The contradiction between the law of universal gravitation and the second laws of Newton and Kepler

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Abstract

The motion of bodies along an ellipse with a constant sector velocity is considered. At perihelion, the velocity and acceleration are greater than at aphelion. The radii are the opposite: at perihelion, the radius is smaller than at aphelion. Therefore, if the force is calculated using Newton's second law, it will not be equal to the force calculated using the law of universal gravitation.

Keywords: Kepler's and Newton's laws.

According to Kepler's first and second laws, the planets move in an ellipse with a constant sector velocity relative to the center of mass, point *C*, Fig. 1. This means that at perihelion, point *P*, the velocity and acceleration are greater than at aphelion, point *A*. At the same time, $r_a > r_p$, where r_a is the radius from the center of mass at aphelion, r_p is the radius from the center of mass at perihelion.

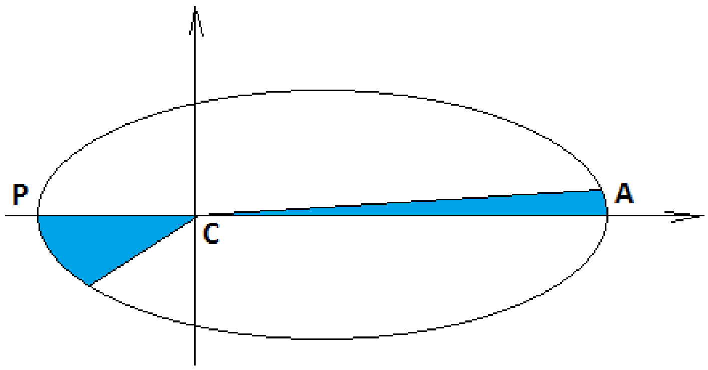

Fig. 1

There are two pairs of objects in the Solar system with a barycenter significantly removed from the centers of the objects. These are Jupiter - the Sun, the Moon - the Earth.

Let's calculate the parameters of the motion of a mathematical point along second-order curves in accordance with Kepler's laws using equation (1) for the Moon - Earth system:

$$\ddot{\varphi} = \frac{2*e*\sin(\varphi(t))*\dot{\varphi}^2}{1-e*\cos(\varphi(t))} \qquad (1)$$

The barycenter is in the left focus. The point starts moving from aphelion.

The acceleration formulas from equation (1) are shown in Appendix [A1]. The properties of the equation are discussed in the article [1].

We construct a graph of forces, Fig. 1, based on the calculation results, Table [A2].

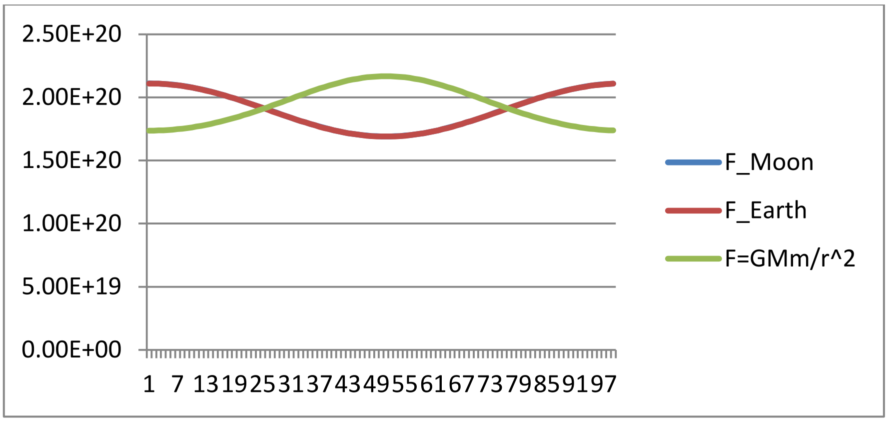

Fig. 2

Modules of average forces:

$$F_{Moon} = m_{Moon}\dot{v}_{n\_Moon} = 1.89934e + 20 \text{ кг·м·с}^{-2} \quad (2)$$

$$F_{Earth} = m_{Earth}\dot{v}_{n\_Earth} = 1.89934e + 20 \text{ кг·м·с}^{-2} \quad (3)$$

$$F = \frac{GMm}{r^2} = 1.93869e + 20 \text{ кг·м·с}^{-2} \quad (4)$$

$$F \approx F_{Moon} = F_{Earth} \quad (5)$$

$$\boldsymbol{F} \neq \boldsymbol{F}_{Moon} = \boldsymbol{F}_{Earth} \quad (6)$$

The forces are approximately equal in magnitude.

