Theoretical Derivation of Coulomb's Law Based on 5D Space. Screening and Renormalization of Coulomb Interaction at Small Distances Between Charges

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Abstract

This work considers a new model of elementary particle interactions based on representing space as an elastic medium with variable density. It is assumed that each elementary particle is a region of local compression or rarefaction of this medium, characterized by a density function $\rho(\mathbf{r})$. The interaction between such density clusters is described as a result of superposition of their individual metric deformations, leading to a change in the total space density and the emergence of interaction energy.

Methodologically, the derivation is conducted through the convolution of the density functions of two compressed regions, expressed via Heaviside functions. As a result of analytical transformations, an expression for the total perturbation of space density is obtained, including three terms: two independent (self-consistent) and one cross-term responsible for the energetic interaction between clusters. It is shown that the cross-term, when expanded, gives four terms, one of which fully coincides in form with Coulomb's law if the concept of "charge" is introduced as $Q = \rho_0 \frac{4\pi R^4}{3}$.

Thus, Coulomb's law in this model is not postulated but arises naturally as a result of the spatial interaction of two density deformations. The first three terms of the expression are interpreted as screening and field normalization effects, analogous to those accounted for in quantum electrodynamics (QED). The introduced parameters R_1, R_2, and ρ_0 preserve the correct dimensionality of expressions and reflect intrinsic properties of the space metric.

The proposed approach opens the possibility of a unified description of electromagnetic, gravitational, and nuclear interactions as manifestations of different regimes of space density deformation, bringing this formalism closer to the construction of a unified geometric field theory.

I Introduction

Electromagnetic and gravitational forces are among the most fundamental interactions known in physics. These forces govern the behavior of matter and energy on scales ranging from subatomic particles to the cosmos. Despite extensive empirical data and theoretical models describing their behavior, their true nature and the material substance from which they arise remain subjects of deep investigation.

From a physical perspective, we understand how these forces act and can predict their effects with high precision. However, questions remain: what exactly are these forces? How are they interconnected? And, most importantly, what is the proto-matter, the fundamental substance from which these forces originate? These questions concern not only physical principles but also philosophical reflections on the nature of reality.

In this paper, we propose a theoretical model that introduces a fifth spatial dimension, called the "space density". We assume that this dimension plays a critical role in the formation of gravitational and electric fields. Our model posits that conventional three-dimensional space combined with time is insufficient to fully explain the origin of these forces. Instead, space itself may possess intrinsic properties that contribute to the formation of these fields. By extending our understanding of space to include an additional dimension, we explore the potential for new interpretations of gravitational and electromagnetic interactions.

II Hypothesis

We assume that electromagnetic and gravitational fields are manifestations of a more fundamental property of space, which we call ”space density”. This property is defined in a five-dimensional system, where the fifth dimension is orthogonal to the traditional three spatial and one temporal dimensions.

In this model, ”space density” represents a measure of how space itself can be compressed or expanded independently of its metric. This density is not analogous to the matter density familiar in three-dimensional space, but rather reflects a fundamental characteristic of space that influences the formation of gravitational and electric fields.

Our hypothesis is based on several key postulates:

- **Space density:** In five-dimensional space, the density $\rho(r)$ characterizes the state of space and can vary, allowing us to talk about space curvature without metric curvature. Let us call this phenomenon first-order space curvature. A similar term is used in the theory of relativity, but within this framework, it will have a slightly different context.
- **Spherical symmetry of perturbations:** The distribution of space density under perturbation is assumed to be spherically symmetric. The space density distribution $\rho(r)$ is assumed to be symmetric about the point at the center of the perturbation.
- **Conservation of space density:** Upon perturbation of a given region of space, the surrounding space can change its density so that the total density of all space remains constant. In other words, in a certain approximation, one can say that the total ”density” of space over the infinite volume of this space must remain constant.
- **Postulate of maximum entropy of space density distribution:** Space tends toward states of maximum entropy, striving for uniform distribution of space density. This principle determines the natural tendency of space to return to a uniform density distribution after perturbations, analogous to thermodynamic principles governing physical systems.

Exploring these postulates in the framework of five-dimensional space, we aim to provide a deeper understanding of the origin of gravitational and electromagnetic fields. This model challenges the traditional notion that these fields are independent and instead proposes that they are interconnected through intrinsic properties of space itself. During the analysis, we obtain completely unexpected results: Coulomb’s law in logarithmic form containing an expression responsible for the correction of elementary charge interactions at distances comparable to their ”classical” physical sizes (this phenomenon is well-studied in QED — screening). The most surprising result is the connection of this mathematical model to the foundation of Quantum Mechanics —

Bohr's Postulate on the quantization of electron states in the hydrogen atom. The solution for the interaction quantity of two space density clusters in logarithmic form has only a complex solution, and it turns out that the complex part of the solution corresponds to the resonance frequency of the interaction quantity of the two clusters. Using this complex part of the solution as an expression for the resonance frequency of two space density clusters, we obtained the resonance condition for one cluster orbiting another, which fully corresponds to the orbital quantization condition derived from Bohr's postulate on electron angular momentum quantization in the hydrogen atom. By analyzing the resulting formulas, we attempt to explain the physical meaning of such an empirically obtained quantity as Planck's constant, which has two values within this mathematical model — the size of the electron, and the ratio of the total energy of the electron in the atom to its imaginary energy. If you are interested in how all this arises from simple representations of space density and its tendency toward maximum entropy, I will begin to outline the main approaches underlying my theory presented in this paper.

III Methodology

3.1 Space Density Distribution Around a Single Compressed Spherical Region of Space

We have two states of the universe: in the first state, the density throughout space is ρ_0 and is constant. In the second state of the system, we have a certain region of space bounded by a sphere $S(R_1)$, which we compress to $S(R_1')$. We need to find the distribution of space density inside the sphere and beyond, based on the established laws operating in our hypothetical universe.

3.1.1 Density Distribution After Compression

The density after compression inside the sphere is defined as $\rho_{\text{inside}} = \rho_0 + \rho_1$, where ρ_1 is the added density determined from the volume relation before and after compression:

$$\rho_0 V(R_1) = \rho_{\text{inside}} V(R_1')$$

Substituting the sphere volumes:

$$\rho_0 \frac{4}{3}\pi R_1^3 = (\rho_0 + \rho_1)\frac{4}{3}\pi R_1'^3$$

Simplifying:

$$\rho_0 R_1^3 = (\rho_0 + \rho_1) R_1'^3$$

$$\rho_1 = \rho_0 \left(\frac{R_1^3}{R_1'^3} - 1 \right)$$

3.1.2 Density Distribution Outside the Sphere

We assume that outside the sphere, the amount of removed space density must equal the amount added inside it, $\rho_1 \cdot V(R_1')$. Therefore, when integrating the perturbation from the surface of the compressed sphere to infinity, the integral must give a finite number, i.e., it must converge, and the integrand function must be convergent. In three-dimensional space, such a function is $1/r^4$. We assume that the distribution of reduced density outside the compressed space region will satisfy this dependence on the distance from the center of perturbation. Then we obtain the following dependence of space density distribution outside the compressed sphere:

$$\Delta\rho_{\text{decrease}}(r) = \frac{A}{r^4}$$

3.1.3 Normalization Coefficient A

To satisfy the conservation of space density, the integral of $\Delta\rho_{\text{decrease}}(r)$ over the volume from R_1' to infinity must equal the added density inside the sphere:

$$\rho_1 V(R_1') = \int_{R_1'}^{\infty} \Delta\rho_{\text{decrease}}(r)\, dV$$

Considering spherical symmetry, in spherical coordinates the integral simplifies to:

$$\rho_1 V(R_1') = \int_{R_1'}^{\infty} \Delta\rho_{\text{decrease}}(r) \cdot 4\pi r^2\, dr$$

Substituting:

$$\rho_1 \frac{4}{3}\pi R_1'^3 = 4\pi \int_{R_1'}^{\infty} \frac{A}{r^4} r^2\, dr$$

Computing the integral:

$$4\pi A \int_{R_1'}^{\infty} \frac{1}{r^2}\, dr = 4\pi A \left[-\frac{1}{r}\right]_{R_1'}^{\infty} = 4\pi A \left(\frac{1}{R_1'} - 0\right) = \frac{4\pi A}{R_1'}$$

Equating the amounts of densities:

$$\rho_1 \frac{4}{3}\pi R_1'^3 = \frac{4\pi A}{R_1'}$$

Solving for A:

$$A = \rho_1 \frac{R_1'^4}{3}$$

Final formula for $\Delta\rho_{\text{decrease}}(r)$:

$$\Delta\rho_{\text{decrease}}(r) = \frac{A}{r^4} = \frac{\rho_1 \frac{R_1'^4}{3}}{r^4}$$

Multiplying numerator and denominator by 4π:

$$\Delta\rho_{\text{decrease}}(r) = \frac{4\pi\rho_1 \frac{R_1'^4}{3}}{4\pi r^4} = \frac{\rho_1 \frac{4}{3}\pi R_1'^4}{4\pi r^4} = \frac{\rho_1 \cdot R_1' \cdot V(R_1')}{4\pi r^4}$$

Thus, we obtain the following formula for the density distribution outside the sphere $\Delta\rho_{\text{decrease}}(r)$:

$$\boxed{\Delta\rho_{\text{decrease}}(r) = \frac{\rho_1 \cdot R_1' \cdot V(R_1')}{4\pi r^4}} \tag{1}$$

Also, noting that the amount of added density in the compressed sphere volume is expressed as:

$$Q = (V(R_1) - V(R_1')) \cdot \rho_0$$

where $V(R_1)$ and $V(R_1')$ are the volumes of spheres with radii R_1 and R_1', respectively. Also, taking into account the formula for ρ_1 — the density of the added quantity inside the sphere:

$$\rho_1 = \frac{Q}{V(R_1')}$$

where $V(R_1')$ is the volume of the sphere after compression.

We can express the obtained formula for the space density distribution $\Delta\rho_{\text{decrease}}(r)$ as:

$$\boxed{\Delta\rho_{\text{decrease}}(r) = \frac{Q \cdot R_1'}{4\pi r^4}} \tag{2}$$

where Q is the amount of density added in the volume $S(R_1')$, R_1' is the radius of the compressed sphere, and r is the distance from the center of the sphere to a point in space in spherical coordinates.

3.1.4 Verification of Space Density Conservation

To satisfy the third law established in our system, the following equality must hold:

$$\int_{R_1'}^{\infty} \Delta\rho_{\text{decrease}}(r)\, dV = \int_{R_1'}^{\infty} \Delta\rho_{\text{decrease}}(r) \cdot 4\pi r^2\, dr = \rho_1 V(R_1')$$

Substituting the expression for $\Delta\rho_{\text{decrease}}(r)$:

$$\int_{R_1'}^{\infty} \frac{\rho_1 \cdot R_1' \cdot V(R_1')}{4\pi r^4} \cdot 4\pi r^2 \, dr = \rho_1 \cdot R_1' \cdot V(R_1') \int_{R_1'}^{\infty} \frac{1}{r^2} \, dr$$

Integrating and substituting the limits:

$$\rho_1 \cdot R_1' \cdot V(R_1') \left[-\frac{1}{r} \right]_{R_1'}^{\infty} = \rho_1 \cdot R_1' \cdot V(R_1') \left(\frac{1}{R_1'} - 0 \right) = \frac{\rho_1 \cdot R_1' \cdot V(R_1')}{R_1'}$$

We obtain:

$$\int_{R_1'}^{\infty} \Delta\rho_{\text{decrease}}(r) \, dV = \rho_1 V(R_1') = \rho_1 \frac{4}{3}\pi R_1'^3$$

Thus, we have confirmed that our space density distribution outside the compressed sphere, proportional to $1/r^4$, is consistent with the third law of space density conservation in the system, taking into account the normalization coefficient A.

