Octonionic Preon Model as an alternative to the Higgs mechanism for the Mass-ratio Prediction of the Weak and Higgs Bosons

Jau Tang[1,*] and Qiang Tang[2]

[1]Institute of Technological Sciences, Wuhan University, Wuhan 430074, China

[2]Anhui University of Science and Technology, Huainan, Anhui 232000, China

[*]Corresponding author: Jau Tang, Wuhantang72@gmail.com

ORCID: 0000-0003-2078-1513

DOI: 10.5281/zenodo.14751111

Abstract

Unlike the conventional treatment of the weak bosons in the Yang-Mills theory, which utilizes the Dirac equation for a point-like particle with no internal degrees of freedom, we propose an octonionic preon model to describe the internal dynamics of this vector0boson family with a photon as a isospin singlet, Z, W^+, and W^- bosons as a triplet. Instead of the Higgs mechanism, their masses are acquired from the internal dynamics of the chiral pair via strong spin-exchange couplings. Assuming couplings involving Gell-Mann's lambda matrices, with no adjustable parameters, we predict m_W/m_Z = sqrt(3)/2~ 0.87 vs. 0.88, a Weinberg angle of 30^0 vs. 29^o, decay width m_H/m_W = sqrt (3)/2~ 0.87 vs. 0.84, and a Higgs boson, as a composite of W and Z bosons, with m_W/m_Z = sqrt(7/3)~ 1.53 vs. 1.56 experimentally. These small discrepancies can be accounted for if weak interaction couplings are included. We use octonion operators to represent these particles and to elucidate their connections, and the topological structures' relations to fiber bare bundles and Hopf fibration. Moreover, we elucidate that our proposed preons are essentially the foundation for ding the hypercomplex algebra, and they are the building blocks for the composite particles that represent the topological structures in the higher-dimensional spacetime.

Keywords: Weak bosons, Preon, Octonion, Higgs mechanism, Yang-Mills theory, Mass gap

1. **Introduction**

In the Standard Model (STM)[1-3] of particle physics, the W and Z vector bosons[4-5], the carriers for the weak force, are considered point-like elementary particles and are not composed of smaller constituents. However, such a line of thinking is logically inconsistent, because an infinitely small point-like object should have neither size nor internal structure, yet , a STM particle is assumed to own spin, charge, color charge, or isospin. In addition, STM could not explain the origins of the three generations[6] of leptons and quarks[7], their mass ratios which follow a simple Koide formula.[8] It is unclear why neutrinos have a small mass[9] and why some particles remain massless even though the Higgs field[10-11] is omnipresent. According to the Yang-Mills theory[12] in the conventional electroweak theory,[13] a point-like particle is assumed, and there are no extra degrees of freedom for the initial massless weak gauge bosons to acquire a mass. The dilemma was rescued decades later by Higgs' proposal of a coupling mechanism to an external scalar Higgs field. Unlike the Higgs Mechanism, we propose a model based on massless chiral preons as a building block for the isospin singlet and triplet vector bosons. According to this model, an interacting massless isospin-1/2 preon pair could form a massless singlet with oppositely aligned spins, and a massive weak boson triplet with parallel spins. The singlet-triplet energy splitting is a result of spin-spin exchange interaction. Therefore, the photon possesses spin-1 and isospin-0, but the weak bosons own spin-1 but isospin-1. Our view is shared by many other researchers who have considered higher-dimensional models as alternatives to the existing theories to explain some unanswered problems facing the Standard Model, such as the mass ratios and the existence of three leptons and quark generations, etc.

Our preon-pair idea originates from the singlet-triplet formation and splitting in molecular physics.[14.15] To explain the formation of a photon as an isospin singlet and a weak boson as a triplet in a family, we propose this family of four is formed by two doublets so that$\mathbf{2} \times \mathbf{2} = \mathbf{3} \oplus \mathbf{1}$, like a singlet-triplet family in quantum theory for chemical bonds, except that the spin is an isospin related to internal degrees of freedom. For two spin-1/2 vectors, $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$. The v0, depending$\boldsymbol{S} = \boldsymbol{s}_1 + \boldsymbol{s}_2$, and one has $\boldsymbol{S}^2 \equiv \boldsymbol{S} \cdot \boldsymbol{S} = \boldsymbol{s}_1 \cdot \boldsymbol{s}_1 + 2\boldsymbol{s}_1 \cdot \boldsymbol{s}_2 + \boldsymbol{s}_2 \cdot \boldsymbol{s}_2$, which leads to $\boldsymbol{S}^2 = S(S+1) =$ 2 or 0, depending on whether two spins are parallel with $S = 1$ or opposite with $S = 0$, where $\boldsymbol{s}_1 \cdot \boldsymbol{s}_1 = \boldsymbol{s}_2 \cdot \boldsymbol{s}_2 = 3/4$ for spin-1/2 particles. For the triplet, there ae three states, $|\uparrow\uparrow\rangle, (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}, |\downarrow\downarrow\rangle$, and for the singlet there is one state $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$. The energy splitting between the triplet and the singlet is caused by the exchange interaction involving $\boldsymbol{s}_1 \cdot \boldsymbol{s}_2$ between two spin-1/2 particles with the eigenenergy given by $E = \Delta\,(1/2 + 2\boldsymbol{s}_1 \cdot \boldsymbol{s}_2)/2$ so that the singlet with S = 0 is at zero energy and the triple with S = 1 is at energy Δ.