$m_{Moon}, \dot{v}_{n\_Moon}$ – mass and normal acceleration of the Moon

$m_{Earth}, \dot{v}_{n\_Earth}$ – mass and normal acceleration of the Earth

m_{Moon} =7.3477e+22 kg, m_{Earth} =5.9726e+24 kg

G = 6.67418·10^{-11} m^3·s^{-2}·kg^{-1} – gravitational constant.

$r = r_{Moon} + r_{Earth}$ – distance between the centers of the Moon and the Earth

r_{Moon} , r_{Earth} – radii from the barycenter to the center of the Moon and the Earth

As we can see from the formulas (2, 3), $F_{Moon} = F_{Earth}$ – Newton's third law in reverse.

$$\boldsymbol{F} \neq \boldsymbol{F}_{Moon} = \boldsymbol{F}_{Earth} \quad (5)$$

The masses (m) and semi-major axes (a) of the Moon and Earth are taken from reference books.

$$\frac{a_1}{a_2} = 81.315; \; \frac{m_2}{m_1} = 81.2853$$

Reasons for the discrepancy between the force graphs in Fig. 2

Newton's law of universal gravitation states that "every object in the universe attracts every other object along a line connecting the centers of mass of the objects, proportional to the mass of each object, and inversely proportional to the square of the distance between the objects," [2].

a) Newton's law of universal gravitation assumes that the forces of interaction between objects are not equal to $F_{Moon} \neq F_{Earth}$.

b) It is also assumed that the forces are directed along the line connecting the centers of mass of the objects. However, when moving along a curve, the centripetal acceleration is directed toward the center of curvature of the trajectory, Fig. 3, 4, more details [1].

$e = 0$ we get a circle and $\frac{\ddot{x}}{\ddot{y}} = \frac{x}{y}$,

a circle is a special case of an ellipse, Fig. 3.

In Figures 3 – 4, the red lines indicate the velocities, and the green ones indicate the accelerations.

The coordinates of the beginning of the velocity and acceleration vectors, the points of the original ellipse (x, y). The coordinates of the end of the velocity vector *(dx+x, dy+y).* The coordinates of the end of the acceleration vector *(ddx+x, ddy+y)*

