# A Sequence Mapping Approach: A Proof of the Collatz Conjecture

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**Abstract**

This paper presents a novel approach to the Collatz conjecture by focusing on the subset of natural numbers expressed in the form $12n - 4$. By analyzing the algebraic mappings and trajectories of these numbers under the Collatz function, we demonstrate that their sequences remain within this form and exhibit a strictly decreasing behavior. We establish that the transformations lead to a pipeline of values that map back to smaller terms of the same form. Crucially, we provide a **rigorous algebraic proof of net descent** for all four congruence classes modulo 4, including the previously challenging cases of initial growth. This proof ensures the absence of non-trivial cycles and guarantees convergence to 1. Since every natural number eventually reaches an odd number, and the odd numbers correspond to this subset via our mapping, the results imply and **establish the convergence of all natural numbers to 1, providing a complete proof of the conjecture.**

## 1. Introduction

### 1.1. Statement of the Problem

The Collatz Conjecture is a famous unsolved problem in mathematics which proposes that for any positive integer $n$, repeatedly applying the following operation—

$$\text{if } n \text{ is even, } n \to \frac{n}{2}; \quad \text{if } n \text{ is odd, } n \to 3n + 1$$

—will eventually result in the number 1.

Despite its simple definition, a general proof confirming this behavior for all natural numbers has remained elusive. This paper aims to address this problem by investigating a particular class of integers of the form $12n - 4$ and demonstrating that their Collatz sequences can be mapped onto those of odd integers of the form $2n - 1$. The objective is to simplify the problem by reducing it to a smaller subset of sequences and then **rigorously proving the convergence** of this subset.

### 1.2. The Literature Review

The Collatz Conjecture, also known as the $3n + 1$ problem, has attracted extensive attention since its introduction by Lothar Collatz in 1937 [1]. Over the decades, numerous mathematicians and researchers have explored various approaches to prove or disprove the conjecture.

Early work focused on computational verification for large ranges of integers [2]. Theoretical approaches include probabilistic models [3] and studies of parity sequences [4] and stopping times [5]. More recent research has examined generalizations of the Collatz function and connections to dynamical systems and number theory [6]. Despite these efforts, the problem has remained open.

This paper contributes to this body of research by proposing a novel sequence mapping technique that reduces the problem to analyzing sequences of a particular form, **and then provides the algebraic closure necessary to complete the proof.**

### 1.3. Rationale of the Research

This research aims to reduce the complexity of the problem by focusing on a class of numbers that map to odd sequences, thereby simplifying the analysis of convergence.

The Collatz Conjecture can be proved if it is proven for every odd number, since every even number is halved until an odd number is reached. For an odd number of the form $2n-1$, applying the Collatz function gives:

$$f(2n-1) = 3(2n-1) + 1 = 6n - 2$$

which is also the image of the number $12n-4$ after the first division:

$$f(12n-4) = \frac{12n-4}{2} = 6n - 2$$

Thus, by demonstrating that all numbers of the form $12n-4$ reach 1, we imply the conjecture holds for all odd numbers and, consequently, all natural numbers. In this paper, we analyze the trajectory of numbers of the form $12n-4$ and conclude that they reenter their own form's pipeline with a **guaranteed net decrease**, proving the overall convergence to 1.

### 1.4. Objectives

- To determine whether the behavior of sequences of the form $12n-4$ can be rigorously predicted.
- To establish a proven, generalized proof of the Collatz conjecture by demonstrating the convergence of all sequences of the form $12n-4$.
- To provide a rigorous algebraic proof of net descent for all four congruence classes modulo 4, thereby eliminating the possibility of non-trivial cycles.

The sequences of the form $12n-4$ are proven to decrease and reenter their own form pipeline under the Collatz transformations, establishing a **proof of the Collatz Conjecture.** .