1. Theory

We shall show that both W and Z bosons possess an internal structure and acquire their rest masses via the exchange interactions between the paired preons. Unlike the Standard Model, our views of these vector bosons as composites are supported by the existence of the photon as an isospin singlet, the weak boson as a triplet, and the Koide mass ratio for charged leptons and quarks. Our dual-preon model for the vector bosons offers an alternative to the conventional electroweak theory which treats them as a point-like object, it also differs from the Higgs mechanism that invokes a spontaneously broken scalar field. We shall show that the interacting preon pairs with opposite chirality have an internal structure with their mass acquired from internal kinetic energy. We shall demonstrate the advantage of this by making a fairly accurate prediction, with no adjustable parameters, of their mass ratios among W, Z, and Higgs bosons, and the decay width of the weak bosons. In the last theory section, we shall elucidate the connections of octonions between octonion's mathematical structures, the representation of these bosons and preons, their underlying symmetry and physical properties.

2.1. Boson-type preon model

In this work about the vector boson family of photons and weak bosons, we first consider a real-value dual-component description for the unified treatment of electroweak interactions. According to this model, there are two types of equations, one for the boson-type preon and the other for the fermion-type preon. A boson-type preon contains two degrees of freedom and can be represented by two real-valued wave functions. A fermion-type preon is a spinor constructed from a pair of boson-type preons. We first consider

$$
\begin{array}{c}
\quad \boldsymbol{f}(t,\boldsymbol{r}) \qquad\qquad \boldsymbol{g}(t,\boldsymbol{r}) \\
\frac{\partial}{\partial t}\begin{pmatrix}\boldsymbol{f}(t,\boldsymbol{r})\\ \boldsymbol{g}(t,\boldsymbol{r})\end{pmatrix} = -(\nabla\times)\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\boldsymbol{f}(t,\boldsymbol{r})\\ \boldsymbol{g}(t,\boldsymbol{r})\end{pmatrix} \\
\boldsymbol{f}(t,\boldsymbol{r}) = \begin{pmatrix}f_1(t,\boldsymbol{r})\\ f_2(t,\boldsymbol{r})\\ f_3(t,\boldsymbol{r})\end{pmatrix}, \boldsymbol{g}(t,\boldsymbol{r}) = \begin{pmatrix}g_1(t,\boldsymbol{r})\\ g_2(t,\boldsymbol{r})\\ g_3(t,\boldsymbol{r})\end{pmatrix} \\
\sigma_t \equiv \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}, {\sigma_t}^2 \equiv -\boldsymbol{I}_2
\end{array} \qquad (2A)
$$

where ε_{ijk} is the Levi-Civita symbol for the cross-product operation. The skew-symmetric matrix σ_t plays a role like the imaginary number in the conventional quantum theory that uses complex wave functions. If one defines a complex wave function one can rewrite the above equation as

$$
i\,\partial/\partial t\,\boldsymbol{\Psi}(t,\boldsymbol{r}) = (\nabla\times)\boldsymbol{\Psi}(t,\boldsymbol{r}), \boldsymbol{\Psi}(t,\boldsymbol{r}) \equiv \boldsymbol{f}(t,\boldsymbol{r}) + i\boldsymbol{g}(t,\boldsymbol{r}) \qquad (2B)
$$

Eqs. (2A-2B) represents the 1st-order differential wave equation for a massless "preon" which has dual-component real-value wave functions. Taking its 2nd-order time derivative, one obtains

$$
\partial^2/\partial t^2\,\boldsymbol{\Psi}(t,\boldsymbol{r}) = -\nabla\times\left(\nabla\times\boldsymbol{\Psi}(t,\boldsymbol{r})\right) = -\nabla\left(\nabla\cdot\boldsymbol{\Psi}(t,\boldsymbol{r})\right) + \nabla^2\boldsymbol{\Psi}(t,\boldsymbol{r}) = \nabla^2\boldsymbol{\Psi}(t,\boldsymbol{r}) \qquad (2C)
$$

where $\nabla\cdot\boldsymbol{\Psi}(t,\boldsymbol{r}) = 0$, or $\nabla\cdot F(t,\boldsymbol{r}) = \nabla\cdot G(t,\boldsymbol{r}) = 0$ was assumed in vacuum-like the conditions for an electric and magnetic field. Eq. (2C) indicates that such a preon is massless. The curl operator in Eqs. (2A) - (2B) coupes cyclically each axial component to two other component. It has a topological structure like three strains of intertwined fiber bundles of Möbius tori, which is a 3D extension of a 2D Möbius strip.[16] This topological structure underlies the three-color concepts for quarks and gluons and is related to a branch of differential topology called the Hopf fibratio.[17\]

2.2. Fermion-type preon model

In addition to the boson-type preon, we now consider the fermion-type preon. Similar to Dirac's operator approach[18,] except that a curl operator is used here, we incorporate four anti-commutative matrices into the momentum operators in Eq. (2B) to obtain

$$
i\frac{\partial}{\partial t}\boldsymbol{A}_0 \otimes \boldsymbol{\Psi}_i(t,\boldsymbol{r}) = \sum_{j,k=1}^{3} \varepsilon_{ijk}\frac{\partial}{\partial x_j}\boldsymbol{A}_j \otimes \boldsymbol{\Psi}_k(t,\boldsymbol{r}), i,j,k = 1,2,3 \qquad (3A)
$$

where ⊗ represents a tensor product and the operator $\boldsymbol{A}_\mu, \mu = 0,1,2,3$, are related to Dirac's gamma matrices which follow anti-commutative relations of $\{\boldsymbol{A}_\mu, \boldsymbol{A}_\nu\} = 2\delta_{\mu\nu}\boldsymbol{I}_4, \mu, \nu = 0,1,2,3$. By taking the 2nd-order time derivative of Eq. (3A) and the use of Eq (3B) we obtain

$$-\boldsymbol{A}_0{}^2 \otimes \frac{\partial^2}{\partial t^2}\boldsymbol{\Psi}_i(t,\boldsymbol{r}) = \sum_{j,k}\sum_{l,m} \varepsilon_{ijk}\,\varepsilon_{klm}\,\boldsymbol{A}_j\boldsymbol{A}_l \otimes \partial^2\boldsymbol{\Psi}_m(t,\boldsymbol{r})/\partial x_j \partial x_l \qquad (3B)$$