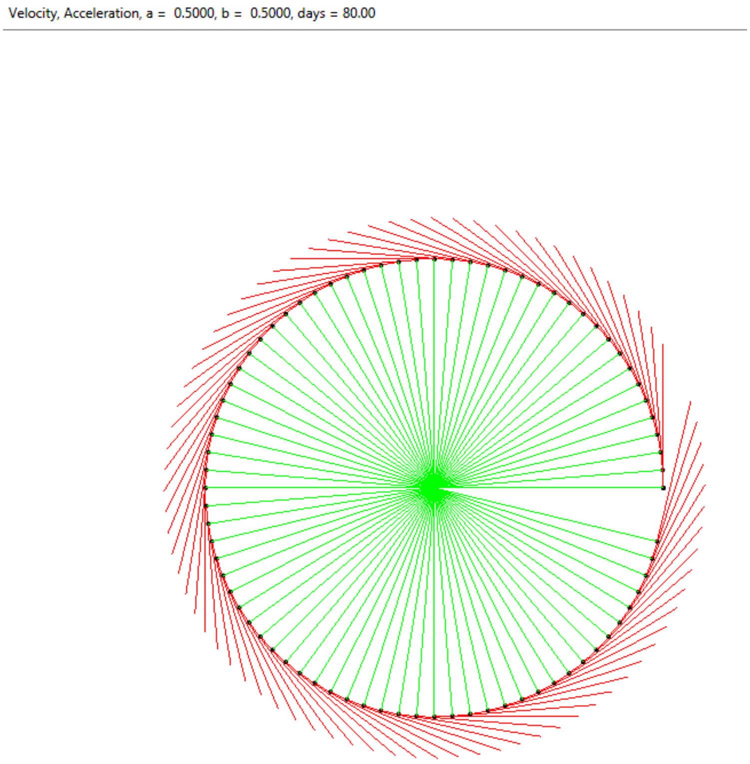

Figure 3

$e \neq 0$, then $\frac{\ddot{x}}{\ddot{y}} \neq \frac{x}{y}$, Figure 4

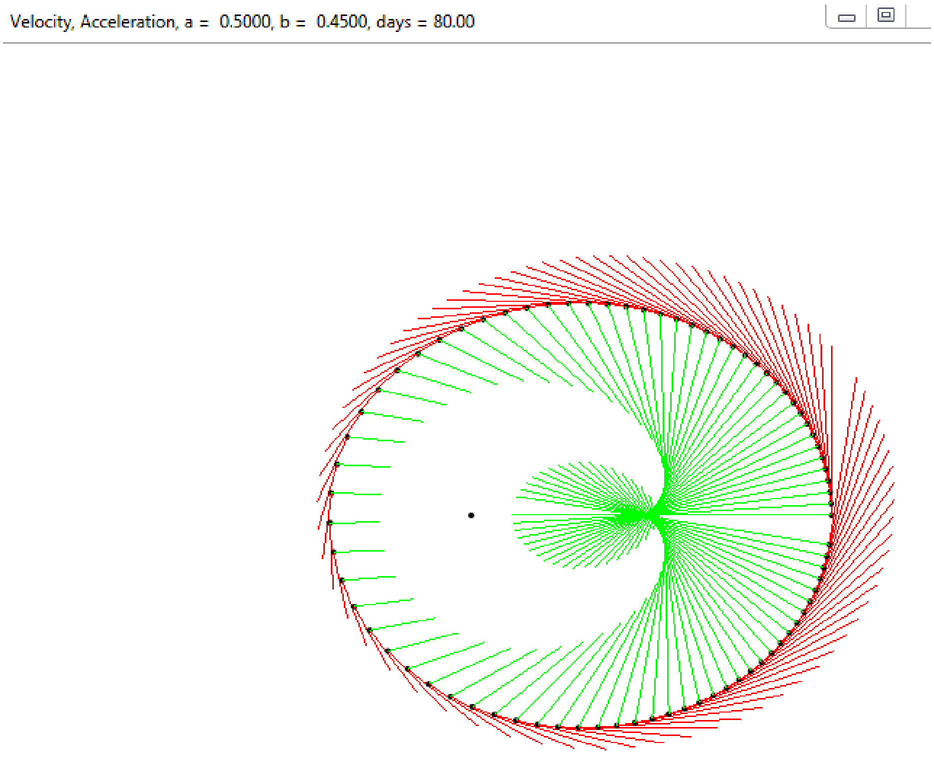

Figure 4

Links

1. Kinematics of the motion of a point along an ellipse, https://www.academia.edu/94295697/Ellipse_kinematics

2. Newton's law of universal gravitation, https://en.wikipedia.org/wiki/Newton%27s_law_of_universal_gravitation

Applications

1. Derivation of formulas for velocity and acceleration using second-order curves:

There is a system of equations for a parametric pendulum (1)

The parameter is time (t).

$$\begin{cases} x = r(\varphi(t)) \cdot cos(\varphi(t)) \\ \boldsymbol{y} = r(\varphi(t)) \cdot sin(\varphi(t)) \end{cases} \tag{1.1}$$

Let's substitute the radius of the ellipse relative to the focus into system (1):

$$r\big(\varphi(t)\big) = \frac{b^2}{a\big(1 - e * cos(\varphi(t))\big)} \tag{1.2}$$

$$\begin{cases} x = \frac{b^2}{a\left(1-e*cos\left(\varphi(t)\right)\right)} \cdot cos\left(\varphi(t)\right) \\ y = \frac{b^2}{a\left(1-e*cos\left(\varphi(t)\right)\right)} \cdot sin\left(\varphi(t)\right) \end{cases} \tag{1.3}$$