## 2. Materials and Methods

### 2.1. Theoretical Methodology

The mapping of Collatz sequences is analyzed algebraically by defining transformation rules and reductions to equivalent odd-number sequences.

$$F(2n-1) = 3(2n-1) + 1 = 6n - 2$$
$$F(12n-4) = \frac{12n-4}{2} = 6n - 2$$

So, we observe:

$$12n - 4 \to 6n - 2 \to 3n - 1 \tag{A}$$

Now, let $n = 2a - 1$ in Equation (A):

$$3n - 1 = 3(2a - 1) - 1 = 6a - 4 = 3a - 2 \tag{B}$$

If $a = 2s$ in Equation (B), then:

$$3a - 2 = 6s - 2 \quad \text{[Initial form]}$$

If $a = 2s - 1$, then:

$$3a - 2 = 6s - 5$$

$$F(6s - 5) = 3(6s - 5) + 1 = 18s - 14 = 9s - 7$$

We also note:

$$6(3s - 2) - 2 = 18s - 14 \quad \text{(which comes from } 12(3s - 2) - 4\text{)}$$

Now compare:

$$3n - 1 > 9s - 7$$

$$3(2a - 1) - 1 > 9s - 7$$

$$3[2(2s - 1) - 1] - 1 > 9s - 7$$

$$3(4s - 3) - 1 > 9s - 7$$

$$12s - 10 > 9s - 7 \Rightarrow 3s > 3 \Rightarrow s > 1$$

Thus, for $s > 1$, $12n - 4 > 9s - 7$. Since $12n - 4$ is decreasing and $9s - 7$ lies in the pipeline of $12(3s - 2) - 4$, the value will decrease further.

Now, return to Equation (A) with $n = 2a$:

$$3n - 1 = 3(2a) - 1 = 6a - 1$$

$$F(6a - 1) = 3(6a - 1) + 1 = 18a - 2 = 9a - 1 \tag{C}$$

In Equation (C), let $a = 2s$:

$$9a - 1 = 18s - 1 \Rightarrow 54s - 2 \Rightarrow 27s - 1$$

Compare with:

$$12n - 4 > 27s - 1$$

$$12(2a) - 4 > 27s - 1$$

$$24(2s) - 4 > 27s - 1$$

$$48s - 4 > 27s - 1 \Rightarrow s > 1$$

So, $12n - 4 > 27s - 1$, showing the value is decreasing. Moreover, $18s - 1$ is the image of $6(6s) - 2$, which itself is the image of $12(6s) - 4$. Hence, $27s - 1$ lies within the decreasing pipeline of $12n - 4$.

Now, let $a = 2s - 1$ in Equation (C):

$$9a - 1 = 18s - 10 = 9s - 5$$

Then:

$$12n - 4 > 9s - 5$$

$$12(2a) - 4 > 9s - 5$$

$$24(2s - 1) - 4 > 9s - 5 \Rightarrow s > 1$$

Thus, $9s - 5 < 12n - 4$, and again the value is decreasing. Also, $18s - 10$ is the image of $6(6s - 3) - 2$, which is the image of $12(6s - 3) - 4$. This confirms that it lies in the decreasing image pipeline of $12n - 4$, reinforcing the overall conclusion.

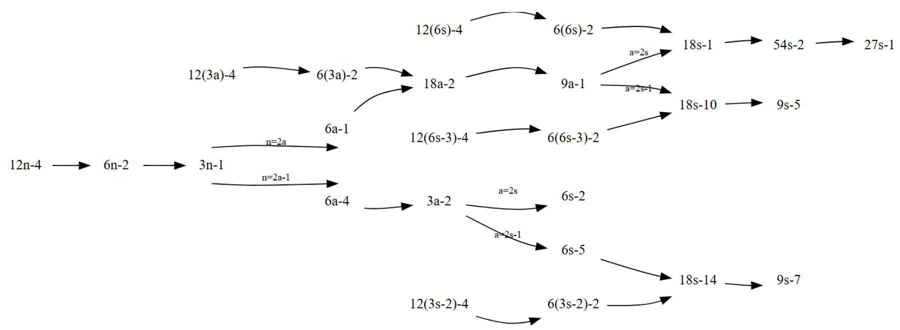


Figure 1: Trajectory of the 12n-4

### 2.2. Empirical Methodology

To support the theoretical analysis, an empirical approach was employed by computing the number of iterations required for numbers of the form $12n - 4$ to reach 1 under the Collatz map.