Using the relations for summing up two Levi-Civita symbols and with the anti-commutative relations, one obtains $\boldsymbol{I}_4 \otimes (\partial^2/\partial t^2 - \nabla^2)\boldsymbol{\Psi}(t,\boldsymbol{r}) = 0$, where $\nabla \cdot \boldsymbol{\Psi}_i(t,\boldsymbol{r})$ vanishes in vacuum. The 2nd-order derivative wave equation for either boson- 0r fermion-type preon leads to Eq. (1) for a massless particle.

In a quantum system, two spin-1/2 particles can form a singlet and triplet, i.e., $\mathbf{2} \times \mathbf{2} = \mathbf{3} \oplus \mathbf{1}$, we use such a pair of massless spin-1/2 preons with an opposite chirality to construct the massless singlet photon and the massive weak boson triplet. In such a composite system with a preon at $\boldsymbol{r}_1$ and the other at $\boldsymbol{r}_2$, one can define the average position $\boldsymbol{r} = (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2$ and the relative position $\boldsymbol{R} = (\boldsymbol{r}_1 - \boldsymbol{r}_2)/2$. For a singlet preon pair without an internal structure, one only needs $\boldsymbol{r}$ to describe its dynamics. Let's consider the following equation for a pair of real-value paired wave functions $(\boldsymbol{f}(t,\boldsymbol{r}), \boldsymbol{g}(t,\boldsymbol{r}))$ and $(\boldsymbol{F}(t,\boldsymbol{r}), \boldsymbol{G}(t,\boldsymbol{r}))$

$$\begin{gathered} -\frac{\partial}{\partial t}\begin{pmatrix}\Psi_1(t,r)\\ \Psi_2(t,r)\end{pmatrix} = (\nabla \times)\begin{pmatrix}\sigma_t & 0\\ 0 & -\sigma_t\end{pmatrix} \\ \Psi_1(t,r) = \begin{pmatrix} f(t,r)\\ g(t,r)\end{pmatrix}, \Psi_1(t,r) = \begin{pmatrix}F(t,r)\\ G(t,r)\end{pmatrix} \end{gathered} \qquad (4A)$$

Using complex-vale wave functions $\Psi_1(t,r) = f(t,r) + i\,g(t,r), \Psi_2(t,r) = F(t,r) + i\,G(t,r)$ the above equation becomes

$$\boldsymbol{i}\frac{\partial}{\partial \boldsymbol{t}}\boldsymbol{\Psi}(\boldsymbol{t},\mathbf{r}) = (\sigma_3 \otimes \mathbf{I}_2)\nabla \times \boldsymbol{\Psi}(\boldsymbol{t},\mathbf{r}), \boldsymbol{\Psi}(\boldsymbol{t},\mathbf{r}) \equiv \begin{pmatrix}\boldsymbol{\Psi}_1(\boldsymbol{t},\mathbf{r})\\ \boldsymbol{\Psi}_2(\boldsymbol{t},\mathbf{r})\end{pmatrix} \qquad (4B)$$

The above equation leads to $\nabla \times \boldsymbol{g} = \partial \boldsymbol{f}/\partial t$, $\nabla \times \boldsymbol{f} = -\partial \boldsymbol{g}/\partial t$, $\nabla \cdot \boldsymbol{f} = 0, \nabla \cdot \boldsymbol{g} = 0$ for the preon. For the preon with the opposite chirality one has $\nabla \times \boldsymbol{F} = \partial \boldsymbol{G}/\partial t, \nabla \times \boldsymbol{G} = -\partial \boldsymbol{F}/\partial t, \nabla \cdot \boldsymbol{F} = 0, \nabla \cdot \boldsymbol{G} = 0$. We can regard a photon as a singlet of an opposite-chirality preon pair with no internal structure. By assigning the electric field $\mathbf{E}(\boldsymbol{t},\mathbf{r})$ to $\boldsymbol{\Psi}_1(\boldsymbol{t},\mathbf{r})$ and the magnetic field $\mathbf{B}(\boldsymbol{t},\mathbf{r})$ to $\boldsymbol{\Psi}_2(\boldsymbol{t},\mathbf{r})$, Eq. (4B) is identical to Maxwell's equation in a vacuum without a source.

2.3. Preon-pair model for weak bosons

Unlike the photon which is an isospin-0 singlet with no internal degrees of freedom, , we construct an isospin triplet from an interacting massless preon pair with opposite chirality. We shall extend the dual-component model to the triplet weak vector bosons. In comparison to the photon singlet, the weak bosons are represented by a triplet. Therefore, we consider them having an internal degree of freedom. For a triplet with isospin-1, we employ Gell-Mann's 3x3 lambda matrices to describe the couplings in their internal dynamics, involving a coordinate vector$R$. To distinguish the two sets of coordinates, we denote ∇_r and ∇_R as the corresponding gradients. We consider the following equation for a pair of massless preon $\boldsymbol{\Psi}_1(\boldsymbol{t},\mathbf{r},\boldsymbol{R})$ and a $\boldsymbol{\Psi}_2(\boldsymbol{t},\mathbf{r},\boldsymbol{R})$ with an opposite chirality