Let's differentiate twice. We'll get the coordinates of velocity and acceleration:

$$\dot{x} = \frac{d}{dt}\Big(r\big(\varphi(t)\big)cos\big(\varphi(t)\big)\Big) = -\frac{b^2*\dot{\varphi}*\sin\big(\varphi(t)\big)}{a\big(e*\cos\big(\varphi(t)\big)-1\big)^2} = \frac{r^2*\dot{\varphi}*\sin\big(\varphi(t)\big)}{e*\cos\big(\varphi(t)\big)-1} \tag{1.4}$$

$$\dot{y} = \frac{d}{dt}\left(\frac{p}{1-e*cos\big(\varphi(t)\big)} sin\big(\varphi(t)\big)\right) = \frac{b^2*\dot{\varphi}*\big(-e+\cos\big(\varphi(t)\big)\big)}{a\big(e*\cos\big(\varphi(t)\big)-1\big)^2} = \frac{r^2*\dot{\varphi}*\big(-e+\dot{\cos}\big(\varphi(t)\big)\big)}{1-e*\cos\big(\varphi(t)\big)} \tag{1.5}$$

$$\ddot{x} = \frac{b^2\Big(\Big(-e*cos\big(\varphi(t)\big)*sin\big(\varphi(t)\big)+sin\big(\varphi(t)\big)\Big)\ddot{\varphi}+\dot{\varphi}^2\Big(e*cos\big(\varphi(t)\big)^2-2e+cos\big(\varphi(t)\big)\Big)\Big)}{a\big(e*cos\big(\varphi(t)\big)-1\big)^3} \tag{1.6}$$

$$\ddot{y} = \frac{-b^2\Big(\Big(-cos\big(\varphi(t)\big)\big(e*cos\big(\varphi(t)\big)-1\big)+e\Big)\ddot{\varphi}+2\dot{\varphi}^2\Big(e^2-\frac{e*cos\left(\varphi(t)\right)+1}{2}\Big)sin\big(\varphi(t)\big)\Big)}{a\big(e*cos\big(\varphi(t)\big)-1\big)^3} \tag{1.7}$$

speed $v = \sqrt{\dot{x}^2+\dot{y}^2} = \frac{b^2*\dot{\varphi}*\sqrt{1+e^2-2e*\cos\varphi(t)}}{a(-1+e*\cos\varphi(t))^2} = \frac{r*\dot{\varphi}*\sqrt{1+e^2-2e*\cos\varphi(t)}}{(1-e*\cos\varphi(t))}$ (1.8)

acceleration $\dot{v} = \sqrt{\ddot{x}^2+\ddot{y}^2} =$

$$b^2\left(\begin{array}{c}\frac{\sqrt{(e^2-2e*cos(\varphi(t))+1)\left(e*cos(\varphi(t))-1\right)^2*\ddot{\varphi}^2}}{a\left(e*cos(\varphi(t))-1\right)^3}+ \\ \frac{\sqrt{4\left(e^2-\frac{3*e*cos(\varphi(t))+1}{2}\right)\dot{\varphi}^2\left(e*cos(\varphi(t))\sin(\varphi(t))-1\right)\ddot{\varphi}}}{a\left(e*cos(\varphi(t))-1\right)^3}- \\ \frac{\sqrt{4\dot{\varphi}^4\left(-cos(\varphi(t))^3e^3+\left(e^4-\frac{e^2}{4}\right)cos(\varphi(t))^2+\left(e^3+\frac{e}{2}\right)cos(\varphi(t))-e^4-\frac{1}{4}\right)}}{a\left(e*cos(\varphi(t))-1\right)^3}\end{array}\right) \tag{1.9}$$

There is a system of equations for a parametric pendulum (1)

The parameter is time (*t*).

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Let's differentiate twice. We'll get the coordinates of velocity and acceleration:

$$\dot{x} = \frac{d}{dt}\Big(r(\varphi(t))cos(\varphi(t))\Big) = -\frac{b^2*\dot{\varphi}*\sin(\varphi(t))}{a(e*\cos(\varphi(t))-1)^2} = \frac{r^2*\dot{\varphi}*\sin(\varphi(t))}{e*\cos(\varphi(t))-1} \tag{1.4}$$

$$\dot{y} = \frac{d}{dt}\left(\frac{p}{1-e*cos(\varphi(t))}sin(\varphi(t))\right) = \frac{b^2*\dot{\varphi}*(-e+\cos(\varphi(t)))}{a(e*\cos(\varphi(t))-1)^2} = \frac{r^2*\dot{\varphi}*(-e+\cos(\varphi(t)))}{1-e*\cos(\varphi(t))} \tag{1.5}$$