IV Expression for the Total Space Density Distribution for a Single Compressed Sphere

Let us write our distribution taking into account boundary conditions, using the Heaviside function. This representation of the space density distribution will be needed to find the total interaction quantity of two clusters, considering the space density added to the first cluster as well as the gradient at the boundary transition — the sphere limiting the first cluster. Why this is important will become clear in the next section of this paper.

4.1 Representation of Space Density Distribution Using the Heaviside Function

The space density distribution, $\rho(r)$, for a single sphere can be expressed using the Heaviside function $H(x)$ for precise description of the density inside and outside the compressed sphere. The main density distribution is defined as:

$$\rho(r) = \begin{cases} \rho_0 + \rho_1, & \text{if } r \leq R_1' \\ \rho_0 - \frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4}, & \text{if } r > R_1' \end{cases}$$

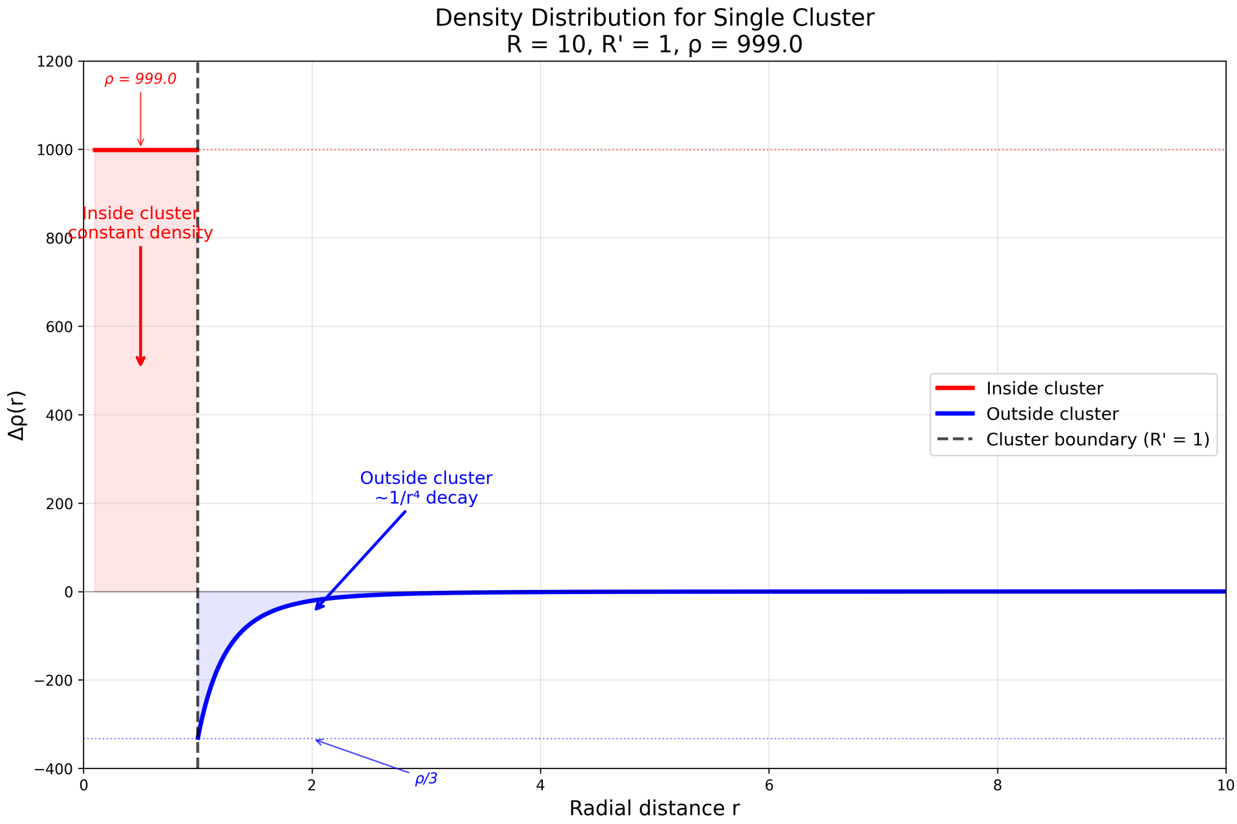

Figure 1: Graphs of space density distribution along a line passing through the center of the compressed sphere

The increase in density $\Delta\rho_{\text{increase}}(r)$ inside the compressed region can be expressed as:

$$\Delta\rho_{\text{increase}}(r) = \begin{cases} \rho_1, & \text{if } r \leq R_1' \\ 0, & \text{if } r > R_1' \end{cases}$$

Similarly, the decrease in density $\Delta\rho_{\text{decrease}}(r)$ outside the sphere:

$$\Delta\rho_{\text{decrease}}(r) = \begin{cases} 0, & \text{if } r \leq R_1' \\ \frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4}, & \text{if } r > R_1' \end{cases}$$

Now we can rewrite these expressions in terms of the Heaviside function $H(x)$:

$$\Delta\rho_{\text{increase}}(r) = \rho_1 H(R_1' - r)$$

$$\Delta\rho_{\text{decrease}}(r) = \frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4} H(r - R_1')$$

Thus, the total density change $\Delta\rho(r)$ is:

$$\Delta\rho(r) = \rho_1 H(R_1' - r) - \frac{R_1' \cdot \rho_1 \cdot V(R_1')}{4\pi r^4} H(r - R_1')$$

4.1.1 Boundary Condition Verification

Now we check the boundary conditions:

1. For $r \leq R'_1$:

$$\Delta\rho(r) = \rho_1 H(R'_1 - r) - \frac{R'_1 \cdot \rho_1 \cdot V_{R1'}}{4\pi r^4} H(r - R'_1)$$

Since $H(R'_1 - r) = 1$ and $H(r - R'_1) = 0$:

$$\Delta\rho(r) = \rho_1 - 0 = \rho_1$$

2. For $r > R'_1$:

$$\Delta\rho(r) = \rho_1 H(R'_1 - r) - \frac{R'_1 \cdot \rho_1 \cdot V_{R1'}}{4\pi r^4} H(r - R'_1)$$

Since $H(R'_1 - r) = 0$ and $H(r - R'_1) = 1$:

$$\Delta\rho(r) = 0 - \frac{R'_1 \cdot \rho_1 \cdot V_{R1'}}{4\pi r^4}$$

Now substituting $V_{R1'} = \frac{4}{3}\pi(R'_1)^3$:

$$\Delta\rho(r) = -\frac{R'_1 \cdot \rho_1 \cdot \frac{4}{3}\pi(R'_1)^3}{4\pi r^4} = -\frac{\rho_1 \cdot R'^4_1}{3r^4}$$

Thus, we obtain the following expression for $\Delta\rho(r)$ in terms of the Heaviside function:

$$\Delta\rho(r) = \rho_1 H(R'_1 - r) - \frac{\rho_1 \cdot R'^4_1}{3r^4} H(r - R'_1) \tag{3}$$

4.2 Verification of the Space Density Conservation Condition

To verify, we take the integral of $\Delta\rho(r)$. Let us integrate $\Delta\rho(r)$ over the entire volume. Recall that $\Delta\rho(r)$ is represented as:

$$\Delta\rho(r) = \rho_1 \left[H(R'_1 - r) - \frac{R'^4_1}{3r^4} H(r - R'_1) \right]$$

Compute the integral:

$$\int_0^\infty \Delta\rho(r) \cdot 4\pi r^2 \, dr$$

Split the integral into two parts, corresponding to $\Delta\rho_{\text{increase}}(r)$ and $\Delta\rho_{\text{decrease}}(r)$:

$$\int_0^\infty \Delta\rho(r) \cdot 4\pi r^2 \, dr = \int_0^\infty \left[\rho_1 H(R'_1 - r) - \frac{\rho_1 \cdot R'^4_1}{3r^4} H(r - R'_1) \right] \cdot 4\pi r^2 \, dr$$

Separate into two individual integrals:

$$\int_0^{\infty} \rho_1 H(R_1' - r) \cdot 4\pi r^2 \, dr - \int_0^{\infty} \frac{\rho_1 \cdot R_1'^4}{3r^4} H(r - R_1') \cdot 4\pi r^2 \, dr$$

Consider the first integral:

$$\int_0^{R_1'} \rho_1 \cdot 4\pi r^2 \, dr = 4\pi\rho_1 \int_0^{R_1'} r^2 \, dr = 4\pi\rho_1 \left[\frac{r^3}{3}\right]_0^{R_1'} = 4\pi\rho_1 \cdot \frac{(R_1')^3}{3} = \frac{4\pi\rho_1 (R_1')^3}{3}$$

Now consider the second integral:

$$\int_{R_1'}^{\infty} \frac{\rho_1 \cdot R_1'^4}{3r^4} \cdot 4\pi r^2 \, dr = \frac{4\pi\rho_1 R_1'^4}{3} \int_{R_1'}^{\infty} \frac{1}{r^2} \, dr = \frac{4\pi\rho_1 R_1'^4}{3} \left[-\frac{1}{r}\right]_{R_1'}^{\infty}$$

Evaluate the limits:

$$\frac{4\pi\rho_1 R_1'^4}{3} \left(-\frac{1}{\infty} + \frac{1}{R_1'}\right) = \frac{4\pi\rho_1 R_1'^4}{3} \cdot \frac{1}{R_1'} = \frac{4\pi\rho_1 R_1'^3}{3}$$

Now sum both results:

$$\int_0^{\infty} \Delta\rho(r) \cdot 4\pi r^2 \, dr = \frac{4\pi\rho_1 (R_1')^3}{3} - \frac{4\pi\rho_1 (R_1')^3}{3} = 0$$

Thus, the integral of $\Delta\rho(r)$ over the entire volume equals zero:

$$\int_0^{\infty} \Delta\rho(r) \cdot 4\pi r^2 \, dr = 0$$

We obtained the expected result, which serves as a verification.

