# Parametrization for cube equal to sum of four cubes

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**Abstract**

In math literature there are not many examples of parametrization for the equation:

($a^3+b^3+c^3+d^3=e^3$). There are two Identities mentioned in ref. # (3). In this paper the author has parametrized the above equation by means of Algebra. By applying certain conditions to the above equation, we have avoided the use of elliptic curve theory in-order to solve the problem.

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The two Identities given in ref. # (3) are shown below:

The first is:

$$(a+b-c)^3(-a+b+c)^3+\ (a-b+c)^3+\ d^3=\ (a+b+c)^3$$

Condition: $(24abc)=\ (d)^3$

For, $(a,b,c)=(9,1,1)$, we get, $(9,-7,9,6)^3=(11)^3$

And the second is:

$$(11x^2+xy+14y^2)^3+\ (12x^2-3xy+13y^2)^3+\ (13x^2+3xy+12y^2)^3 \\ +\ (14x^2-xy+11y^2)^3=\ (20x^2+20y^2)^3$$

For $(x,y)=(1,1)$, we get: $(11,12,13,14)^3=(20)^3$

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Consider the below equation:

$a^3+b^3+c^3+d^3=e^3$ ------------------ (1)

we assume the conditions :

$e=(b+c)$ & $7(a+d)=9(b+c)$

from eqn (1) we have:

$$(a^3+d^3)=(e)^3-(b^3+c^3)$$

$$=(b+c)^3-(b^3+c^3)=3bc(b+c)$$

Hence: $(a+d)(a^2-ad+d^2)=3bc(b+c)$

Hence:

$$(7*3)*(a+d)(a^2-ad+d^2)=(7*3)*3bc(b+c)$$

Since: $7(a+d)=9(b+c)$ ----(2a) we get:

$$3(a^2-ad+d^2)=7bc \quad --(2b)$$

from, eqn.(2a) & (2b) we have:

$(a*d)=(\frac{1}{441})*(243b^2+143bc+243c^2)$ ------- (3)

also: we have: $(a+d)=\left(\frac{9}{7}\right)(b+c)$ ------------- (4)

Solving, simultaneous eqn. (3) & (4)

we get: $a=(\frac{1}{42})(27(b+c)+w)$ &

$d=(\frac{1}{42})(27(b+c)-w)$

where: $(w)^2=886bc-243c^2-243b^2$ -------------- (5)

eqn. (5) is satisfied at:

$$(b,c,w)=(19,9,210)$$

Hence we parametrize eqn (5) at (b,c)=(19,9) & we get:

$$[(mb),(mc)]=\begin{bmatrix}(14647k^2-9234k+2187),\\(4617k^2-4374k+3357)\end{bmatrix}$$

& w=210 & $m=(243k^2-886k+243)$

Since: $a=\left(\frac{1}{42}\right)(27(b+c)+w)$,

$d=(\frac{1}{42})(27(b+c)-w)$

Substituting for (b,c,w) we get:

$$ma = (13599k^2 - 13178k + 4779)$$

$$md = (11169k^2 - 4318k + 2349)$$

& since e=(b+c) we have:

$$me = 56(344k^2 - 243k + 99)$$

Hence for k=0,we get:

$$(a, b, c, d,) = (531,261,243,373)$$

$$\& \, e = (56 * 11) = 616$$

Hence; $(a, b, c, d, e)^3 = ( 531,261,243,373,616)^3$

Hence the parametric form of eqn (1) is:

$$a = 13599k^2 - 13178k + 4779$$

$$b = 14647k^2 - 9234k + 2187$$

$$c = 9(513k^2 - 486k + 373)$$

$$d = 11169k^2 - 4318k + 2349$$

$$e = 56(344k^2 - 243k + 99)$$

If we put, $(k = \frac{m}{n})$ in the above & then take, $n = 0$,

we get the below numerical solution:

$(a, b, c, d)^3$= $(13599,14647,4617,11169)^3 = (56 * 344)^3 = (19264)^3$

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