$$\boldsymbol{i}\frac{\partial}{\partial \boldsymbol{t}}\boldsymbol{\Psi}(\boldsymbol{t},\mathbf{r},\boldsymbol{R}) = (\sigma_3 \otimes \mathbf{I}_3)\nabla_r \times \boldsymbol{\Psi}(\boldsymbol{t},\mathbf{r},\boldsymbol{R}) + \left(\sum_{\boldsymbol{n}=1}^{3} \boldsymbol{Q_n}\sigma_1 \otimes \boldsymbol{\Lambda_n} + \boldsymbol{Q}\sigma_2 \otimes \boldsymbol{\Lambda}_9\right)\nabla_{\boldsymbol{R}} \times \boldsymbol{\Psi}(\boldsymbol{t},\mathbf{r},\boldsymbol{R})$$
$$\boldsymbol{\Psi}(\boldsymbol{t},\mathbf{r},\boldsymbol{R}) = \begin{pmatrix}\boldsymbol{\Psi}_1(\boldsymbol{t},\mathbf{r},\boldsymbol{R})\\ \boldsymbol{\Psi}_2(\boldsymbol{t},\mathbf{r},\boldsymbol{R})\end{pmatrix} \tag{5A}$$

where the operators $\boldsymbol{\Lambda}_n$ are related to a subset of Gell-Mann's 3x3 SU(3) generator[1-2] matrices which are defined below[18]

$$\boldsymbol{\Lambda}_1 = \begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, \boldsymbol{\Lambda}_2 = \begin{pmatrix}0 & -i & 0\\ i & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, \boldsymbol{\Lambda}_3 = \begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix}, \boldsymbol{\Lambda}_8 = \frac{1}{\sqrt{3}}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{pmatrix}$$
$$\boldsymbol{\Lambda}_0 = \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}, \boldsymbol{\Lambda}_9 \equiv \frac{2}{\sqrt{3}}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix} = \boldsymbol{\Lambda}_8 - \sqrt{\sum_{k=1,2,3} {\boldsymbol{\Lambda}_k}^2} \tag{5B}$$

where$[\boldsymbol{\Lambda}_k, \boldsymbol{\Lambda}_9] = \{\boldsymbol{\Lambda}_{k,}, \boldsymbol{\Lambda}_9\} = 0, \{\boldsymbol{\Lambda}_{i,}, \boldsymbol{\Lambda}_j\} = 2\delta_{ij}\boldsymbol{\Lambda}_0, i,j = 1,2,3$. The set of three matrices $\boldsymbol{\Lambda}_{i,} i = 1,,2,3$ belongs to SU(2) group, together with$\boldsymbol{\Lambda}_0$ are a subset of SU(3) generators used in Gell-Mann's quark model. These three operators$\sigma_3 \otimes \boldsymbol{I}_3$, and $\sigma_2 \otimes \boldsymbol{\Lambda}_9$ anti-commute, and $Q_1 = Q\sin\theta\cos\phi$, $Q_2 = Q\sin\theta\sin\phi$, $Q_3 = Q\cos\theta$. By taking the 2nd-order time derivative of Eq. (5A) and by Fourier transformation, one obtains

$$(\omega^2 - k^2)(\boldsymbol{I}_2 \otimes \boldsymbol{I}_3) = \boldsymbol{M}^2, \boldsymbol{M}^2 = Q^2K^2\boldsymbol{I}_2 \otimes \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4/3 \end{pmatrix} \tag{6}$$

We have used the operator $\boldsymbol{\Lambda}_9$ for the Z bosom so that it is invariant under SU(2) rotation by $\boldsymbol{\Lambda}_k$ of the W bosons, i.e., $[\boldsymbol{\Lambda}_{k,}\boldsymbol{\Lambda}_9] = \{\boldsymbol{\Lambda}_{k,}\boldsymbol{\Lambda}_9\} = 0. k = 1, 2, 3$. The equality relation is used for the curl operator $-(\nabla \times)(\nabla \times)\boldsymbol{\Psi} = -\nabla(\nabla \cdot \boldsymbol{\Psi}) + \nabla^2\boldsymbol{\Psi} = \nabla^2\boldsymbol{\Psi}$ and the divergence $\nabla_r \cdot \boldsymbol{\Psi}(t, \boldsymbol{r}, R) = \nabla_R \cdot \boldsymbol{\Psi}(t, \boldsymbol{r}, R) = 0$ in vacuum. The coupling strength Q for the singlet-triplet splitting is related to the spin-spin exchange interaction ting energy is between the isospin-1/2 preon pair with the energy $E = \Delta\,(1/2 + 2\boldsymbol{s}_1 \cdot \boldsymbol{s}_2)/2 = S(S+1)/2$ so that the singlet-triplet gap is Δ.. The further splitting between the W boson doubles and the Z boson is due to the coupling involving $(1/2 - 2\,\boldsymbol{s}_{1z}\boldsymbol{s}_{2z})$. In our model we employ $\boldsymbol{\Lambda}_8$, the 8th Gell-Mann matrix, which is a diagonal matrix with a trace $tr(\boldsymbol{\Lambda}_8{}^2) = 2$. This value corresponds to the square of the spin-1 angular momentum $\boldsymbol{S}^2 = S(S+1) = S_x{}^2 + S_y{}^2 + S_z{}^2 = 2\boldsymbol{I}_3$ where $\boldsymbol{S}_y{}^2 = \boldsymbol{\Lambda}_{8,22}{}^2 = 1/3$, $S_x{}^2 = \boldsymbol{\Lambda}_{8,11}{}^2 = 1/3$, and $S_z{}^2 = \boldsymbol{\Lambda}_{8,33}{}^2 = 4/3$. This result implies the spin-spin exchange coupling is anisotropic which is common for magnetic materials in solid state physics.