V Interaction Quantity of Two Compressed Space Density Spheres

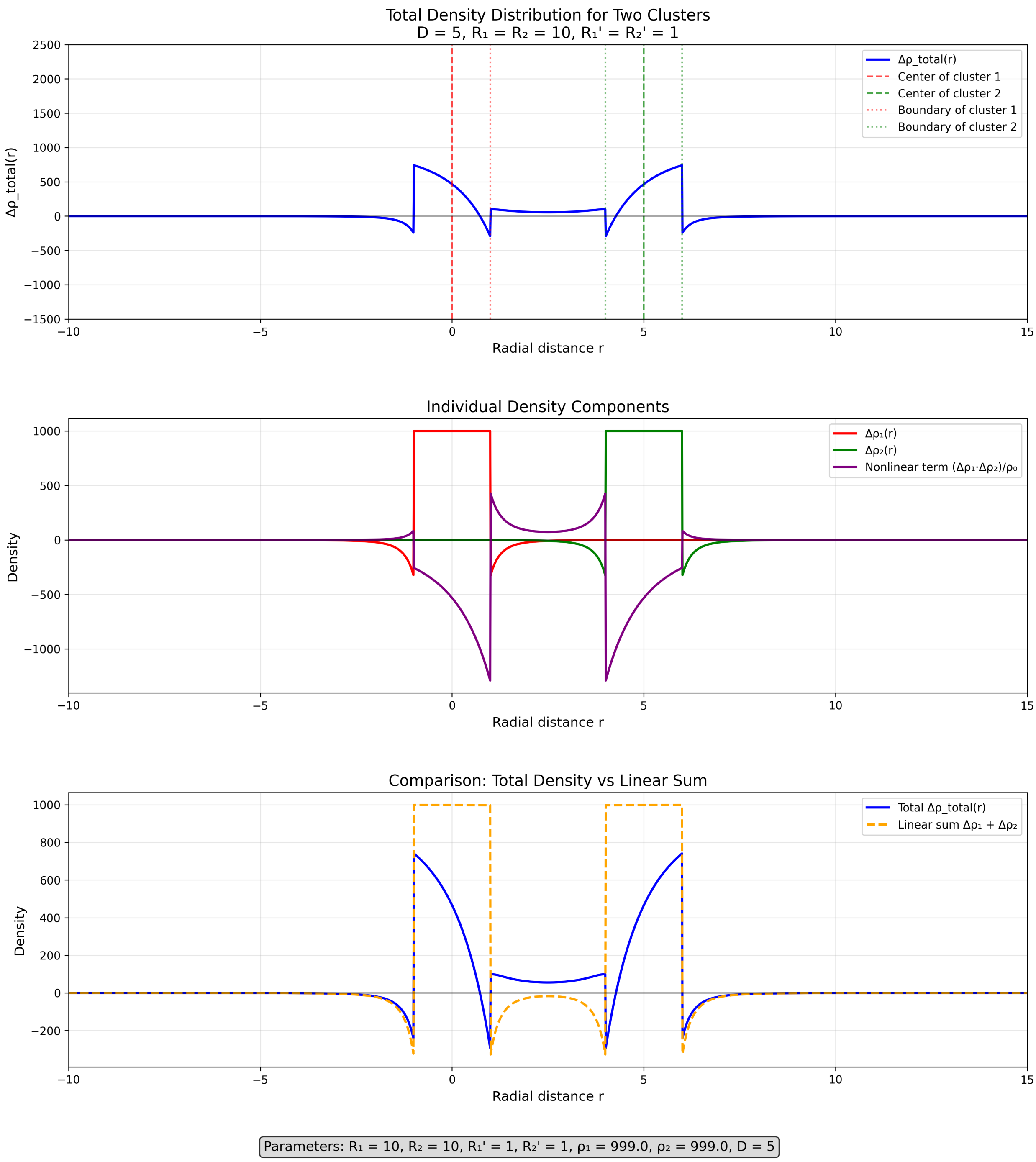

Figure 2: Graph of the total density distribution for two clusters.

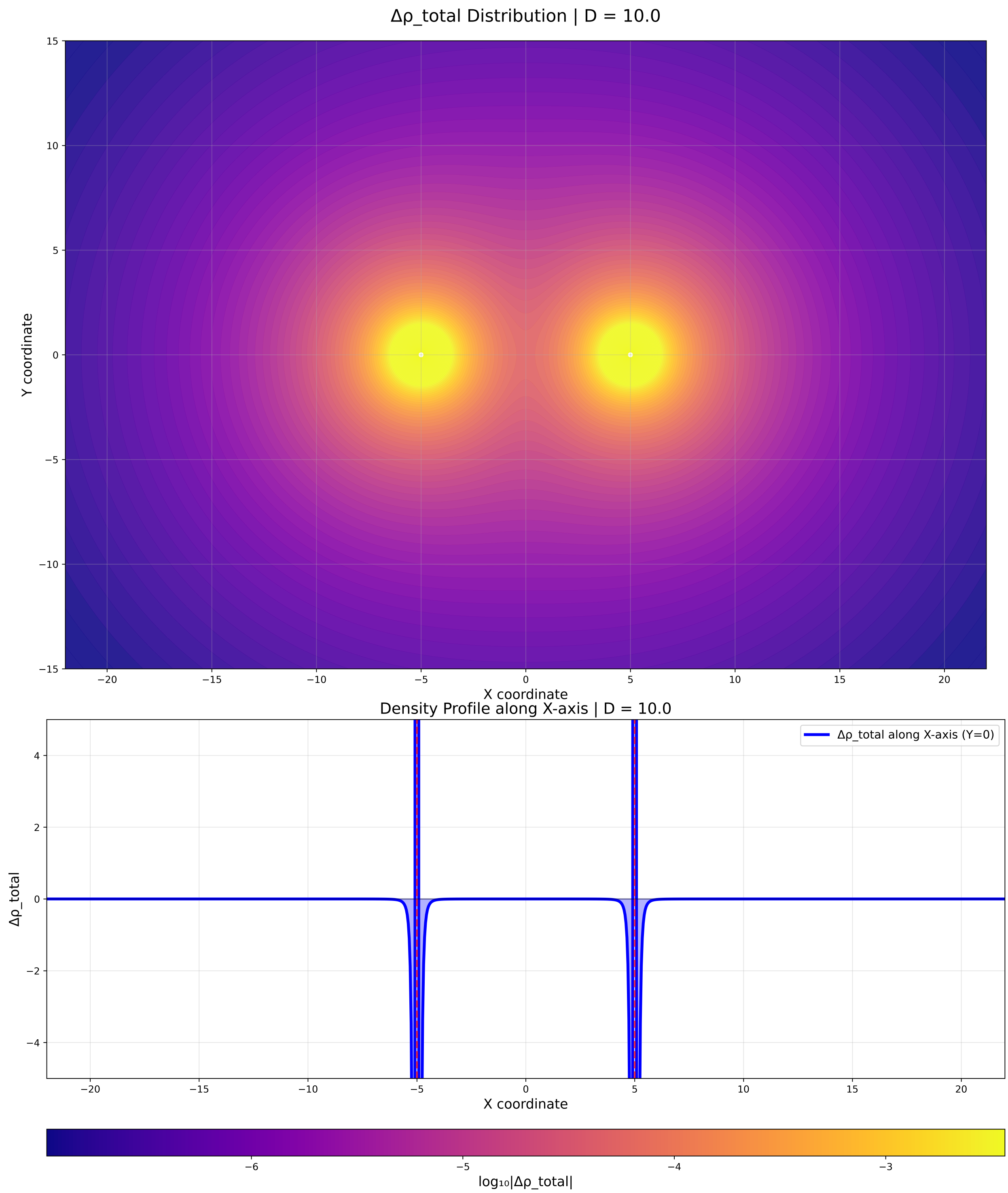

Figure 3: Visualization of space density distribution for two spherical clusters

5.1 Total Density via Heaviside Functions

5.1.1 For a Single Cluster

For a single space density cluster, the disturbance distribution with respect to the radial distance r_1 is defined as:

$$\Delta\rho_1(\mathbf{r}_1) = \rho_1 H(R_1' - r_1) - \frac{\rho_1 R_1'^4}{3r_1^4} H(r_1 - R_1'),$$

where:

- ρ_1 is the amplitude of the density disturbance of the cluster;
- R_1' is the radius of the deformed space region;
- $H(x)$ is the Heaviside function, separating the internal and external regions.

5.1.2 For Two Clusters via Curvature Coefficients

Curvature Coefficient for the First Cluster

$$K_1(\mathbf{r}_1) = 1 + \frac{\Delta\rho_1(\mathbf{r}_1)}{\rho_0} = 1 + \frac{1}{\rho_0}\Big[\rho_1 H(R_1' - r_1) - \frac{\rho_1 R_1'^4}{3r_1^4} H(r_1 - R_1')\Big].$$

Curvature Coefficient for the Second Cluster

$$K_2(\mathbf{r}_1) = 1 + \frac{\Delta\rho_2(\mathbf{r}_1 - \mathbf{D})}{\rho_0} = 1 + \frac{1}{\rho_0}\Big[\rho_2 H(R_2' - |\mathbf{r}_1 - \mathbf{D}|) - \frac{\rho_2 R_2'^4}{3|\mathbf{r}_1 - \mathbf{D}|^4} H(|\mathbf{r}_1 - \mathbf{D}| - R_2')\Big],$$

where **D** is the vector displacement of the second cluster's center relative to the first.

5.1.3 Total Curvature Coefficient and Total Density Change

The total space curvature coefficient created by both clusters is defined as the product of their individual coefficients:

$$K_{\text{total}}(\mathbf{r}_1) = K_1(\mathbf{r}_1) \cdot K_2(\mathbf{r}_1).$$

Accordingly, the total disturbance of the space density is:

$$\Delta\rho_{\text{total}}(\mathbf{r}_1) = \rho_0\big[K_{\text{total}}(\mathbf{r}_1) - 1\big] = \rho_0\left[\left(1 + \frac{\Delta\rho_1(\mathbf{r}_1)}{\rho_0}\right)\left(1 + \frac{\Delta\rho_2(\mathbf{r}_1 - \mathbf{D})}{\rho_0}\right) - 1\right].$$

5.1.4 Expanded Expression

$$\Delta\rho_{\text{total}}(\mathbf{r}_1) = \Delta\rho_1(\mathbf{r}_1) + \Delta\rho_2(\mathbf{r}_1 - \mathbf{D}) + \frac{\Delta\rho_1(\mathbf{r}_1)\,\Delta\rho_2(\mathbf{r}_1 - \mathbf{D})}{\rho_0}.$$

The third (nonlinear) term reflects the *mutual distortion of space metric* arising from the superposition of distortions from both clusters. This term is responsible for the emergence of energy exchange between them.

5.1.5 Complete Form with Heaviside Functions

$$
\begin{aligned}
\Delta\rho_{\text{total}}(\mathbf{r}_1) = \rho_1 H(R_1' - r_1) - \frac{\rho_1 R_1'^4}{3r_1^4} H(r_1 - R_1') \\
+ \rho_2 H(R_2' - |\mathbf{r}_1 - \mathbf{D}|) - \frac{\rho_2 R_2'^4}{3|\mathbf{r}_1 - \mathbf{D}|^4} H(|\mathbf{r}_1 - \mathbf{D}| - R_2') \\
+ \frac{1}{\rho_0}\Big[\rho_1 H(R_1' - r_1) - \frac{\rho_1 R_1'^4}{3r_1^4} H(r_1 - R_1')\Big] \\
\times \Big[\rho_2 H(R_2' - |\mathbf{r}_1 - \mathbf{D}|) - \frac{\rho_2 R_2'^4}{3|\mathbf{r}_1 - \mathbf{D}|^4} H(|\mathbf{r}_1 - \mathbf{D}| - R_2')\Big].
\end{aligned} \tag{4}
$$

VI Integral of the Total Density Change in Three-Dimensional Space

Consider the volumetric integral of the total space density change:

$$I := \int_{\mathbb{R}^3} \Delta\rho_{\text{total}}(\mathbf{r}_1)\, d^3 r_1.$$

Substituting the decomposition

$$\Delta\rho_{\text{total}}(\mathbf{r}_1) = \Delta\rho_1(\mathbf{r}_1) + \Delta\rho_2(\mathbf{r}_1 - \mathbf{D}) + \frac{\Delta\rho_1(\mathbf{r}_1)\,\Delta\rho_2(\mathbf{r}_1 - \mathbf{D})}{\rho_0},$$

we obtain the integral decomposition:

$$I = I_1 + I_2 + \frac{1}{\rho_0} J,$$

where

$$I_1 := \int_{\mathbb{R}^3} \Delta\rho_1(\mathbf{r}_1)\, d^3 r_1, \qquad I_2 := \int_{\mathbb{R}^3} \Delta\rho_2(\mathbf{r}_1 - \mathbf{D})\, d^3 r_1,$$

$$J := \int_{\mathbb{R}^3} \Delta\rho_1(\mathbf{r}_1)\,\Delta\rho_2(\mathbf{r}_1 - \mathbf{D})\, d^3 r_1.$$

Our goal is to show that $I_1 = I_2 = 0$, while $J \neq 0$; this implies that the total volumetric integral $I \neq 0$ and the postulate of the conservation of the "quantity" of space density is violated in the three-dimensional case.