$$\ddot{x} = \frac{b^2\Big(\big(-e*cos(\varphi(t))*sin(\varphi(t))+sin(\varphi(t))\big)\ddot{\varphi}+\dot{\varphi}^2\big(e*cos(\varphi(t))^2-2e+cos(\varphi(t))\big)\Big)}{a(e*cos(\varphi(t))-1)^3} \tag{1.6}$$

$$\ddot{y} = \frac{-b^2\Big(\big(-cos(\varphi(t))(e*cos(\varphi(t))-1)+e\big)\ddot{\varphi}+2\dot{\varphi}^2\big(e^2-\frac{e*cos(\varphi(t))+1}{2}\big)sin(\varphi(t))\Big)}{a(e*cos(\varphi(t))-1)^3} \tag{1.7}$$

speed $v = \sqrt{\dot{x}^2+\dot{y}^2} = \frac{b^2*\dot{\varphi}*\sqrt{1+e^2-2e*\cos\varphi(t)}}{a(-1+e*\cos\varphi(t))^2} = \frac{r*\dot{\varphi}*\sqrt{1+e^2-2e*\cos\varphi(t)}}{(1-e*\cos\varphi(t))}$ (1.8)

acceleration $\dot{v} = \sqrt{\ddot{x}^2+\ddot{y}^2} =$

$$b^2\left(\frac{\sqrt{(e^2-2e*cos(\varphi(t))+1)(e*cos(\varphi(t))-1)^2*\ddot{\varphi}^2}}{a(e*cos(\varphi(t))-1)^3} + \frac{\sqrt{4\big(e^2-\frac{3*e*cos(\varphi(t))+1}{2}\big)\dot{\varphi}^2(e*cos(\varphi(t))\sin(\varphi(t))-1)\ddot{\varphi}}}{a(e*cos(\varphi(t))-1)^3} - \frac{\sqrt{4\dot{\varphi}^4\big(-cos(\varphi(t))^3e^3+\big(e^4-\frac{e^2}{4}\big)cos(\varphi(t))^2+\big(e^3+\frac{e}{2}\big)cos(\varphi(t))-e^4-\frac{1}{4}\big)}}{a(e*cos(\varphi(t))-1)^3}\right) \tag{1.9}$$

2. Table 1. Calculations of the motion of the Moon and Earth around the barycenter. The barycenter is located at the left focus. The motion begins from aphelion.

φ_i – angle, i = 0, 1, ..., 99.
r_{Moon} – radius from the center of the Moon to the barycenter, r_{Earth} – radius from the center of the Earth to the barycenter.

$\dot{v}_1, \dot{v}_2$ – corresponding normal accelerations.

m_{Moon}, m_{Earth} – masses of the Moon and the Earth.

Moon - Earth						
N	φ	r_{Moon}	r_{Earth}	$m_{Moon}\dot{v}_1$	$m_{Earth}\dot{v}_2$	$G\frac{m_1m_2}{r^2}$
0	0	0.002712	3.34E-05	2.11E+20	2.11E+20	1.74E+20
1	0.056323	0.002712	3.34E-05	2.11E+20	2.11E+20	1.74E+20
2	0.112667	0.002711	3.33E-05	2.11E+20	2.11E+20	1.74E+20
3	0.169053	0.00271	3.33E-05	2.11E+20	2.11E+20	1.74E+20
4	0.225502	0.002708	3.33E-05	2.10E+20	2.10E+20	1.74E+20