According to Einstein's mass-energy relation, $E^2 - p^2 - m_0{}^2 = 0$, we obtain tan effective mass-squared diagonal matrix $\boldsymbol{M}^2$. The above result implies that the exchange coupling between the paired isospin-1/2 preons has an anisotropic spin-spin coupling, often seen in magnetic materials, instead of Heisenberg's simplest isotropic coupling. Such an anisotropy breaks the triplet degeneracy with $\boldsymbol{M}^2$ with $m^2 = Q^2K^2\,(2S(S+1) - S_z{}^2)/3 = Q^2K^2\,(2\boldsymbol{S} \cdot \boldsymbol{S} - S_z{}^2)/3$, which leads to $m = 0$ for the photon singlet $|0{,}0\rangle$, $m = QK$ for the $W^{\pm}$ bosons $|1, \pm 1\rangle$, and $m = QK2/\sqrt{3}$ for the Z boson $|1{,}0\rangle$. One can express the mass-square as $m^2 = Q^2K^2(1 + (4\boldsymbol{s}_1 \cdot \boldsymbol{s}_2 - (s_{1z} + s_{2z})^2)/3)$. showing the anisotropic exchange interaction between the spin-1/2 preon pair. Our model predicts a mass ratio $m_W/m_Z = \sqrt{3}/2 \sim 0.8660$, and a Weinberg angle θ_W of 30^0. Given the rest mass for Z boson m_Z =91.1776 Gev/c^2. For W boson m_W = 80.377 Gev/c^2,[19] one has $m_W/m_Z = 0.8815$ and $\theta_W = 29^o$. According to the Higgs mechanism, these parameters require experimental measurements, yet they are derived theoretically from our model with no adjustable parameters.

2.5. Higgs boson as a composite particle

We question the commonly accepted notion of the Higgs boson as an elementary God's particle because it is heavier than W, Z bosons, leptons, and some quarks. To show that it is a composite particle with$m_H = \sqrt{m_Z{}^2 + m_W{}^2}$, we consider the mixing between Z and W bosons in the following equation that involves only time dependence

$$\begin{gathered} i\,\partial \boldsymbol{\Psi}_H/\partial t = (m_W\sigma_1 + m_Z\sigma_2)\boldsymbol{\Psi}_H + \xi\sigma_3\boldsymbol{\Psi}_H, \quad \boldsymbol{\Psi}_H = \begin{pmatrix}\boldsymbol{\Psi}_W \\ \boldsymbol{\Psi}_Z\end{pmatrix} \\ -\partial^2\boldsymbol{\Psi}_H/\partial t^2 = \left(m_W{}^2 + m_Z{}^2 + \xi^2\right)\boldsymbol{\Psi} \end{gathered} \tag{7}$$

where $\boldsymbol{\Psi}_W = (\boldsymbol{\Psi}_{W^+} - \boldsymbol{\Psi}_{W^-})/\sqrt{2}$ is a coherent state of $\boldsymbol{W}^{\pm}$. One obtains $m_H{}^2 = m_W{}^2 + m_Z{}^2$ if $\xi = 0$, indicating the Higgs boson is a composite. W$m_W/m_Z = \sqrt{3}/2$, we predict $m_H/m_W = \sqrt{7/3} \sim 1.528$, which agrees with the experimental ratio of 1,558 with a ~ 2% error.

2.6. The decay width of the weak bosons

To explain the cause for the small error between the theoretical and experimental values, and the decay of the W and Z bosons, we consider the modified equation

$$\begin{gathered} i\partial\,\boldsymbol{\Psi}(t,\boldsymbol{r},R)/\partial t = (\sigma_3 \otimes \boldsymbol{I}_3)\nabla_r \times \boldsymbol{\Psi}(t,\boldsymbol{r},R) \\ +\left(\sum_{n=1}^{3} Q_n\sigma_1 \otimes \boldsymbol{\Lambda}_n + (\alpha_0 + i\alpha)\sigma_1 \otimes \boldsymbol{\Lambda}_0 + \left(\beta_0 + i\beta\right)\sigma_2 \otimes \boldsymbol{\Lambda}_9\right)\nabla_R \times \boldsymbol{\Psi}(t,\boldsymbol{r},R) \end{gathered} \tag{8A}$$

he above equation differs from Eq. (6A) in two additional terms involving $(\alpha_0 + i\alpha) \otimes \boldsymbol{\Lambda}_0$ and $(\beta_0 + i\beta)\sigma_2 \otimes \boldsymbol{\Lambda}_9$, which represents a small perturbation with a non-Hermitian matrix for weak interactions. By taking the 2nd-order time derivative of the modified equation and the Fourier transform, one obtains a 6x6 matrix equation om Eq. (8A). By matrix diagonalization and solving the eigenvalue problem, we obtain

$$\begin{gathered} m_W = (1+\alpha_0)QK\sqrt{1-\alpha^2}(1 + (2\alpha/(1-\alpha^2))^2)^{1/4} cos\left(\phi_W/2\right) \\ \Gamma_W/m_W = tan\left(\phi_W/2\right), \phi_W = tan^{-1}\left(2\,\alpha/\sqrt{1-\alpha^2}\right) \\ m_Z = \left(2/\sqrt{3}\right)\left(1+\beta_0\right)QK\sqrt{1-\beta^2}\left(1 + \left(2\beta/\left(1-\beta^2\right)\right)^2\right)^{1/4} cos\left(\phi_Z/2\right) \\ \Gamma_Z/m_Z = tan\left(\phi_Z/2\right), \phi_Z = tan^{-1}\left(2\,\beta/\sqrt{1-\beta^2}\right) \end{gathered} \tag{8B}$$