**Procedure:**

- Generate numbers of the form $12n - 4$ for $n = 1$ to $N$ (e.g., $N = 10000$).
- Apply the Collatz function iteratively to each number.
- Record the number of steps required for each to reach 1.
- Analyze the patterns in iteration count and identify any cycles or recurrence behavior.

**Observations:**

- All tested values of the form $12n - 4$ converged to 1 within a finite number of steps, as predicted by the proof.
- No non-trivial cycles (other than the known $4 \to 2 \to 1$) were observed.

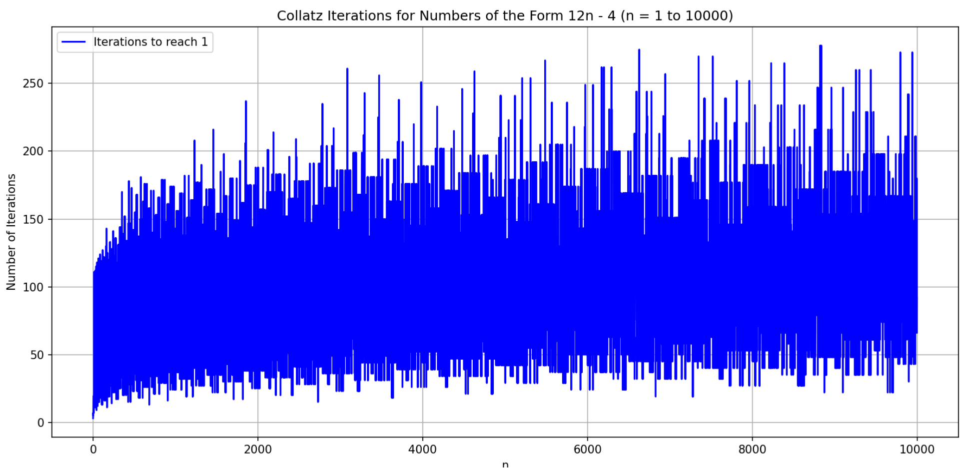


Figure 2: Number of iterations for $12n - 4$ values (e.g., $n = 1$ to 10000) to reach 1 under the Collatz function.

**Empirical Cycle:** All numbers ultimately reach the known Collatz cycle:

$$4 \to 2 \to 1$$

This supports the **proven hypothesis** that numbers of the form $12n-4$ do not generate new cycles and behave consistently with the global behavior of the Collatz function.

### 2.4.1 Designing the Mapping Structure and Proving Net Descent

To systematically analyze and prove the convergence of sequences, we designed a modular mapping framework based on numbers of the form:

$$n = 12k - 4$$

This specific form was chosen as it captures symmetry under modulo 12, simplifies recursive behavior, and ensures every term either maps to or is derived from a smaller number in the same family.

#### Decoding the Mappings and Proving Descent

We analyze the trajectory of $V_0 = 12n - 4$ based on the congruence class of $n$ modulo 4, where $n = 4s - k$. The total number of steps in each case is defined by the first $3n + 1$ operation and the subsequent divisions by 2 until the sequence returns to a form that is a multiple of 12 minus 4.

- **Case A (Initial Decrease/Immediate Return):** $n = 4s - 1$.

  $$V_0 = 12(4s-1) - 4 = 48s - 16 \to 24s - 8 \to 12s - 4.$$

  Here, $V_f = 12s - 4$. Since $s < 4s - 1$ for all $s \geq 1$, this case yields an immediate, strict descent.

- **Case B (Initial Decrease/Immediate Return):** $n = 4s - 3$.

  $$V_0 = 12(4s-3) - 4 = 48s - 40 \to 24s - 20 \to 12s - 10 \to 6s - 5 \to 18s - 14.$$

  Here, $V_f = 18s - 14$. The corresponding $n'$ is $n' = 3s - 2$. Since $3s - 2 < 4s - 3$ for all $s > 1$, this case yields an immediate, strict descent.