5	0.282034	0.002706	3.33E-05	2.10E+20	2.10E+20	1.74E+20
6	0.338671	0.002703	3.32E-05	2.10E+20	2.09E+20	1.75E+20
7	0.395432	0.0027	3.32E-05	2.09E+20	2.09E+20	1.75E+20
8	0.452339	0.002697	3.32E-05	2.08E+20	2.08E+20	1.76E+20
9	0.509411	0.002692	3.31E-05	2.08E+20	2.08E+20	1.76E+20
10	0.566669	0.002688	3.31E-05	2.07E+20	2.07E+20	1.77E+20
11	0.624133	0.002683	3.30E-05	2.06E+20	2.06E+20	1.77E+20
12	0.681821	0.002677	3.29E-05	2.05E+20	2.05E+20	1.78E+20
13	0.739753	0.002671	3.29E-05	2.05E+20	2.04E+20	1.79E+20
14	0.797947	0.002665	3.28E-05	2.04E+20	2.03E+20	1.80E+20
15	0.856423	0.002659	3.27E-05	2.03E+20	2.02E+20	1.81E+20
16	0.915197	0.002652	3.26E-05	2.01E+20	2.01E+20	1.82E+20
17	0.974286	0.002644	3.25E-05	2.00E+20	2.00E+20	1.83E+20
18	1.03371	0.002637	3.24E-05	1.99E+20	1.99E+20	1.84E+20
19	1.09348	0.002629	3.23E-05	1.98E+20	1.98E+20	1.85E+20
20	1.15361	0.002621	3.22E-05	1.97E+20	1.97E+20	1.86E+20
21	1.21412	0.002613	3.21E-05	1.95E+20	1.95E+20	1.87E+20
22	1.27502	0.002604	3.20E-05	1.94E+20	1.94E+20	1.88E+20
23	1.33632	0.002596	3.19E-05	1.93E+20	1.93E+20	1.90E+20
24	1.39804	0.002587	3.18E-05	1.92E+20	1.92E+20	1.91E+20
25	1.46018	0.002578	3.17E-05	1.90E+20	1.90E+20	1.92E+20
26	1.52275	0.002569	3.16E-05	1.89E+20	1.89E+20	1.94E+20
27	1.58575	0.00256	3.15E-05	1.88E+20	1.88E+20	1.95E+20
28	1.6492	0.002551	3.14E-05	1.86E+20	1.86E+20	1.96E+20
29	1.7131	0.002542	3.13E-05	1.85E+20	1.85E+20	1.98E+20
30	1.77744	0.002533	3.12E-05	1.84E+20	1.84E+20	1.99E+20
31	1.84223	0.002525	3.10E-05	1.83E+20	1.82E+20	2.00E+20
32	1.90746	0.002516	3.09E-05	1.81E+20	1.81E+20	2.02E+20
33	1.97314	0.002508	3.08E-05	1.80E+20	1.80E+20	2.03E+20
34	2.03924	0.0025	3.07E-05	1.79E+20	1.79E+20	2.04E+20
35	2.10578	0.002492	3.06E-05	1.78E+20	1.78E+20	2.06E+20
36	2.17272	0.002484	3.06E-05	1.77E+20	1.77E+20	2.07E+20
37	2.24006	0.002477	3.05E-05	1.76E+20	1.76E+20	2.08E+20
38	2.30779	0.00247	3.04E-05	1.75E+20	1.75E+20	2.09E+20
39	2.37588	0.002464	3.03E-05	1.74E+20	1.74E+20	2.10E+20
40	2.44432	0.002458	3.02E-05	1.73E+20	1.73E+20	2.11E+20
41	2.51307	0.002452	3.02E-05	1.72E+20	1.72E+20	2.12E+20
42	2.58213	0.002447	3.01E-05	1.72E+20	1.72E+20	2.13E+20
43	2.65145	0.002443	3.00E-05	1.71E+20	1.71E+20	2.14E+20
44	2.72101	0.002439	3.00E-05	1.71E+20	1.70E+20	2.15E+20
45	2.79078	0.002435	3.00E-05	1.70E+20	1.70E+20	2.15E+20
46	2.86072	0.002433	2.99E-05	1.70E+20	1.70E+20	2.16E+20
47	2.93081	0.00243	2.99E-05	1.69E+20	1.69E+20	2.16E+20
48	3.00101	0.002429	2.99E-05	1.69E+20	1.69E+20	2.17E+20
49	3.07128	0.002428	2.99E-05	1.69E+20	1.69E+20	2.17E+20
50	3.14159	0.002428	2.99E-05	1.69E+20	1.69E+20	2.17E+20
51	3.2119	0.002428	2.99E-05	1.69E+20	1.69E+20	2.17E+20
52	3.28217	0.002429	2.99E-05	1.69E+20	1.69E+20	2.17E+20

