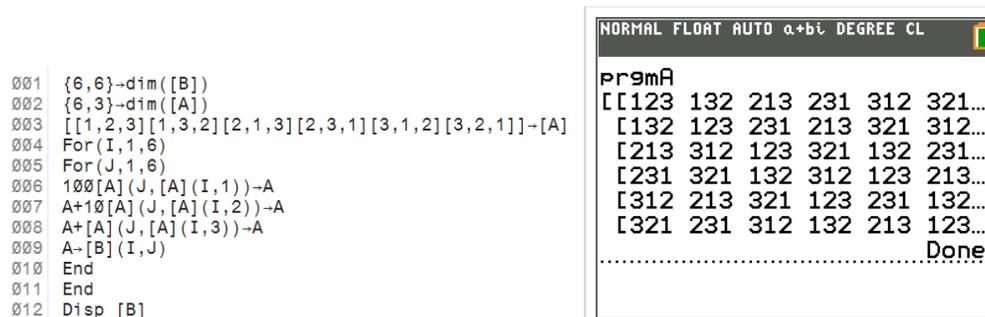


Figure 1: Left: Code for generating  $S_3$  Cayley table; Right, Output of the code.

Let's try a couple of checks. How about  $(312) \times (231)$ . Our mental algorithm goes like this: 1 goes to 3 and then 3 goes to 1; write (1; next 2 goes to 1 and 1 goes 2; continue writing for (12; finally, 3 goes to 2 and 2 goes to 3; write (123). We find this on the table. How about  $(231) \times (132)$ ? Our mental algorithm gives (321) and the table confirms this.

## Conclusion

Try modifying the code and making a Cayley table for  $S_4$ . Congratulations you have been introduced to the world of abstract groups.

## References

- [1] Birkoff, G., MacLane, S. (1977). *A Survey of Modern Algebra*, 4th ed. New York: Macmillan.