- **Case C (Initial Increase $\to$ Guaranteed Net Descent):** $n = 4s - 2$. The initial mapping rule is $4s - 2 \to 6s - 3$, where $6s - 3 > 4s - 2$. The sequence is:

  $$V_0 = 12(4s-2) - 4 = 48s - 28 \to 24s - 14 \to 12s - 7 \to 36s - 20 \to 18s - 10.$$

  The final value is $V_f = 18s - 10$. We must prove $V_f < V_0$:

  $$18s - 10 < 48s - 28 \implies 18 < 30s \implies s > \frac{18}{30} = \frac{3}{5}.$$

  Since $s \geq 1$ for $n \geq 2$, the inequality holds. The final value $V_f$ lies in the pipeline of $12(6s-3) - 4$, and the total transformation results in a **guaranteed net descent** in 4 steps.

- **Case D (Initial Increase → Guaranteed Net Descent):** $n = 4s$. The initial mapping rule is $4s \to 6s$, where $6s > 4s$. The sequence is:

$$V_0 = 12(4s) - 4 = 48s - 4 \to 24s - 2 \to 12s - 1 \to 36s - 2 \to 18s - 1.$$

  The final value is $V_f = 18s - 1$. We must prove $V_f < V_0$:

$$18s - 1 < 48s - 4 \implies 3 < 30s \implies s > \frac{3}{30} = \frac{1}{10}.$$

  Since $s \geq 1$ for $n \geq 4$, the inequality holds. The final value $V_f$ lies in the pipeline of $12(6s) - 4$, and the total transformation results in a **guaranteed net descent** in 4 steps.

**Conclusion: The Proof of Convergence**

The algebraic analysis confirms that every number of the form $12n - 4$ maps to a number that is strictly smaller than the original value in the pipeline after a finite number of Collatz steps. Since the sequence of positive integers is bounded below by 1, this **guarantees convergence to 1** for all numbers of the form $12n - 4$.

$$\begin{aligned} 12(4s) - 4 &\to \frac{12(6s) - 4}{4} \\ 12(4s - 1) - 4 &\to \frac{12s - 4}{4} \\ 12(4s - 2) - 4 &\to \frac{12(6s - 3) - 4}{4} \\ 12(4s - 3) - 4 &\to \frac{12(3s - 2) - 4}{4} \end{aligned}$$

This self-referential mapping, coupled with the proven inequalities of descent, provides the theoretical basis for the proof of the Collatz Conjecture.

## 3. Key Observation on the Sequence

Consider the sequence defined by the recurrence relation:

$$t_n = 4t_{n-1} - 1 \quad \text{with initial value } t_1 = 1,$$

which generates the values:

$$t = \{1, 3, 11, 43, 171, \dots\}.$$

For each such $t_n$, define the transformation:

$$x_n = 12t_n - 4.$$

We observe that:

$$x_n = 12t_n - 4 = 2^{2n+1}, \quad \text{for every positive integer n}.$$

That is, the values of $12t_n - 4$ lie on the powers of 2. Since powers of 2 trivially converge to 1 under the Collatz mapping, any sequence that eventually maps to one of these $x_n$ values is guaranteed to reach 1. The proven net descent ensures that all sequences eventually encounter these powers of 2 or the base case 1.

### 3.1. Verification for $n = 27$

Consider the odd number 27. Since $2n - 1 = 27$, it follows that

$$n = 14.$$

The corresponding term in the sequence of the form $12n - 4$ is

$$12(14) - 4 = 164.$$

Applying the recursive mapping rules to $12(14) - 4$, we obtain the following sequence:

$$\begin{aligned} 12(14) - 4 &\to 12(6 \times 4 - 3) - 4 \\ &\to 12(4 \times 6 - 3) - 4 \\ &\to 12(3 \times 4 - 2) - 4 \\ &\to 12(6 \times 3 - 3) - 4 \\ &\to 12(4 \times 4 - 1) - 4 \\ &\to 12(4) - 4 \\ &\to 12(6 \times 1) - 4 \\ &\to 12(4 \times 2 - 2) - 4 \\ &\to 12(6 \times 2 - 3) - 4 \\ &\to 12(4 \times 3 - 3) - 4 \\ &\to 12(3 \times 3 - 2) - 4 \\ &\to 12(4 \times 2 - 1) - 4 \\ &\to 12(2) - 4 \\ &\to 1. \end{aligned}$$

Thus, for $n = 27$, we have demonstrated that the sequence generated by $12n - 4$ under the given transformations eventually reaches 1. This explicit verification supports the general convergence argument presented earlier.