For a small$\alpha$and βwe obtain the ratios for the mass and decay width of the W and Z bosons as

$$m_W/m_Z = \left(\sqrt{3}(1+\alpha_0)/2\left(1+\beta_0\right)\right)QK\left(cos\varphi_W/cos\varphi_Z\right) \approx \sqrt{3}/2$$
$$\Gamma_W/\Gamma_Z = \left(\sqrt{3}(1+\alpha_0)/2\left(1+\beta_0\right)\right)\left(sin\varphi_W/sin\varphi_Z\right) \approx \sqrt{3}/2\left(\varphi_W/\varphi_Z\right) \quad (8C)$$

If $\alpha_0 = \beta_0 = 1, \alpha = \beta \to 0$ for the ideal case $\Gamma_W/\Gamma_Z = \sqrt{3}/2 = 0.866$ is predicted, with no adjustable parameters, as compared to the experimental $(\Gamma_W/\Gamma_Z)_{exp}/2.495 = 0.836 \pm 0.017$.[20] The small errors from the experiments can be accounted for if one includes a perturbation term involvingα andβ. Based on the experiments, we obtain $\Gamma_W/m_W \approx \alpha = 0.026$ which is very close to$\Gamma_Z/m_Z \approx \beta = 0.027$. This weak force with a strength ratio of ~ 3% breaks the SU(3) symmetry to cause a small mass shift and decay.

2.7 Relations to octonion algebra

Here, we explain the intricate connections between the concept of preons in our chiral preon-pair model and the mathematical structure of octonion algebra. As illustrated in Fig. 1, Octonion algebra contains a unity element and seven anti-commutative imaginary operators. It consists of a 4-element quaternion algebra with one unity and three anti-commutative imaginary elements, and another quaternion-like quartet but with four anti-commutative imaginary elements. Octonions can be constructed from 4D quaternions by the Cayley0Dickson scheme. Quaternion algebra has wide applications, including the description of 4D Minkowski space, special relativity, and Maxwell's equation for electromagnetism. The other quartet of the 8D octonions contains a pseudo-scalar time operator and a spinor set of spatial operators to describe a particle's internal dynamics.

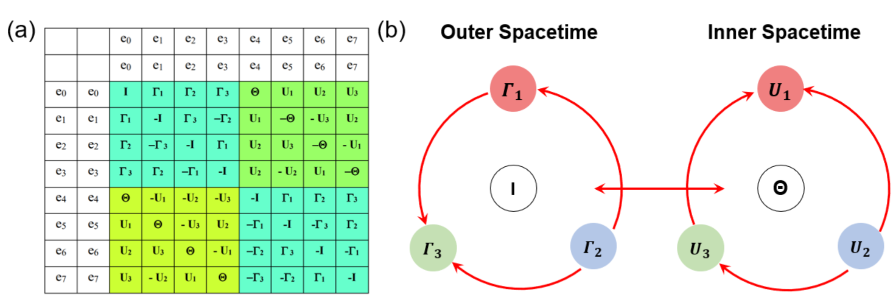

Fig. 1. (a) Multiplication table of 8-element octonions. It consists of a quaternion $\{\mathbf{I}, \mathbf{\Gamma}_1, \mathbf{\Gamma}_2, \mathbf{\Gamma}_3\}$, (an identity element **I** and a spinor triplet, and a set of four anti-commutative imaginary operators. (**B**) The four-element quaternion set $\{\mathbf{I}, \mathbf{\Gamma}_1, \mathbf{\Gamma}_2, \mathbf{\Gamma}_3\}$ describe the exterior 4D spacetime, while the other set $\{\mathbf{\Theta}, \mathbf{U}_1, \mathbf{U}_2, \mathbf{U}_3\}$ describe the internal spacetime.

The basis elements of octonion algebra have been used to construct eight Gell-Mann lambda matrices, the SU(3) generators for description of quarks. Here, we show how to construct these eight lambda matrices using octonions.[22] We first define three pairs of fermion creation and annihilation operators, as $\alpha_1 = (-\boldsymbol{U}_2 + i\boldsymbol{U}_1)/2$, $\alpha_2 = (-\boldsymbol{\Gamma}_3 + i\boldsymbol{\Gamma}_1)/2$, $\alpha_3 = (-\boldsymbol{U}_3 + i\boldsymbol{\Gamma}_2)/2$, $\alpha_1^+ = (\boldsymbol{U}_2 + i\boldsymbol{U}_1)/2$, $\alpha_2^+ = (\boldsymbol{\Gamma}_3 + i\boldsymbol{\Gamma}_1)/2$, $\alpha_3^+ = (\boldsymbol{U}_3 + i\boldsymbol{\Gamma}_2)/2$, which satisfy the anti-commutation relations $\{\alpha_i, \alpha_j\} = \{\alpha_i^+, \alpha_j^+\} = 0, \{\alpha_i, \alpha_j^+\} = \delta_{ij}$.