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6.1 Zeros of the First Two Integrals

For the first cluster

$$\Delta\rho_1(\mathbf{r}_1) = \rho_1 H(R_1' - r_1) - \frac{\rho_1 R_1'^4}{3r_1^4} H(r_1 - R_1'),$$

introduce spherical coordinates centered at the first cluster: $d^3r_1 = 4\pi r_1^2 dr_1$. Then

$$I_1 = 4\pi \left[\int_0^{R_1'} \rho_1 r_1^2 \, dr_1 - \int_{R_1'}^{\infty} \frac{\rho_1 R_1'^4}{3r_1^4} r_1^2 \, dr_1 \right].$$

Compute the integrals separately:

$$\int_0^{R_1'} r_1^2 \, dr_1 = \frac{R_1'^3}{3}, \qquad \int_{R_1'}^{\infty} \frac{1}{r_1^2} \, dr_1 = \frac{1}{R_1'}.$$

Hence,

$$I_1 = 4\pi\rho_1 \left(\frac{R_1'^3}{3} - \frac{R_1'^3}{3} \right) = 0.$$

Similarly, for I_2 we just change variables $\mathbf{r}' = \mathbf{r}_1 - \mathbf{D}$, obtaining

$$I_2 = \int_{\mathbb{R}^3} \Delta\rho_2(\mathbf{r}') \, d^3r' = 0.$$

Thus,

$$I_1 = I_2 = 0.$$

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6.2 Nonlinear Term J

Decompose the product $\Delta\rho_1(\mathbf{r}_1)\Delta\rho_2(\mathbf{r}_1 - \mathbf{D})$ into four terms corresponding to the internal (I) and external (O) regions of each cluster:

$$\Delta\rho_1 = \underbrace{\rho_1 H(R_1' - r_1)}_{\mathrm{I}_1} + \underbrace{-\frac{\rho_1 R_1'^4}{3r_1^4} H(r_1 - R_1')}_{\mathrm{O}_1},$$

$$\Delta\rho_2 = \underbrace{\rho_2 H(R_2' - |\mathbf{r}_1 - \mathbf{D}|)}_{\mathrm{I}_2} + \underbrace{-\frac{\rho_2 R_2'^4}{3|\mathbf{r}_1 - \mathbf{D}|^4} H(|\mathbf{r}_1 - \mathbf{D}| - R_2')}_{\mathrm{O}_2}.$$

Then

$$J = J_{\mathrm{II}} + J_{\mathrm{IO}} + J_{\mathrm{OI}} + J_{\mathrm{OO}},$$

where, for example,

$$J_{\mathrm{OO}} = \int_{\substack{r_1 > R_1' \\ |\mathbf{r}_1 - \mathbf{D}| > R_2'}} \frac{\rho_1 R_1'^4}{3r_1^4} \cdot \frac{\rho_2 R_2'^4}{3|\mathbf{r}_1 - \mathbf{D}|^4} \, d^3r_1.$$

Since $(\mathrm{O}_1)(\mathrm{O}_2) > 0$ in the overlapping external region, and this region has nonzero volume, we have

$$J_{\mathrm{OO}} > 0.$$

Hence,

$$J = \int_{\mathbb{R}^3} \Delta\rho_1(\mathbf{r}_1)\Delta\rho_2(\mathbf{r}_1 - \mathbf{D})\, d^3r_1 \neq 0 \quad (\rho_1, \rho_2 > 0).$$

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6.3 Conclusion: Violation of Integral Conservation in 3D

Since $I_1 = I_2 = 0$, but $J \neq 0$, we have

$$I = I_1 + I_2 + \frac{1}{\rho_0}J = \frac{1}{\rho_0}J \neq 0. \tag{5}$$

Thus, the total volumetric integral of the density change, computed using the correct multiplicative combination rule for the curvature coefficients, is nonzero in three-dimensional space. This means that **within this model, in 3D the integral** $\int_{\mathbb{R}^3} \Delta\rho_{\text{total}}\, d^3r$ **is not conserved**, i.e., the simple form of the postulate of conservation of the "quantity" of space density is violated.

VII Transition to Five-Dimensional Space and Introduction of Interaction Operators

In three-dimensional space, the integral of the total density perturbation

$$I = \int_{\mathbb{R}^3} \Delta\rho_{\text{total}}(\mathbf{r})\, d^3r \tag{6}$$

turned out to be nonzero, indicating the **impossibility of strict density conservation** when two curvature fields overlap within the 3D model.

Moreover, the analytic evaluation of the cross integral

$$J = \int_{\mathbb{R}^3} \Delta\rho_1(\mathbf{r})\, \Delta\rho_2(\mathbf{r} - \mathbf{D})\, d^3r \tag{7}$$

in 3D space is **unsolvable in closed form**, making it impossible to obtain an exact expression for the total perturbation.

To overcome these limitations and correctly describe the interaction, we introduce a **minimal five-dimensional structure of space**, allowing the two three-dimensional subspaces to be treated as independent but partially coupled along a single coordinate. This transition provides a symmetric description of interactions and restores the integral balance when the metric is properly chosen.

1. Construction of Five-Dimensional Space

Consider a space consisting of two three-dimensional subspaces, denoted as $\mathbb{R}^3_1$ and $\mathbb{R}^3_2$.

- The first subspace is defined by the coordinates:

$$(r_{x_1}, r_{y_1}, r_{z_1}, 0, 0),$$

- The second subspace by the coordinates:

$$(0, 0, r_{z_2}, r_{x_2}, r_{y_2}).$$

All coordinates are orthogonal to each other, **except for the components** r_{z_1} **and** r_{z_2}, which are connected along a common direction — the Z axis. They are related via the displacement

$$r_{z_2} = r_{z_1} - D, \tag{8}$$

where D is the distance between the centers of the two density clusters along the common axis.

Thus, the position vectors of the subspaces in five-dimensional space are:

$$\mathbf{r}_1 = (r_{x_1}, r_{y_1}, r_{z_1}, 0, 0), \tag{9}$$

$$\mathbf{r}_2 = (0, 0, r_{z_2}, r_{x_2}, r_{y_2}). \tag{10}$$

2. Scalar Product and Direct Interaction Operator

The scalar product of the radius vectors in five-dimensional space:

$$\mathbf{r}_1 \cdot \mathbf{r}_2 = r_{z_1} r_{z_2}. \tag{11}$$

Based on this, we introduce the **direct interaction operator** between the subspaces:

$$T_{12} = \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{|\mathbf{r}_1|\,|\mathbf{r}_2|} = \frac{r_{z_1} r_{z_2}}{|\mathbf{r}_1|\,|\mathbf{r}_2|}. \tag{12}$$

The operator T_{12} determines the degree of projection of subspace $\mathbb{R}^3_1$ onto subspace $\mathbb{R}^3_2$ along the common Z-axis. It describes **directed interaction** — the influence of the first coordinate system on the second in the five-dimensional metric.

Comment for the reader. The transition to a five-dimensional description allows separating the interaction across subspaces while preserving the overall geometric connection. This formalizes the nonlinear term in the Lagrangian not as a "multiplication of functions in a single volume," but as a scalar contraction in different subspaces.

3. Reverse Interaction Operator

Similarly, a **reverse interaction operator** is introduced, accounting for the projection of the second subspace onto the first:

$$T_{21} = \frac{(r_{z_2} + D)(r_{z_1} - D)}{|\mathbf{r}_2 + D|\,|\mathbf{r}_1 - D|}. \tag{13}$$

Here, the mutual relations are used:

$$r_{z_1} = r_{z_2} + D, \tag{14}$$

$$r_{z_2} = r_{z_1} - D. \tag{15}$$

Thus, T_{21} represents the action of T_{12} in the reverse direction, modeling the backward projection $\mathbb{R}^3_2 \to \mathbb{R}^3_1$.

4. Operator Asymmetry under Integration

Although the numerical values of T_{12} and T_{21} may coincide, their action under integration is different, because:

- The integral over the first subspace is performed along $(r_{x_1}, r_{y_1}, r_{z_1})$,
- The integral over the second subspace is performed along $(r_{x_2}, r_{y_2}, r_{z_2})$, and the projections onto the Z-axis have different orientations.

This **geometric asymmetry of the operators** ensures the appearance of a cross-term in the Lagrangian, which accounts not just for the product of perturbations, but for their spatial alignment in different subspaces. Thus, the transition to 5D allows a correct description of the interaction between density clusters as a result of **mutual projections of metric curvatures**.

The five-dimensional construction eliminates the problem of the non-conserved integral from the three-dimensional model, because now the densities belong to different subspaces, and their interaction is described not by a simple sum or product, but by the projection operators T_{12} and T_{21}, which correctly align the metrics.

VIII Conservation of Space Density in 5D under Multiplication by the Direct Interaction Operator T_{12}

Define

$$W_{12} := \int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \Delta\rho_{\text{total}}(\mathbf{r}_1,\mathbf{r}_2)\, T_{12}(\mathbf{r}_1,\mathbf{r}_2)\, d^3r_1\, d^3r_2, \tag{16}$$

where in the 5D formulation the decomposition

$$\Delta\rho_{\text{total}}(\mathbf{r}_1,\mathbf{r}_2) = \Delta\rho_1(\mathbf{r}_1) + \Delta\rho_2(\mathbf{r}_2) + \frac{1}{\rho_0}\,\Delta\rho_1(\mathbf{r}_1)\,\Delta\rho_2(\mathbf{r}_2) \tag{17}$$

is used, and the direct interaction operator is defined as

$$T_{12}(\mathbf{r}_1,\mathbf{r}_2) = \frac{\mathbf{r}_1\cdot\mathbf{r}_2}{|\mathbf{r}_1|\,|\mathbf{r}_2|} = \frac{r_{z_1} r_{z_2}}{|\mathbf{r}_1|\,|\mathbf{r}_2|}. \tag{18}$$

Substituting the decomposition $\Delta\rho_{\text{total}}$ into W_{12} and splitting the integral into three terms:

$$W_{12} = W^{(1)} + W^{(2)} + W^{(\times)}, \tag{19}$$

where

$$W^{(1)} = \int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \Delta\rho_1(\mathbf{r}_1)\, T_{12}(\mathbf{r}_1,\mathbf{r}_2)\, d^3r_1\, d^3r_2, \tag{20}$$

$$W^{(2)} = \int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \Delta\rho_2(\mathbf{r}_2)\, T_{12}(\mathbf{r}_1,\mathbf{r}_2)\, d^3r_1\, d^3r_2, \tag{21}$$

$$W^{(\times)} = \frac{1}{\rho_0}\int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \Delta\rho_1(\mathbf{r}_1)\,\Delta\rho_2(\mathbf{r}_2)\, T_{12}(\mathbf{r}_1,\mathbf{r}_2)\, d^3r_1\, d^3r_2. \tag{22}$$

For further transformations it is convenient to separate angular and radial variables in each subspace. Denote standard spherical coordinates in the first subspace as (r_1,θ_1,φ_1), and in the second as (r_2,θ_2,φ_2). Then:

$$\frac{r_{z_1}}{|\mathbf{r}_1|} = \cos\theta_1, \qquad \frac{r_{z_2}}{|\mathbf{r}_2|} = \cos\theta_2, \tag{23}$$

and the volume element is

$$d^3r_1 = r_1^2\, dr_1\, d\Omega_1, \qquad d^3r_2 = r_2^2\, dr_2\, d\Omega_2, \tag{24}$$

where $d\Omega_i = \sin\theta_i\, d\theta_i\, d\varphi_i$ is the infinitesimal solid angle on the sphere S^2.