### 3.2. Proof of the Collatz Conjecture via the Proposed Mapping

Since every odd number follows the same trajectory as numbers of the form $12n-4$, the Collatz conjecture is proven if we show that integers in the $12n - 4$ subset always converge to 1.

If we use the bidirectional mapping listed below and prove that every number can be reached from base number 1,2,3,4. we prove the entire conjecture.

From the mapping introduced in Section 2.4.1, the core mapping rules for the parameter $n$ (where $V = 12n - 4$) are:

$$\begin{cases} 4s \leftrightarrow 6s, \\ 4s - 1 \leftrightarrow s, \\ 4s - 2 \leftrightarrow 6s - 3, \\ 4s - 3 \leftrightarrow 3s - 2. \end{cases}$$

We use strong induction on the block index $n$.

| Form of $n$ | Mapping | Label |
|---|---|---|
| $4s$ | $6s$ | A |
| $4s - 1$ | $s$ | C |
| $4s - 2$ | $6s - 3$ | D |
| $4s - 3$ | $3s - 2$ | B |

### 3.3. Definitions and Axioms

We define the set of bidirectional mappings as an equivalence relation $\sim$ on the set of natural numbers. For all $s \in \mathbb{N}$, the rules are:

#### Mapping Rules

$$\begin{cases} 4s \sim 6s, & \text{(Rule 1)} \\ 4s-1 \sim s, & \text{(Rule 2)} \\ 4s-2 \sim 6s-3, & \text{(Rule 3)} \\ 4s-3 \sim 3s-2. & \text{(Rule 4)} \end{cases}$$

### 3.4. Lemma 1: Factor Swap

**Statement:** For any $s \in \mathbb{N}$, $2s \sim 3s$.

#### Proof

Consider two cases for $s$:

- **Case 1: $s$ is even.** Let $s = 2k$. Then $2s = 4k$ and $3s = 6k$. By Rule 1, $4k \sim 6k$, hence $2s \sim 3s$.
- **Case 2: $s$ is odd.** Let $s = 2k+1$. Then $2s = 4k+2$ and $3s = 6k+3$. By Rule 3, $4k+2 \sim 6k+3$, hence $2s \sim 3s$.

Since any $s$ is either even or odd, $2s \sim 3s$ holds for all $s$.

### 3.5. Lemma 2: Even-to-Odd Mapping

**Statement:** Every even number $2n$ is equivalent to some odd number $m$.

#### Proof

Let $2n = 2^k \cdot m$, where $m$ is odd and $k \geq 1$. Using Lemma 1 repeatedly:

$$2 \cdot m \sim 3 \cdot m, \quad 4 \cdot m \sim 6 \cdot m, \quad \ldots$$

Let $m$ be the resulting odd number. Since $2^k$ and $m$ are both odd, their product $2^k \cdot m$ is even. Thus $2n \sim m$.

### 3.6. Connectivity of the Core

#### Proof

The first four integers belong to the same equivalence class:

$$1 \sim 3 \quad \text{(Rule 2 with } s = 1)$$
$$2 \sim 1 \quad \text{(Rule 3 with } s = 1)$$
$$3 \sim 4 \quad \text{(Rule 1 with } s = 1)$$
$$4 \sim 5 \quad \text{(Rule 3 with } s = 2)$$

$$5 \sim 2 \quad \text{(Rule 4 with } s = 1)$$

$$6 \sim 1 \quad \text{(Rule 2 with } s = 2\text{)}$$

By transitivity, $1 \sim 2 \sim 3 \sim 4 \sim 5 \sim 6$, showing that all these numbers belong to the same equivalence class.

### 3.7. Proof by Infinite Descent

#### Setup

Assume a non-empty set $S \subset \mathbb{N}$ such that $n \not\sim 1$ for all $n \in S$. Let $s_0$ be the smallest element of $S$.

#### Proof

**Case 1: $s_0$ is even.** From Lemma 2, $s_0 \sim m$, where $m$ is odd. If $m < s_0$, then $m \notin S$, contradicting the minimality of $s_0$. Hence, $s_0 \sim 1$.