53	3.35236	0.00243	2.99E-05	1.69E+20	1.69E+20	2.16E+20
54	3.42245	0.002433	2.99E-05	1.70E+20	1.70E+20	2.16E+20
55	3.4924	0.002435	3.00E-05	1.70E+20	1.70E+20	2.15E+20
56	3.56217	0.002439	3.00E-05	1.71E+20	1.70E+20	2.15E+20
57	3.63173	0.002443	3.00E-05	1.71E+20	1.71E+20	2.14E+20
58	3.70105	0.002447	3.01E-05	1.72E+20	1.72E+20	2.13E+20
59	3.77011	0.002452	3.02E-05	1.72E+20	1.72E+20	2.12E+20
60	3.83886	0.002458	3.02E-05	1.73E+20	1.73E+20	2.11E+20
61	3.9073	0.002464	3.03E-05	1.74E+20	1.74E+20	2.10E+20
62	3.97539	0.00247	3.04E-05	1.75E+20	1.75E+20	2.09E+20
63	4.04312	0.002477	3.05E-05	1.76E+20	1.76E+20	2.08E+20
64	4.11046	0.002484	3.06E-05	1.77E+20	1.77E+20	2.07E+20
65	4.1774	0.002492	3.06E-05	1.78E+20	1.78E+20	2.06E+20
66	4.24393	0.0025	3.07E-05	1.79E+20	1.79E+20	2.04E+20
67	4.31004	0.002508	3.08E-05	1.80E+20	1.80E+20	2.03E+20
68	4.37572	0.002516	3.09E-05	1.81E+20	1.81E+20	2.02E+20
69	4.44095	0.002525	3.10E-05	1.83E+20	1.82E+20	2.00E+20
70	4.50574	0.002533	3.12E-05	1.84E+20	1.84E+20	1.99E+20
71	4.57008	0.002542	3.13E-05	1.85E+20	1.85E+20	1.98E+20
72	4.63398	0.002551	3.14E-05	1.86E+20	1.86E+20	1.96E+20
73	4.69742	0.00256	3.15E-05	1.88E+20	1.88E+20	1.95E+20
74	4.76043	0.002569	3.16E-05	1.89E+20	1.89E+20	1.94E+20
75	4.823	0.002578	3.17E-05	1.90E+20	1.90E+20	1.92E+20
76	4.88514	0.002587	3.18E-05	1.92E+20	1.92E+20	1.91E+20
77	4.94685	0.002596	3.19E-05	1.93E+20	1.93E+20	1.90E+20
78	5.00816	0.002604	3.20E-05	1.94E+20	1.94E+20	1.88E+20
79	5.06906	0.002613	3.21E-05	1.95E+20	1.95E+20	1.87E+20
80	5.12957	0.002621	3.22E-05	1.97E+20	1.97E+20	1.86E+20
81	5.1897	0.002629	3.23E-05	1.98E+20	1.98E+20	1.85E+20
82	5.24947	0.002637	3.24E-05	1.99E+20	1.99E+20	1.84E+20
83	5.30889	0.002644	3.25E-05	2.00E+20	2.00E+20	1.83E+20
84	5.36798	0.002652	3.26E-05	2.01E+20	2.01E+20	1.82E+20
85	5.42676	0.002659	3.27E-05	2.03E+20	2.02E+20	1.81E+20
86	5.48523	0.002665	3.28E-05	2.04E+20	2.03E+20	1.80E+20
87	5.54343	0.002671	3.29E-05	2.05E+20	2.04E+20	1.79E+20
88	5.60136	0.002677	3.29E-05	2.05E+20	2.05E+20	1.78E+20
89	5.65905	0.002683	3.30E-05	2.06E+20	2.06E+20	1.77E+20
90	5.71651	0.002688	3.31E-05	2.07E+20	2.07E+20	1.77E+20
91	5.77377	0.002692	3.31E-05	2.08E+20	2.08E+20	1.76E+20
92	5.83084	0.002697	3.32E-05	2.08E+20	2.08E+20	1.76E+20
93	5.88775	0.0027	3.32E-05	2.09E+20	2.09E+20	1.75E+20
94	5.94451	0.002703	3.32E-05	2.10E+20	2.09E+20	1.75E+20
95	6.00114	0.002706	3.33E-05	2.10E+20	2.10E+20	1.74E+20
96	6.05768	0.002708	3.33E-05	2.10E+20	2.10E+20	1.74E+20
97	6.11412	0.00271	3.33E-05	2.11E+20	2.11E+20	1.74E+20
98	6.17051	0.002711	3.33E-05	2.11E+20	2.11E+20	1.74E+20