Substituting $T_{12} = \cos\theta_1\cos\theta_2$ into the expressions for $W^{(1)}, W^{(2)}, W^{(\times)}$ yields factorization over angles and radii.

For $W^{(1)}$:

$$W^{(1)} = \int_0^\infty\int_0^\infty \left[\Delta\rho_1(r_1)\, r_1^2\, dr_1\right]\left[r_2^2\, dr_2\right] \times \left[\int_{S^2}\cos\theta_1\, d\Omega_1\right]\left[\int_{S^2}\cos\theta_2\, d\Omega_2\right]. \tag{25}$$

Similarly for $W^{(2)}$:

$$W^{(2)} = \int_0^\infty \int_0^\infty \left[r_1^2\, dr_1\right] \left[\Delta\rho_2(r_2)\, r_2^2\, dr_2\right] \\ \times \left[\int_{S^2} \cos\theta_1\, d\Omega_1\right] \left[\int_{S^2} \cos\theta_2\, d\Omega_2\right]. \tag{26}$$

For the cross-term $W^{(\times)}$ we have full factorization:

$$W^{(\times)} = \frac{1}{\rho_0} \left(\int_0^\infty \Delta\rho_1(r_1)\, r_1^2\, dr_1 \int_{S^2} \cos\theta_1\, d\Omega_1 \right) \\ \times \left(\int_0^\infty \Delta\rho_2(r_2)\, r_2^2\, dr_2 \int_{S^2} \cos\theta_2\, d\Omega_2 \right). \tag{27}$$

Key observation: on the sphere it holds that

$$\int_{S^2} \cos\theta\, d\Omega = \int_0^{2\pi} d\varphi \int_0^\pi \cos\theta\, \sin\theta\, d\theta \\ = 2\pi \left[\tfrac{1}{2}\sin^2\theta\right]_0^\pi = 0. \tag{28}$$

Therefore, each angular integral term is zero:

$$\int_{S^2} \cos\theta_1\, d\Omega_1 = 0, \qquad \int_{S^2} \cos\theta_2\, d\Omega_2 = 0. \tag{29}$$

It follows directly that:

$$W^{(1)} = 0, \qquad W^{(2)} = 0, \qquad W^{(\times)} = 0, \tag{30}$$

hence:

$$W_{12} = 0. \tag{31}$$

Thus, when using the direct interaction operator T_{12} in the five-dimensional formalism, the integral of the total perturbation weighted by T_{12} vanishes. This demonstrates the restoration of the volumetric “conservation of density” law in the proposed 5D model: projection mechanisms between subspaces

IX Definition of the Integral Expression W_{21} with the Reverse Interaction Operator in 5D

Define the integral:

$$W_{21} := \int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \Delta\rho_{\text{total}}(\mathbf{r}_1,\mathbf{r}_2)\, T_{21}(\mathbf{r}_1,\mathbf{r}_2)\, d^3r_1\, d^3r_2, \tag{32}$$

where the reverse interaction operator is defined as:

$$T_{21}(\mathbf{r}_1,\mathbf{r}_2) = \frac{(r_{z_2}+D)(r_{z_1}-D)}{|\mathbf{r}_2+D|\,|\mathbf{r}_1-D|}. \tag{33}$$

Substitute the decomposition of the total perturbation:

$$\Delta\rho_{\text{total}}(\mathbf{r}_1,\mathbf{r}_2) = \Delta\rho_1(\mathbf{r}_1) + \Delta\rho_2(\mathbf{r}_2) + \frac{1}{\rho_0}\,\Delta\rho_1(\mathbf{r}_1)\,\Delta\rho_2(\mathbf{r}_2), \tag{34}$$

into the integral W_{21}. We obtain the decomposition into three terms:

$$\begin{aligned} W_{21} = &\underbrace{\int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \Delta\rho_1(\mathbf{r}_1)\, T_{21}(\mathbf{r}_1,\mathbf{r}_2)\, d^3r_1\, d^3r_2}_{W_{21}^{(1)}} \\ &+ \underbrace{\int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \Delta\rho_2(\mathbf{r}_2)\, T_{21}(\mathbf{r}_1,\mathbf{r}_2)\, d^3r_1\, d^3r_2}_{W_{21}^{(2)}} \\ &+ \underbrace{\frac{1}{\rho_0}\int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \Delta\rho_1(\mathbf{r}_1)\,\Delta\rho_2(\mathbf{r}_2)\, T_{21}(\mathbf{r}_1,\mathbf{r}_2)\, d^3r_1\, d^3r_2}_{W_{21}^{(\times)}}. \end{aligned} \tag{35}$$

9.1 Substitution of Expressions for Individual Density Perturbations

Let the local density perturbations in each subspace be:

$$\Delta\rho_1(\mathbf{r}_1) = \rho_1\, H(R_1' - r_1) - \frac{\rho_1 R_1'^4}{3r_1^4}\, H(r_1 - R_1'), \tag{36}$$

$$\Delta\rho_2(\mathbf{r}_2) = \rho_2\, H(R_2' - r_2) - \frac{\rho_2 R_2'^4}{3r_2^4}\, H(r_2 - R_2'), \tag{37}$$

where $H(x)$ is the Heaviside function, R_i' are the effective radii of the clusters, and ρ_i are their internal densities.

Then each term of the integral can be written explicitly:

1. First term:

$$W_{21}^{(1)} = \int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \left[\rho_1\, H(R_1' - r_1) - \frac{\rho_1 R_1'^4}{3r_1^4}\, H(r_1 - R_1')\right]$$

$$\times \frac{(r_{z_2}+D)(r_{z_1}-D)}{|\mathbf{r}_2+D|\,|\mathbf{r}_1-D|}\, d^3r_1\, d^3r_2. \tag{38}$$

2. Second term:

$$W_{21}^{(2)} = \int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \left[\rho_2\, H(R_2'-r_2) - \frac{\rho_2 R_2'^4}{3r_2^4}\, H(r_2-R_2')\right] \times \frac{(r_{z_2}+D)(r_{z_1}-D)}{|\mathbf{r}_2+D|\,|\mathbf{r}_1-D|}\, d^3r_1\, d^3r_2. \tag{39}$$

3. Cross-term:

$$W_{21}^{(\times)} = \frac{1}{\rho_0}\int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \left[\rho_1\, H(R_1'-r_1) - \frac{\rho_1 R_1'^4}{3r_1^4}\, H(r_1-R_1')\right] \times \left[\rho_2\, H(R_2'-r_2) - \frac{\rho_2 R_2'^4}{3r_2^4}\, H(r_2-R_2')\right] \times \frac{(r_{z_2}+D)(r_{z_1}-D)}{|\mathbf{r}_2+D|\,|\mathbf{r}_1-D|}\, d^3r_1\, d^3r_2. \tag{40}$$

This is the final expression for the integral W_{21} as an integral over $\mathbb{R}^3 \times \mathbb{R}^3$ accounting for the reverse interaction operator and the explicit functions $\Delta\rho_1$ and $\Delta\rho_2$.

9.2 Cross-Term of the Integral W_{21} as a Product of Integrals over Subspaces

Consider the cross-term of the integral $W_{21}^{(\times)}$:

$$W_{21}^{(\times)} = \frac{1}{\rho_0}\int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \Delta\rho_1(\mathbf{r}_1)\,\Delta\rho_2(\mathbf{r}_2)\, T_{21}(\mathbf{r}_1,\mathbf{r}_2)\, d^3r_1\, d^3r_2, \tag{41}$$

where the interaction operator is:

$$T_{21}(\mathbf{r}_1,\mathbf{r}_2) = \frac{(r_{z_2}+D)(r_{z_1}-D)}{|\mathbf{r}_2+D|\,|\mathbf{r}_1-D|}. \tag{42}$$

For convenience, decompose the integrand into two separate integrals over each subspace. Define:

$$I_{\mathbb{R}_1^3} = \int_{\mathbb{R}^3} \Delta\rho_1(\mathbf{r}_1)\frac{r_{z_1}-D}{|\mathbf{r}_1-D|}\, d^3r_1, \tag{43}$$

$$I_{\mathbb{R}_2^3} = \int_{\mathbb{R}^3} \Delta\rho_2(\mathbf{r}_2)\frac{r_{z_2}+D}{|\mathbf{r}_2+D|}\, d^3r_2. \tag{44}$$

Then the cross-term $W_{21}^{(\times)}$ can be written in factorized form:

$$W_{21}^{(\times)} = \frac{1}{\rho_0}\, I_{\mathbb{R}_1^3}\, I_{\mathbb{R}_2^3}. \tag{45}$$

Substituting the expressions for the local density perturbations, we obtain:

$$I_{\mathbb{R}^3_1} = \int_{\mathbb{R}^3} \left[\rho_1 \, H(R'_1 - r_1) - \frac{\rho_1 R'^4_1}{3r_1^4} \, H(r_1 - R'_1) \right] \frac{r_{z_1} - D}{|\mathbf{r}_1 - D|} \, d^3 r_1, \tag{46}$$

$$I_{\mathbb{R}^3_2} = \int_{\mathbb{R}^3} \left[\rho_2 \, H(R'_2 - r_2) - \frac{\rho_2 R'^4_2}{3r_2^4} \, H(r_2 - R'_2) \right] \frac{r_{z_2} + D}{|\mathbf{r}_2 + D|} \, d^3 r_2. \tag{47}$$

Thus, the cross-term of the integral $W_{21}^{(\times)}$ reduces to the product of two three-dimensional integrals, each describing the contribution of its subspace. The factorization emphasizes a fundamental feature of the five-dimensional formalism: the interaction between two density clusters is represented as the result of **the joint action of integrals over two three-dimensional subspaces**, which restores the symmetry of the system and ensures correct conservation of the total density after integration.

9.3 Calculation of the Integral $I_{\mathbb{R}^3_1}$

Switch to spherical coordinates $(r_1, \theta_1, \varphi_1)$ centered at the first cluster:

$$r_{x_1} = r_1 \sin\theta_1 \cos\varphi_1, \quad r_{y_1} = r_1 \sin\theta_1 \sin\varphi_1, \quad r_{z_1} = r_1 \cos\theta_1.$$

The volume element then reads:

$$d^3 r_1 = r_1^2 \sin\theta_1 \, dr_1 \, d\theta_1 \, d\varphi_1,$$

and the fraction in the integrand can be written as:

$$\frac{r_{z_1} - D}{|\mathbf{r}_1 - D|} = \frac{r_1 \cos\theta_1 - D}{\sqrt{r_1^2 - 2Dr_1 \cos\theta_1 + D^2}}.$$

Substituting all this into the integral, we obtain:

$$\begin{aligned} I_{\mathbb{R}^3_1} = & \int_0^\infty r_1^2 \, dr_1 \int_0^\pi \sin\theta_1 \, d\theta_1 \int_0^{2\pi} d\varphi_1 \\ & \times \left[\rho_1 \, H(R'_1 - r_1) - \frac{\rho_1 R'^4_1}{3r_1^4} \, H(r_1 - R'_1) \right] \frac{r_1 \cos\theta_1 - D}{\sqrt{r_1^2 - 2Dr_1 \cos\theta_1 + D^2}}. \end{aligned} \tag{48}$$

The integral over the azimuthal angle is trivial:

$$\int_0^{2\pi} d\varphi_1 = 2\pi.$$

Hence,

$$\begin{aligned} I_{\mathbb{R}^3_1} = 2\pi & \int_0^\infty r_1^2 \, dr_1 \int_0^\pi \sin\theta_1 \\ & \times \left[\rho_1 \, H(R'_1 - r_1) - \frac{\rho_1 R'^4_1}{3r_1^4} \, H(r_1 - R'_1) \right] \frac{r_1 \cos\theta_1 - D}{\sqrt{r_1^2 - 2Dr_1 \cos\theta_1 + D^2}} \, d\theta_1. \end{aligned} \tag{49}$$