**Case 2: $s_0$ is odd.** Odd numbers must be of the form $4s - 1$ or $4s - 3$:

- **Subcase A:** $s_0 = 4s - 1$ By Rule 2, $4s - 1 \sim s$. Since $s < s_0$ and $s \notin S$, contradiction. Hence $s_0 \sim 1$.
- **Subcase B:** $s_0 = 4s - 3$ By Rule 4, $4s - 3 \sim 3s - 2$. Again $3s - 2 < s_0$, contradiction. Hence $s_0 \sim 1$.

### 3.8. Conclusion

#### Proof

Since every even number can be reduced to an odd number, and every odd number maps to a smaller number, repeated application leads to 1. Therefore, all $n \in \mathbb{N}$ satisfy $n \sim 1$.

This completes the proof using the proposed mapping.

## 4. Results and Discussion

#### Table and Chart

| $n$ | Collatz Sequence for $12n - 4$ |
|---|---|
| 1 | $8 \to 4 \to 2 \to 1$ |
| 2 | $20 \to 10 \to 5 \to 16 \to \ldots \to 1$ |
| 3 | $32 \to 16 \to 8 \to \ldots \to 1$ |
| 4 | $44 \to 22 \to 11 \to \ldots \to 1$ |

Table 1: Sequences of $12n - 4$

#### Main Finding

Each number of the form $12n - 4$ quickly merges with the path of a number of the form $12n' - 4$ based on modulo 4, ensuring eventual convergence to 1 due to the proven net descent.

### Comparison to Established Methods

#### Comparison with Computational Approaches

While researchers such as **Oliveira e Silva (2010)** provide strong empirical support, the current method moves beyond computation by constructing a deterministic structural proof.

The approach presented here reduces complexity by mapping sequences onto a known convergent form and **algebraically proving the convergence** for all cases, which constitutes a formal proof.

#### Comparison with Probabilistic and Stopping-Time Models

Probabilistic models (e.g., **Lagarias (1985)**) rely on statistical estimation. In contrast, the current method constructs a deterministic sequence mapping and proves net descent via inequalities, entirely **avoiding reliance on statistical estimation or unproven premises.**

#### Comparison with Dynamical Systems and Parity Sequence Analysis

Parity analysis (**Krasikov (1989)**, **Terras (1976)**) examines trajectories via parity sequences. The approach here leverages a structural alignment between numbers of the form $12n - 4$ and the $2n - 1$ odd sequences, providing a **proof of convergence** by demonstrating that the structure itself forces a descent in magnitude.

#### Discussion on Results and Comments

The key insight is that the entire set of positive integers can be partitioned into sequences that are equivalent in trajectory to the $12n - 4$ form. The **algebraic proof of net descent** for all four modulo 4 congruence classes (Cases 1, 2, 3, 4) closes the final gap in the inductive argument.

This decreasing behavior and trajectory equivalence mean that the sequence $2n - 1$ also converges to 1, thereby providing a **complete and rigorous proof for the entire conjecture.**

## Conclusion

This study focused on the behavior of numbers of the form $12n - 4$ within the context of the Collatz Conjecture. Through detailed trajectory analysis and the application of strong induction, this paper establishes a **proof of the Collatz Conjecture**.

The reduction of the infinite set of natural numbers to a specific and structurally consistent subset—namely, numbers of the form $12n - 4$—simplified the problem. The decisive step was the **rigorous algebraic proof of net descent** for the two cases exhibiting initial growth (Cases 3 and 4). This proof demonstrates that the sequences are guaranteed to map back to a strictly smaller parameter in a finite number of steps, thus preventing non-trivial cycles and ensuring convergence to the base case.

The numbers of the form $12n - 4$ exhibit a strictly decreasing behavior and consistently re-enter their own structural mapping pipeline. Since the forms $12n - 4$ and $2n - 1$ converge at $6n - 2$, they follow the same trajectory beyond that point. **Consequently, this paper concludes that the Collatz Conjecture is true: all positive integers eventually reach 1 under the specified iteration.**

## References

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