Using Dirac's notation for the bra and ket, one can define a tensor product $|i\rangle\langle j| \equiv \alpha_i{}^+\alpha_j$ to construct the following eight SU(3) generators, which are related to Gell-Mann's lambda matrices $\boldsymbol{\Lambda}_k$ as

$$\begin{aligned}
\boldsymbol{\Lambda}_1 &= |2\rangle\langle 1| + |1\rangle\langle 2| = i(\boldsymbol{U}_3 - \boldsymbol{U}_2)/2 \\
\boldsymbol{\Lambda}_2 &= -i|1\rangle\langle 2| + i|2\rangle\langle 1| = -i\,(\boldsymbol{U}_1 - \boldsymbol{\Theta}_1)/2 \\
\boldsymbol{\Lambda}_3 &= |1\rangle\langle 1| - |2\rangle\langle 2| = i\,(\boldsymbol{\Gamma}_3 - \boldsymbol{\Gamma}_2)/2 \\
\boldsymbol{\Lambda}_4 &= |1\rangle\langle 3| + |3\rangle\langle 1| = i(\boldsymbol{\Theta}_1 - \boldsymbol{\Gamma}_2)/2 \\
\boldsymbol{\Lambda}_5 &= -i|1\rangle\langle 3| + i|3\rangle\langle 1| = --\,i\,(\boldsymbol{\Gamma}_1 + \boldsymbol{U}_3)/2 \\
\boldsymbol{\Lambda}_6 &= |2\rangle\langle 3| + |3\rangle\langle 2| = -i\,(\boldsymbol{\Gamma}_1 + \boldsymbol{U}_2)/4 \\
\boldsymbol{\Lambda}_7 &= -i|2\rangle\langle 3| + i|3\rangle\langle 2| = -i\,(\boldsymbol{\Theta}_1 - \boldsymbol{\Gamma}_3)/2 \\
\boldsymbol{\Lambda}_8 &= (|1\rangle\langle 1| + |2\rangle\langle 2| - 2|3\rangle\langle 3|)/\sqrt{3} = i\,(\boldsymbol{\Gamma}_3 + \boldsymbol{\Gamma}_2 - 2\boldsymbol{U}_2)/2.
\end{aligned} \tag{10}$$

The creation and annihilation operators used for constructing the Gell-Mann's SU(3) lambda matrices are based on th pairing of two SU(2) spinor sets, $\{\mathbf{\Gamma}_1, \mathbf{\Gamma}_2, \mathbf{\Gamma}_3\}$ and $\{\mathbf{U}_1, \mathbf{U}_2, \mathbf{U}_3\}$ in the quartets of the octonion algebra, physically representing a pair of spin-exchange preons. Our use of Gell-Mann's generators in describing weak bosons implies that these weak boson' large mass originates from strong force for the spin-exchange coupling between the chiral preon pair. We can use the octonion basis elements to construct the boson-type creation and annihilation operators for the photon and gluons, or for fermion-type operators for leptons and quarks. In Table 1, we assign a pair of preons to the neutral Z boson and two charged W bosons. In our ideal model presented in Sec 2.3., the Weinberg angle θ_W is precisely 30 degrees, and a very small deviation from the experiments is due to the inclusion of weak interactions

Table 1. Relations of the weak vector bosons to the octonion-based SU(3) operators

Boson	**Octonion-based Operator**	**Role**
$\mathbf{W}_1$	$\mathbf{\Lambda}_1$	Weak iso-spin generator, x-component
$\mathbf{W}_2$	$\mathbf{\Lambda}_2$	Weak iso-spin generato,r y-component
$\mathbf{W}_3$	$\mathbf{\Lambda}_3$	Weak iso-spin generator, z-component
Y	$\mathbf{\Theta}$	Hypercharge operator
Z	$\mathbf{\Lambda}_9$	Neutral weak boson Z
$\mathbf{W}^+$	$\frac{1}{\sqrt{2}}\left(\mathbf{\Lambda}_1 + \boldsymbol{i}\,\mathbf{\Lambda}_2\right)$	Charged W boson W^+
$\mathbf{W}^-$	$\frac{1}{\sqrt{2}}\left(\mathbf{\Lambda}_1 - \boldsymbol{i}\,\mathbf{\Lambda}_2\right)$	Charged W boson W^-

3. Discussion and conclusions

We propose a chiral preon model to offer physical insights into the electroweak interactions of elementary parties and to derive the mass ratios of the weak and Higgs bosons. Based on Einstein's Pythagorean energy reaction $E = {P_1}^2 + {P_2}^2 + {P_3}^2 + {m_0}^2$ and Dirac's approach of treating E and P_{1k} as operators in 4D spacetime, we treat mass as an operator $\boldsymbol{M}$ in the 5th dimension to describe a particle's internal dynamic energy. We use the Gell-Mann SU(3) generators to describe spin-spin exchange couplings to represent the strong nuclear force. Using such a paired chiral preon model, we determine the masses of the photon singlet, Z and W weak boson triplet, and the Higgs boson as composite particles. This analysis indicates that the mass of the weak bosons is acquired from the strong internal spin-exchange couplings between the chiral preon pair. We show that the symmetrical breaking by weak interaction could result in decay and a small deviation of our predictions. Our model differs from the conventional Standard Model based on the Yang-Mils and Higgs mechanisms. With no adjustable parameters, we theoretically derive $m_W/m_Z = \sqrt{3}/2 \sim 0.87$ vs. 0.88, a Weinberg angle of 30^0 vs. 29^o, decay width $\Gamma_W/\Gamma_Z = \sqrt{3}/2 \sim 0.87$vs. 0.84 ± 0.02, and a Higgs boson as a composite particle of W and Z bosons with

$m_H/m_W = \sqrt{7/3} \sim 1.53$ vs. 1.56, experimentally. We have also shown that these small discrepancies can be accounted for if we include the weak interaction couplings. Therefore, this

preon-model offers better physical insights into the origins of these masses, while the Higgs mechanism could only provide ad hoc and qualitative explorations with less prediction power.