9.4 Separation into Inner and Outer Regions

1. Inner region $(r_1 < R_1')$**:**

$$I^{(\text{in})}_{\mathbb{R}^3_1} = 2\pi\rho_1 \int_0^{R_1'} r_1^2\,dr_1 \int_0^{\pi} \sin\theta_1 \times \frac{r_1\cos\theta_1 - D}{\sqrt{r_1^2 - 2Dr_1\cos\theta_1 + D^2}}\,d\theta_1. \tag{50}$$

2. Outer region $(r_1 > R_1')$**:**

$$I^{(\text{out})}_{\mathbb{R}^3_1} = -\frac{2\pi\rho_1 R_1'^4}{3} \int_{R_1'}^{\infty} \frac{dr_1}{r_1^2} \int_0^{\pi} \sin\theta_1 \times \frac{r_1\cos\theta_1 - D}{\sqrt{r_1^2 - 2Dr_1\cos\theta_1 + D^2}}\,d\theta_1. \tag{51}$$

Thus,

$$I_{\mathbb{R}^3_1} = I^{(\text{in})}_{\mathbb{R}^3_1} + I^{(\text{out})}_{\mathbb{R}^3_1}. \tag{52}$$

Integral over the angle θ_1

Let $u = \cos\theta_1$, then $\sin\theta_1 d\theta_1 = -du$, $u \in [-1, 1]$. The integral over θ_1 can be rewritten as

$$J_1(r_1) = \int_{-1}^{1} \frac{r_1 u - D}{\sqrt{r_1^2 - 2Dr_1 u + D^2}}\,du. \tag{53}$$

Make the substitution $t = r_1^2 + D^2 - 2r_1 D u \Rightarrow dt = -2r_1 D\,du$, with limits $u = -1 \Rightarrow t = (r_1 + D)^2$, $u = 1 \Rightarrow t = (r_1 - D)^2$. Then

$$J_1(r_1) = \frac{1}{4r_1 D} \int_{(r_1-D)^2}^{(r_1+D)^2} \frac{r_1^2 - D^2 - t}{\sqrt{t}}\,dt. \tag{54}$$

Computing the antiderivative gives:

$$\int \frac{r_1^2 - D^2 - t}{\sqrt{t}} dt = 2(r_1^2 - D^2)\sqrt{t} - \frac{2}{3}t^{3/2}. \tag{55}$$

Substitute the limits $(r_1 - D)^2$ and $(r_1 + D)^2$:

$$J_1(r_1) = \frac{1}{2r_1 D^2} \left[(r_1^2 - D^2)((r_1 + D) - |r_1 - D|) - \frac{1}{3}((r_1 + D)^3 - |r_1 - D|^3) \right]. \tag{56}$$

Split by regions:

$$\begin{cases} r_1 < D: & J_1(r_1) = -2\left(1 - \frac{r_1^2}{3D^2}\right), \\ r_1 > D: & J_1(r_1) = -\frac{4}{3}\frac{D}{r_1}. \end{cases}$$

Volume part

Multiply by $2\pi r_1^2$ for the volume measure:

$$A_1(r_1) = 2\pi r_1^2 J_1(r_1) = \begin{cases} -4\pi r_1^2 \left(1 - \frac{r_1^2}{3D^2}\right), & r_1 < D, \\ -\frac{8\pi}{3} D r_1, & r_1 > D. \end{cases} \tag{57}$$

Separation of the integral according to the Heaviside function

Taking into account the inner and outer parts of $\Delta\rho_1(\mathbf{r}_1)$, we get a one-dimensional integral:

$$I_{\mathbb{R}_1^3} = \int_0^{R_1'} A_1(r_1)\, dr_1 - \frac{R_1'^4}{3} \int_{R_1'}^{\infty} \frac{A_1(r_1)}{r_1^4}\, dr_1. \tag{58}$$

Splitting the integral $I_{\mathbb{R}_1^3}$ by the position of R_1 relative to D

Case 1: $R_1 < D$

1. Inner integral:

$$\int_0^{R_1} A_1(r_1)\, dr_1 = \int_0^{R_1} -4\pi r_1^2 \left(1 - \frac{r_1^2}{3D^2}\right) dr_1 \tag{59}$$

$$= -4\pi \left(\frac{R_1^3}{3} - \frac{R_1^5}{15D^2}\right). \tag{60}$$

2. Outer integral:

$$\int_{R_1}^{\infty} \frac{A_1(r_1)}{r_1^4}\, dr_1 = \int_{R_1}^{D} -4\pi \left(\frac{1}{r_1^2} - \frac{1}{3D^2}\right) dr_1 + \int_D^{\infty} -\frac{8\pi}{3}\frac{D}{r_1^3} dr_1 \tag{61}$$

$$= -4\pi \left(\frac{1}{R_1} - \frac{1}{3D} + \frac{R_1}{3D^2}\right). \tag{62}$$

3. Combine $I_{\mathbb{R}_1^3}$:

$$I_{\mathbb{R}_1^3} = \rho_1 \left[-4\pi \left(\frac{R_1^3}{3} - \frac{R_1^5}{15D^2}\right) - \frac{R_1^4}{3}\left(-4\pi \left(\frac{1}{R_1} - \frac{1}{3D} + \frac{R_1}{3D^2}\right)\right)\right] \tag{63}$$

$$= \rho_1 \frac{4\pi R_1^4}{45D^2}(8R_1 - 15D), \quad R_1 < D. \tag{64}$$

Case 2: $R_1 \geq D$

1. Inner integral:

$$\int_0^{R_1} A_1(r_1)\, dr_1 = \int_0^{D} -4\pi r_1^2 \left(1 - \frac{r_1^2}{3D^2}\right) dr_1 + \int_D^{R_1} -\frac{8\pi}{3} D r_1 dr_1 \tag{65}$$

$$= -\frac{8\pi D}{9} R_1^2 + \frac{4\pi}{15} D^3. \tag{66}$$

2. Outer integral:

$$\int_{R_1}^{\infty} \frac{A_1(r_1)}{r_1^4} dr_1 = \int_{R_1}^{\infty} -\frac{8\pi}{3} \frac{D}{r_1^3} dr_1 = -\frac{4\pi D}{3R_1^2}. \tag{67}$$

3. Combine $I_{\mathbb{R}_1^3}$:

$$I_{\mathbb{R}_1^3} = \rho_1 \frac{4\pi D}{45}(3D^2 - 10R_1^2), \quad R_1 \geq D. \tag{68}$$

Final expression for the integral

$$\boxed{I_{\mathbb{R}_1^3} = \begin{cases} \rho_1 \dfrac{4\pi R_1^4}{45D^2}(8R_1 - 15D), & R_1 < D, \\ \rho_1 \dfrac{4\pi D}{45}(3D^2 - 10R_1^2), & R_1 \geq D. \end{cases}} \tag{69}$$

9.5 Calculation of the integral $I_{\mathbb{R}_2^3}$ and verification of the relation $J_2 = -J_1$

Consider the integral over the second subspace:

$$I_{\mathbb{R}_2^3} = \int_{\mathbb{R}^3} \Delta\rho_2(\mathbf{r}_2) \frac{r_{z_2} + D}{|\mathbf{r}_2 + D|} d^3 r_2,$$

where

$$\Delta\rho_2(\mathbf{r}_2) = \rho_2 H(R_2' - r_2) - \frac{\rho_2 R_2'^4}{3r_2^4} H(r_2 - R_2').$$

Switch to spherical coordinates

$$r_{z_2} = r_2 \cos\theta_2, \quad d^3 r_2 = r_2^2 \sin\theta_2 \, dr_2 \, d\theta_2 \, d\varphi_2,$$

$$\frac{r_{z_2} + D}{|\mathbf{r}_2 + D|} = \frac{r_2 \cos\theta_2 + D}{\sqrt{r_2^2 + 2Dr_2\cos\theta_2 + D^2}}.$$

Integration over φ_2 gives a factor of 2π. Define the angular function:

$$J_2(r_2) = \int_0^{\pi} \frac{r_2 \cos\theta_2 + D}{\sqrt{r_2^2 + 2Dr_2\cos\theta_2 + D^2}} \sin\theta_2 \, d\theta_2 = \int_{-1}^{1} \frac{r_2 u + D}{\sqrt{r_2^2 + 2Dr_2 u + D^2}} du,$$

where $u = \cos\theta_2$.

Calculation of $J_2(r)$

Let $t = r^2 + D^2 + 2rDu \Rightarrow dt = 2rD\, du$. For $u = -1 \Rightarrow t = (r - D)^2$, for $u = 1 \Rightarrow t = (r + D)^2$. Then

$$J_2(r) = \frac{1}{2rD} \int_{(r-D)^2}^{(r+D)^2} \frac{t - (r^2 - D^2)}{2\sqrt{t}} dt = \frac{1}{4rD} \int_{(r-D)^2}^{(r+D)^2} \frac{t - (r^2 - D^2)}{\sqrt{t}} dt.$$

Taking the antiderivative:

$$\int \frac{t-(r^2-D^2)}{\sqrt{t}}\,dt = \frac{2}{3}t^{3/2} - 2(r^2-D^2)\sqrt{t},$$

substitute the limits:

$$J_2(r) = \frac{1}{2rD^2}\left[\frac{1}{3}\big((r+D)^3 - |r-D|^3\big) - (r^2-D^2)\big((r+D)-|r-D|\big)\right].$$

Splitting by regions

- For $r < D$, $|r-D| = D-r$:

$$J_2(r) = 2\left(1 - \frac{r^2}{3D^2}\right).$$

- For $r > D$, $|r-D| = r-D$:

$$J_2(r) = \frac{4}{3}\frac{D}{r}.$$

Comparing with previously computed $J_1(r)$:

$$J_1(r) = \begin{cases} -2\left(1-\dfrac{r^2}{3D^2}\right), & r < D, \\ -\dfrac{4}{3}\dfrac{D}{r}, & r > D, \end{cases}$$

we obtain

$$\boxed{J_2(r) = -J_1(r)}.$$

Calculation of the radial part of the integral $I_{\mathbb{R}^3_2}$

Substitute the angular part $J_2(r_2)$ into the radial integral:

$$I_{\mathbb{R}^3_2} = 2\pi\int_0^\infty r_2^2\,dr_2\left[\rho_2 H(R_2'-r_2) - \frac{\rho_2 R_2'^4}{3r_2^4}H(r_2-R_2')\right]J_2(r_2).$$

Introduce the volume measure function:

$$A_2(r_2) = 2\pi r_2^2 J_2(r_2) = -A_1(r_2),$$

where $A_1(r_1)$ was used for the integral $I_{\mathbb{R}^3_1}$. Then

$$I_{\mathbb{R}^3_2} = \int_0^{R_2'} A_2(r_2)dr_2 - \frac{R_2'^4}{3}\int_{R_2'}^\infty \frac{A_2(r_2)}{r_2^4}dr_2.$$

Splitting by the position of R_2 relative to D

Case 1: $R_2 < D$

$$\int_0^{R_2} A_2(r_2)dr_2 = \int_0^{R_2} -A_1(r_2)dr_2 = -\int_0^{R_2} A_1(r_2)dr_2,$$

$$\int_{R_2}^{\infty} \frac{A_2(r_2)}{r_2^4}dr_2 = -\int_{R_2}^{\infty} \frac{A_1(r_2)}{r_2^4}dr_2.$$

