Serial Formula of Periodic Numbers

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ABSTRACT

ENGLISH

We learned in school that $2n$ corresponds to even numbers and that $2n-1$ corresponds to odd numbers. In this document, we'll see a formula for periodic numbers, almost as simple as the first two.

Part 1 - Objective

The expression for a periodic number as a fraction is brief, but it doesn't show the period. The idea is to obtain a formula that satisfies the following conditions.

- Be brief.
- Explicitly show the period of the number.

Let's start by looking at some cases.

Case 1 - Completely Periodic Decimal Part

Example:

$$\frac{22}{7} = 3.\overbrace{142857} \tag{1}$$

In this case, before the decimal point there is an integer and then a sequence of decimals that is periodic from the first digit. We can formulate this case in the following way.

$$\mathbf{Z_p = E + p \sum_{i=1}^{i=\infty} 10^{-di}} \tag{2}$$

Z_p $\rightarrow$ periodic number
E $\rightarrow$ integer part of the periodic number
$p \rightarrow =$ period written as an integer
$d =$ number of digits in the period

If p is written as an integer, then $p > 0$ is equal to $Z_p > 0$. And it is $p < 0$ is equal to $Z_p < 0$.

That is, the period has the same sign as the integer part of the periodic number.

In this example, the serial expression takes the following form.

$$\frac{22}{7} = 3 + 142857 \sum_{i=1}^{i=\infty} 10^{-6i} \tag{3}$$

If we expand this form, we obtain the decimal number corresponding to the analyzed fraction.

$$\frac{22}{7} = 3 + 142857 \cdot 10^{-6} + 142857 \cdot 10^{-12} + 142857 \cdot 10^{-18} + 142857 \cdot 10^{-24} + (...) \tag{4}$$

$$\frac{22}{7} = 3 + \overbrace{142857}\overbrace{142857}\overbrace{142857}... \tag{5}$$

$$\frac{22}{7} = 3.\overbrace{142857}\overbrace{142857}\overbrace{142857}... \tag{6}$$

Case 2 - Decimal part that begins aperiodic

Example:

$$\frac{345}{44} = 7{,}84\overbrace{090} \tag{7}$$

In this case, Z_p is the sum of an aperiodic part and a periodic series.

$$Z_p = A + S \tag{8}$$

A $\rightarrow$ aperiodic part
S $\rightarrow$ periodic series of decimals

In this case, we cannot use formula (2), since the period begins at the third decimal place, not the first. How do we construct the formula? The construction is based on the same concept, adding an order factor.

- Step 1. Select the aperiodic part. In the example, it is 7.84.

$$A = 7.84 \tag{9}$$

  A $\rightarrow$ aperiodic part

- Step 2. Select the period. In the example, it is 0.90.

$$P = 090 \tag{10}$$

  P $\rightarrow$ period

  We see in expression (7) that the first period begins at the third decimal place. and what What follows is the following series of periods.

$$S = \sum_{i=1}^{i=\infty} 10^{-c+ni}\ P$$

Common factor P.

$$S = P\ \sum_{i=1}^{i=\infty} 10^{-(c+di)} \tag{11}$$

$c \rightarrow$ number of aperiodic decimals

In case (7) equation (10) has the following development.

$$S = 090\ \sum_{i=1}^{i=\infty} 10^{-(2+3i)} \tag{12}$$

$$S = 090\ \sum_{i=1}^{i=\infty} 10^{-(2+3i)} \tag{13}$$

$$S = 090\left(10^{-5} + 10^{-8} + 10^{-11} + ...\right) \tag{14}$$

- Step 3. Join the parts In (8) we express S as indicated by (11).

$$Z_p = A + P\ \sum_{i=1}^{i=\infty} 10^{-(c+di)} \tag{15}$$

Part 2 - Commentary and Reflection

- Are formulas (2) and (15) more than just a type of notation?
- Do these formulas have any operational use?

I don't have answers to these questions. I just think it's a good idea to recognize types of periodic numbers and formulate the structure of each type. For now, I can't comment further, and there's still virgin ground for further analysis.

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August 24, 2025

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