The main reason why the Yang-Mills theory requires the Higgs mechanism for mass acquisition is that the Yang-Mills theory invoks the Dirac equation fpr to a point-like particle with no internal degrees of freedom. For a particle to acquire a rest mass, the Higgs mechanism is postulated for the nonlinear coupling of the particle with an external Higgs scalar field. However, if one assumes the particle has a finite size and charge distribution, this would lead to broken U(1) Lorentz gauge, and an electron would acquire a mass. without the need of the Higgs mechanism with a nonlinear coupling to a Higgs scalar field. According to our preon model, the weak bosons, Higgs bosons, leptons, and quarks in the Standard Model are not point-like entities and could own an internal structure, their effective rest masses are contributed by their internal kinetic dynamics and intrinsic rest masses. Our view is shared by many researchers who have considered higher dimensional alternatives,[21-24] such as octonions, sedenions, or Clifford algebra to ex tend the treatments of particle physics beyond the Standard Model. Particularly, Tarazawa *et al*. have proposed that the Standard Model's elementary particles, such as the Higgs boson and quarks, are a composite of more fundamental constituents.[25-27]

Our model of a preon pair is based on dualism. Mathematically speaking, a quartet can be constructed from two doubles, leading to a family of a singlet and a triplet as $\mathbf{2} \times \mathbf{2} = \mathbf{3} \oplus \mathbf{1}$, two triplets form an octet and a single as $3 \times 3 = 8 \oplus \mathbf{1}$. Physically speaking in this work, 2 represents a chiral preon pair, 3 represents a triple for the weak boson triplet, and 1 is the photon singlet. For the octet, 8 represents an octet of gluons and 1 as a Higgs boson. The number 8 was used in Gell-Mann's "eight-fold way" quark model. We believe one can use the chiral preon pair to construct the fundamental building block for all composite particles. From a boson-type preon pair with real-valued wave functions, a spinor preon can be constructed. Then, layer by layer, a quaternion can be built from paired spinor-type preons, an octonion from paired quaternions, a sedenion from paired octonions. Our view is also supported by the mass ratio between the top quark and the Higgs boson which follows an empirical formula we obtained as$m_T/m_H = 4\sqrt{3}/5 \pm 0.01\%$. Unlike Yang-Mills theory, which assumes point-like particles, our paired rpreon model incorporates a curl operator and Gell-Mann's matrices. To describe a fermion-type preon we follow Dirac's approach in using four anti-commutative operators, our model involves generators of $U(1) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(3)$, which is a subgroup of SU(5) proposed by Georgi-Glashow[28], and SO(10) by Gieorgi[29] for the grand unification theory, and is related to the sedenion algebra model we reported.[30] In contrast, octonion algebra contains SU(3). In our previous work, we showed the link of sedenion algebra to SU(5).[22] In Sec. 2.7, we showed the intricate connections between the octonions, or sedenions more generally, and the mathematical representations for the preons as constituents for the vector boson family or other elementary particles. Therefore, those studies based on octonion or sedenion framework beyond the Standard Model description are essentially equivalent to the preon models. In particular, we show how a massless particle by coupling to the sedenion operators could acquire an effective mass from the kinetic energy in its internal dynamics. Together with this work, our mass-acquiring mechanism has answered the mass-gap problem.[31] The strong nuclear force reflects the spin-spin coupling because the paired spin-1/2 preons of opposite chirality behave like two magnetic dipoles. The dipolar interaction energy has $1/r^3$ dependence, yet the Coulomb interaction has 1/r dependence. Using an upper limit of quark's size of ~ 10^{-15} m, for two electrons with 10^{-15} m separation, the energy due to magnetic dipolar interaction is ~ 36 GeV vs. 1.4 MeV due to Coulomb interaction. This magnetic dipolar energy can reach TeV at ~ 0.4 fm. Unlike dyons with magnetic charge, preons are not magnetic monopoles, the magnetic dipolar interaction between the paired preons could be the origin of the strong interaction. Although we focus on the weak bosons here, we have recently expanded the preon model with $U(1) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(3)$ generators to incorporate the sedenion algebra to treat leptons and quarks.

The dual-preon model for weak bosons can be formulated using the octonion algebra formalism. We have elucidated the physical implications of the quaternion a sub-algebra which represents the external Minkowski space for the photon which is the iso-spin singlet state, while the other quartet of four anti-commutative operators describe the internal dynamics of weak bosons, which belong to the iso-spin triplet. We have shown in Table 1 the assignment of these bosons (photon, Z and W bosons, Higgs boson) to the octonion operators to elucidate the physical meanings of these operators. Therefore, there is a deep connection between the octonion's mathematical structure and the Standard Model particles. The presence of three generations of leptons and quarks are deeply rooted in sedenion algebra. Sedenion algebra is shown to contain three sub-octonion algebras, and each contains a quartet of anti-commutative spinor sets and a common quaternion set. Therefore, there are four degrees of freedom for the 4D exterior Minkowski space, and twelve degrees of freedom for the internal spacetime, with each temporal and spatial axis represented by a spinor triplet. Such a sedenion mathematical structure could be potentially used as a framework to construct grand unification theory beyond the Standard Model. We have preliminarily obtained mass relationships among other Standard Model particles. All masses can be expressed in the electron's mass and simple scaling factors involving the fine structure constant α, for example, $m_H/m_e = 3(\pi/\alpha)^2/4$. Our more generalized model and treatments of all other elementary particles, based on preon as a building block, will be published elsewhere.

Acknowledgment:

The corresponding author JT has already retired and has no funding organization.

Data Availability Statement

There are only analytical equation derivations, but no computer numerical simulations. The numerical values of the particles' masses and rations were obtained straightforwardly by plugging the experimental values from the literature into the derived equations. All reasonable questions about the data or derivations can be requested by contacting the corresponding author.

Author Contributions

J. T. initiated the project, derived the equations, and wrote the manuscripts. Q. T. prepared the reference list and revised the manuscripts.

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