Hence,

$$I_{\mathbb{R}_2^3} = -I_{\mathbb{R}_1^3} = \rho_2 \frac{4\pi R_2^4}{45D^2}(15D - 8R_2), \quad R_2 < D.$$

Case 2: $R_2 \geq D$

$$\int_0^{R_2} A_2(r_2)dr_2 = -\int_0^{R_2} A_1(r_2)dr_2, \quad \int_{R_2}^{\infty} \frac{A_2(r_2)}{r_2^4}dr_2 = -\int_{R_2}^{\infty} \frac{A_1(r_2)}{r_2^4}dr_2,$$

then

$$I_{\mathbb{R}_2^3} = -I_{\mathbb{R}_1^3} = \rho_2 \frac{4\pi D}{45}(10R_2^2 - 3D^2), \quad R_2 \geq D.$$

Final expression for the second integral

$$\boxed{I_{\mathbb{R}_2^3} = \begin{cases} \rho_2 \dfrac{4\pi R_2^4}{45D^2}(15D - 8R_2), & R_2 < D, \\ \rho_2 \dfrac{4\pi D}{45}(10R_2^2 - 3D^2), & R_2 \geq D. \end{cases}}$$

9.6 Final expression of the cross term $W_{21}^{\times}$ for the case $D > R_1, R_2$

1. Expansion of brackets for $I_{R_1^3}$ at $D > R_1$

$$I_{R_1^3} = \rho_1 \frac{4\pi R_1^4}{45D^2}(8R_1 - 15D) = \rho_1 \frac{32\pi R_1^5}{45D^2} - \rho_1 \frac{4\pi R_1^4}{3D}.$$

2. Similarly for $I_{R_2^3}$ at $D > R_2$ (with correct sign)

$$I_{R_2^3} = -\left(\rho_2 \frac{32\pi R_2^5}{45D^2} - \rho_2 \frac{4\pi R_2^4}{3D}\right) = \rho_2 \frac{4\pi R_2^4}{3D} - \rho_2 \frac{32\pi R_2^5}{45D^2}.$$

3. Expression through charges Q_1, Q_2

$$Q_1 = \rho_1 \frac{4\pi R_1^3}{3}, \quad Q_2 = \rho_2 \frac{4\pi R_2^3}{3},$$

$$I_{R_1^3} = \frac{8}{15}\frac{Q_1 R_1^2}{D^2} - \frac{Q_1 R_1}{D}, \quad I_{R_2^3} = -\left(\frac{8}{15}\frac{Q_2 R_2^2}{D^2} - \frac{Q_2 R_2}{D}\right) = \frac{Q_2 R_2}{D} - \frac{8}{15}\frac{Q_2 R_2^2}{D^2}.$$

4. Cross term $W_{21}^{\times}$

$$W_{21}^{\times} = \frac{1}{\rho_0} I_{R_1^3} I_{R_2^3} = \frac{1}{\rho_0}\left(\frac{8}{15}\frac{Q_1 R_1^2}{D^2} - \frac{Q_1 R_1}{D}\right)\left(\frac{Q_2 R_2}{D} - \frac{8}{15}\frac{Q_2 R_2^2}{D^2}\right).$$

5. Final compact expression

$$\boxed{W_{21}^{\times} = \frac{1}{\rho_0}\left(\frac{8 Q_1 R_1^2}{15 D^2} - \frac{Q_1 R_1}{D}\right)\left(\frac{Q_2 R_2}{D} - \frac{8 Q_2 R_2^2}{15 D^2}\right), \quad D > R_1, R_2.}$$

6. Full multiplication of brackets

$$W_{21}^{\times} = \frac{1}{\rho_0}\Bigg[-\left(\frac{8 Q_1 R_1^2}{15 D^2}\right)\left(\frac{8 Q_2 R_2^2}{15 D^2}\right) + \left(\frac{8 Q_1 R_1^2}{15 D^2}\right)\left(\frac{Q_2 R_2}{D}\right) + \left(\frac{Q_1 R_1}{D}\right)\left(\frac{8 Q_2 R_2^2}{15 D^2}\right) - \left(\frac{Q_1 R_1}{D}\right)\left(\frac{Q_2 R_2}{D}\right)\Bigg].$$

7. Each term explicitly

$$W_{21}^{\times} = -\frac{64 Q_1 Q_2 R_1^2 R_2^2}{225 \rho_0 D^4} + \frac{8 Q_1 Q_2 R_1^2 R_2}{15 \rho_0 D^3} + \frac{8 Q_1 Q_2 R_1 R_2^2}{15 \rho_0 D^3} - \frac{Q_1 Q_2 R_1 R_2}{\rho_0 D^2}.$$

9.7 Proof of equality of symmetric integrals and vanishing of self-energy terms $W_{21}^{(1)} = -W_{21}^{(2)}$ for $\rho_1 = \rho_2,\ R_1' = R_2'$

Start from definitions (see above):

$$W_{21}^{(1)} = \int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \Delta\rho_1(\mathbf{r}_1)\, T_{21}(\mathbf{r}_1, \mathbf{r}_2)\, d^3 r_1\, d^3 r_2,$$

$$W_{21}^{(2)} = \int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \Delta\rho_2(\mathbf{r}_2)\, T_{21}(\mathbf{r}_1, \mathbf{r}_2)\, d^3 r_1\, d^3 r_2,$$

where

$$T_{21}(\mathbf{r}_1, \mathbf{r}_2) = \frac{(r_{z_2} + D)(r_{z_1} - D)}{|\mathbf{r}_2 + D|\,|\mathbf{r}_1 - D|}.$$

Change the order of integration and take out factors depending on one variable separately:

$$W_{21}^{(1)} = \left(\int_{\mathbb{R}^3} \Delta\rho_1(\mathbf{r}_1)\frac{r_{z_1} - D}{|\mathbf{r}_1 - D|}\, d^3r_1\right)\left(\int_{\mathbb{R}^3} \frac{r_{z_2} + D}{|\mathbf{r}_2 + D|}\, d^3r_2\right) = I_{\mathbb{R}^3_1}\, C,$$

$$W_{21}^{(2)} = \left(\int_{\mathbb{R}^3} \Delta\rho_2(\mathbf{r}_2)\frac{r_{z_2} + D}{|\mathbf{r}_2 + D|}\, d^3r_2\right)\left(\int_{\mathbb{R}^3} \frac{r_{z_1} - D}{|\mathbf{r}_1 - D|}\, d^3r_1\right) = I_{\mathbb{R}^3_2}\, C,$$

where we introduce notations

$$I_{\mathbb{R}^3_1} = \int_{\mathbb{R}^3} \Delta\rho_1(\mathbf{r}_1)\frac{r_{z_1} - D}{|\mathbf{r}_1 - D|}\, d^3r_1, \quad I_{\mathbb{R}^3_2} = \int_{\mathbb{R}^3} \Delta\rho_2(\mathbf{r}_2)\frac{r_{z_2} + D}{|\mathbf{r}_2 + D|}\, d^3r_2,$$

and the common geometric factor (the same when swapping indices)

$$C = \int_{\mathbb{R}^3} \frac{r_z + D}{|\mathbf{r} + D|}\, d^3r = \int_{\mathbb{R}^3} \frac{r_z - D}{|\mathbf{r} - D|}\, d^3r$$

From previous calculations we established (angular part)

$$J_2(r) = -J_1(r) \quad \Longrightarrow \quad A_2(r) = -A_1(r),$$

and for $\rho_1 = \rho_2$ and $R'_1 = R'_2$ this gives

$$I_{\mathbb{R}^3_2} = -\, I_{\mathbb{R}^3_1}.$$

Substituting this into the expressions for $W_{21}^{(1)}$ and $W_{21}^{(2)}$ we get

$$W_{21}^{(1)} = I_{\mathbb{R}^3_1}\, C, \qquad W_{21}^{(2)} = I_{\mathbb{R}^3_2}\, C = (-I_{\mathbb{R}^3_1})\, C = -W_{21}^{(1)}.$$

Hence

$$\boxed{W_{21}^{(1)} + W_{21}^{(2)} = 0}$$

for $\rho_1 = \rho_2,\ R'_1 = R'_2$.

Analogy with the cross term $W_{21}^{(\times)}$ and physical meaning

Previously we wrote the cross term in factorized form

$$W_{21}^{(\times)} = \frac{1}{\rho_0}\, I_{\mathbb{R}^3_1}\, I_{\mathbb{R}^3_2} = \frac{1}{\rho_0}\, I_{\mathbb{R}^3_1}\, (-I_{\mathbb{R}^3_1}) = -\frac{1}{\rho_0}\, I^2_{\mathbb{R}^3_1}.$$

Thus, under symmetry $\rho_1 = \rho_2,\ R'_1 = R'_2$ the individual (linear) contributions $W_{21}^{(1)}$ and $W_{21}^{(2)}$ completely cancel each other, and the remaining cross term is expressed through the square of the finite integral $I_{\mathbb{R}^3_1}$ and gives the finite contribution $-\frac{1}{\rho_0} I^2_{\mathbb{R}^3_1}$.

Unlike the standard situation in the ordinary Lagrangian (where the “self-energy” of like point charges formally diverges), in the proposed five-dimensional formalism:

- linear (self-interaction) contributions of two identical clusters mutually cancel ($W_{21}^{(1)} + W_{21}^{(2)} = 0$);
- the cross (cooperative) contribution $W_{21}^{(\times)}$ remains, which is finite and proportional to $I^2_{\mathbb{R}^3_1}/\rho_0$.

Thus, in this model the "energy of two like charges" does not diverge due to self-cumulative terms, but either completely cancels (linear parts) or remains a finite mutual contributio

X Physical meaning of the terms of the total density perturbation integral

In our approach, the integral of the total density perturbation $\Delta\rho_{\text{total}}$ in 5D space has several key components, each carrying a separate physical interpretation.

10.1 Energy of self-density clusters

The first two terms of the integral correspond to the energy of the *density clusters themselves* ρ_1 and ρ_2 in their own subspaces. In our model they **cancel out** upon volume integration:

$$\int_{R_1^3} \Delta\rho_1 \, d^3r_1 = 0, \qquad \int_{R_2^3} \Delta\rho_2 \, d^3r_2 = 0,$$

which means that the "energy of a cluster" in isolation does not create interactions and does not produce divergent values. This is crucial: due to the finite radii R_1 and R_2 of the charges, the paradox of infinite energy characteristic of point charges in classical theory does not arise.

10.2 Cross term: real interaction

The cross term of the integral forms the real interaction between the two density clusters. In the 5D model this term has the form:

$$W_{21}^{\times} = \frac{1}{\rho_0} \int_{R_1^3} \int_{R_2^3} \Delta\rho_1(\mathbf{r}_1) \, \Delta\rho_2(\mathbf{r}_2) \, T_{21} \, d^3r_1 \, d^3r_2,$$

where T_{21} is the inverse interaction operator, taking into account the relative positions of the two subspaces. This term coincides with the product of the potentials of two charges divided by the dielectric permittivity ρ_0. Thus, the cross term of the integral is a **rigorous theoretical justification of Coulomb interaction**, previously introduced in the Lagrangian only as an empirical guess.

10.3 Screening and pre-normalization of the field

The remaining terms of the integral fully describe the process of *screening and pre-normalization of the field*. They model how the space density is distributed around each charge and correct the field at small distances, on the order of 10 electron radii. These terms ensure smooth field behavior at $D \sim R_1', R_2'$ and eliminate classical paradoxes of infinite energy and field discontinuities.

10.4 Graphical representation of the field

These properties can be demonstrated in a graph of the field dependence on the distance D between charges with finite radius $R_1' = R_2' = 0.1$ (in arbitrary units). At small distances $D \lesssim 10R$ the field **smooths out** due to the screening and normalizing terms, whereas at large distances $D \gg R$ the classical $1/D^2$ dependence of Coulomb interaction appears, fully determined by the cross term.

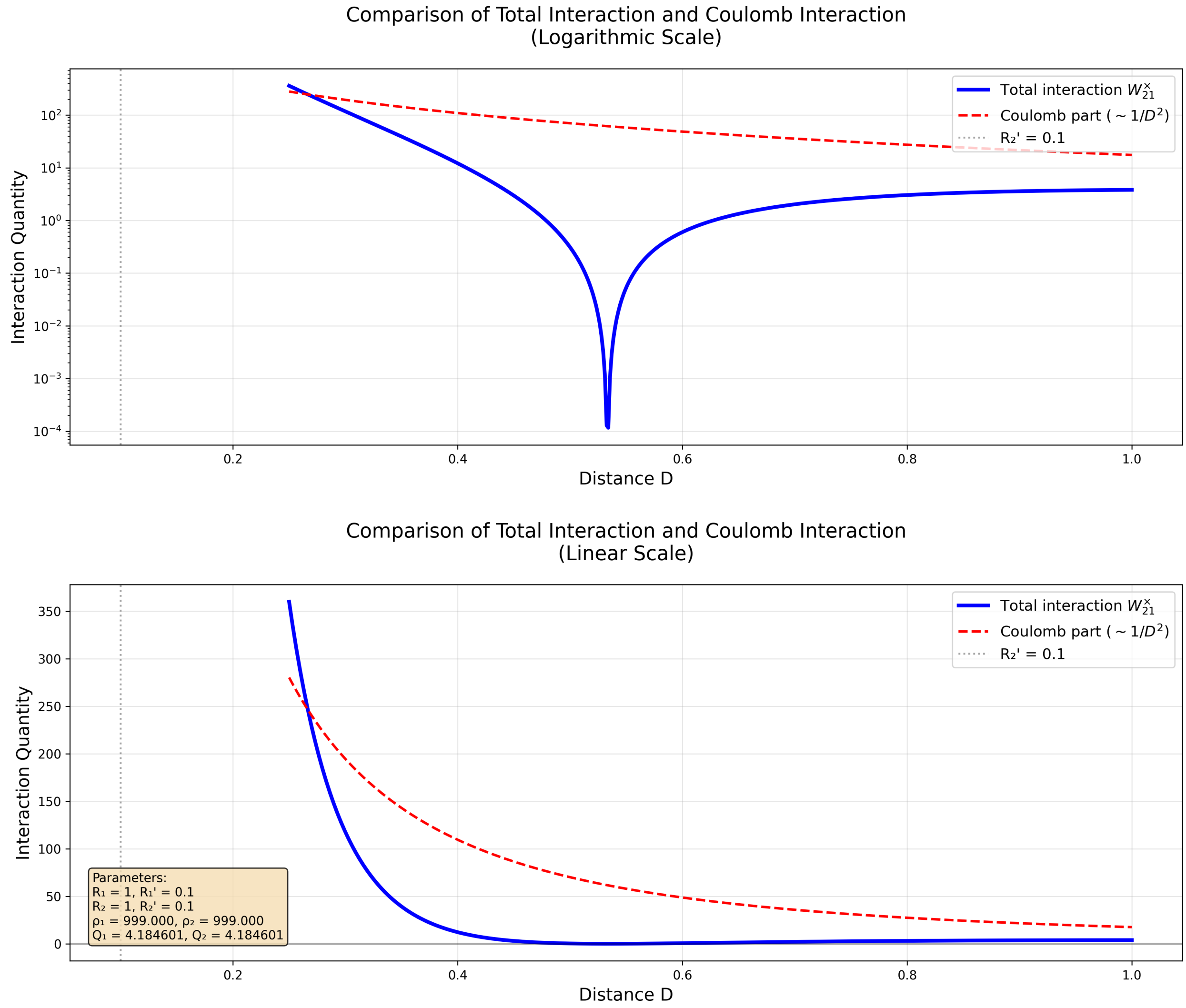

Figure 4: Visualization of the reduction of interaction magnitude at distances comparable to the size of the elementary charge (charge size 0.1 in arbitrary units, D range up to 1)

10.5 Final understanding of energy structure

Thus, the integral in our model not only gives a numerical value of interaction, but also **splits the energy into three physical components**:

1. self-energy of the clusters (cancels out);
2. real interaction of two charges (cross term, Coulomb energy);
3. corrective terms for screening and pre-normalization (ensure physical adequacy of the field at small distances).

This structure allows one to rigorously derive the Lagrangian for a two-charge system *from the geometry of 5D space and the density distribution*, without additional empirical postulates. Moreover, the finite radii of the charges R_1, R_2 create a natural limit $D > R$, eliminating the problem of infinite energy and making the model fully self-consistent.

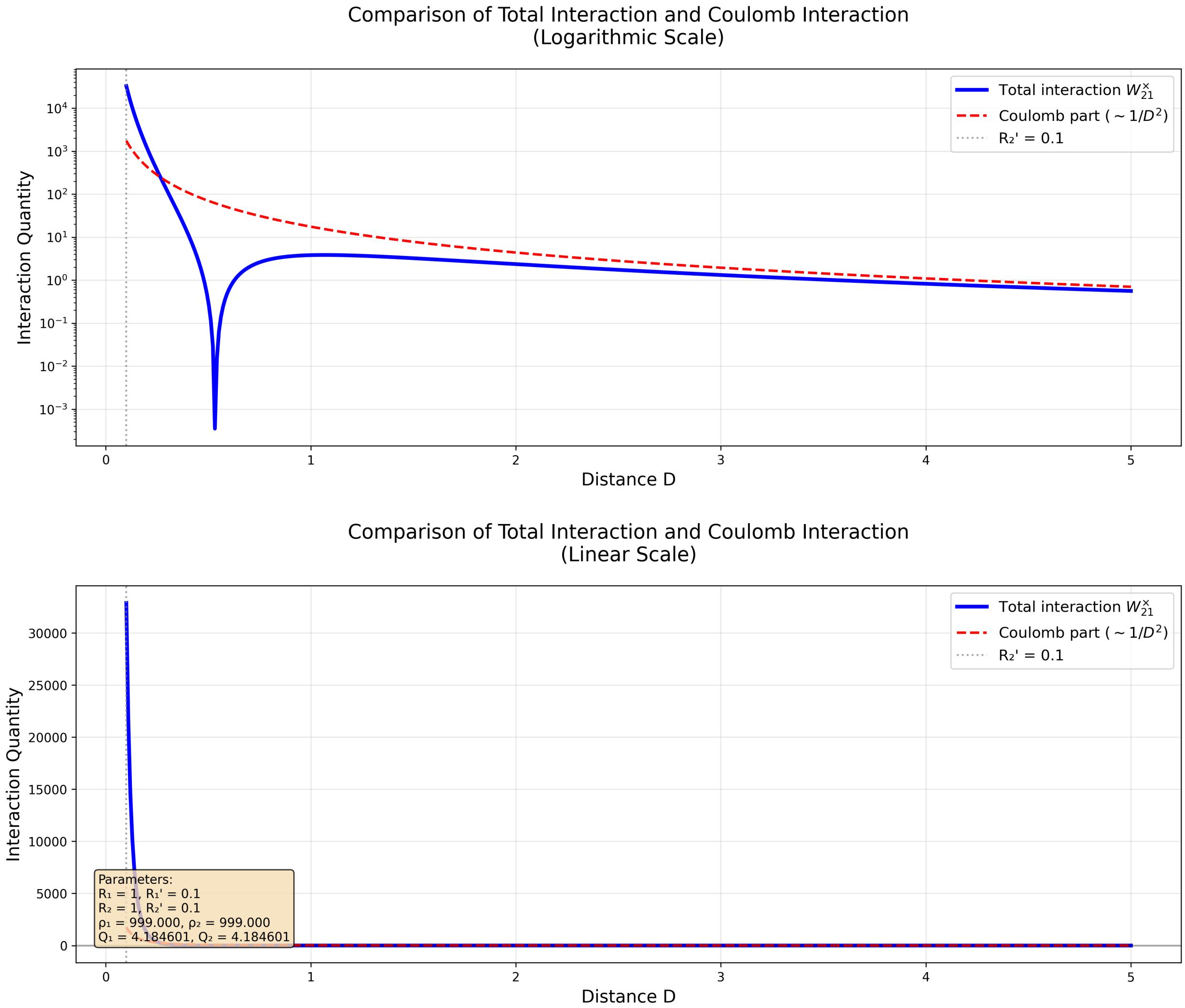

Figure 5: Visualization of the reduction of interaction magnitude at distances comparable to the size of the elementary charge (charge size 0.1 in arbitrary units, D range up to 5)

10.6 Conclusion

The obtained model:

- provides a physically rigorous explanation of Coulomb's law through the cross term of the integral;
- explains the process of screening and field normalization observed in quantum electrodynamics;
- eliminates classical paradoxes of point charges;
- links the Lagrangian of electromagnetic interaction to the actual geometry and density of space.

The graphical representation of the field dependence on D clearly demonstrates all these effects and confirms the correctness of the constructed model.

XI Conclusion and Summary

Conclusion and Summary

In this work, we considered a five-dimensional model of interaction between two density clusters representing electric charges and investigated integrals of density perturbation weighted by the direct and inverse interaction operators. The main results and conclusions can be formulated as follows:

1. It was shown that the integral of the total density perturbation weighted by the direct interaction operator T_{12} completely vanishes after integration over all angles of both subspaces. This demonstrates the restoration of the volumetric "density conservation" law in the five-dimensional formalism, analogous to the principle of charge conservation.

2. For the integral with the inverse operator T_{21}, an explicit factorization into subspace integrals was performed. The cross term $W_{21}^{(\times)}$ was represented as the product of one-dimensional radical integrals $I_{R_1^3}$ and $I_{R_2^3}$.

3. The integrals $I_{R_1^3}$ and $I_{R_2^3}$ were computed analytically in the form of one-dimensional radical integrals and reduced to compact expressions through the "charges" Q_1 and Q_2 under the condition $D > R_1, R_2$:

$$I_{R_1^3} = \frac{8Q_1R_1^2}{15D^2} - \frac{Q_1R_1}{D}, \quad I_{R_2^3} = -\frac{8Q_2R_2^2}{15D^2} + \frac{Q_2R_2}{D}.$$

4. It was shown that the linear contributions $W_{21}^{(1)}$ and $W_{21}^{(2)}$ completely cancel each other under cluster symmetry ($\rho_1 = \rho_2,\ R_1' = R_2'$), i.e.

$$W_{21}^{(1)} + W_{21}^{(2)} = 0.$$

   This is radically different from the usual Lagrangian formulation for point charges, where "self-energy" formally diverges. In the proposed model, linear self-energies vanish, leaving only the finite cooperative cross term $W_{21}^{(\times)}$.

5. The cross term $W_{21}^{(\times)}$ gives a finite contribution to the system energy and describes the effective interaction of two like density clusters. Thus, the five-dimensional formalism ensures both symmetry of interaction and finiteness of energy, bypassing the classical problem of divergence of point charge self-energy.

Overall, the obtained model demonstrates an interesting mechanism of "self-correction" of the energy of like charges due to the structure of five-dimensional space and cross integral interactions. This opens the prospect for further analysis of stable configurations of density clusters, as well as the construction of resonance models of atomic systems based purely on the spatial properties of charges, without introducing formal infinities or requiring additional regularizations.

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