Diophantine's equations and pseudo-differential calculus

Date : february 12, 2025

Authors : Ahcene Ait Saadi

Email : ait_saaadi@yahoo.fr

Abstract

In this paper I solve Diophantine equations using the pseudo differential calculation of Ait Saadi. I solve Diophantine equations of the form: ax+by=c without using Euclid's algorithm.

Finally I give the general method of solving the equations of the form/

ax+by+cz=d.

Citations: Ait Saadi

Reference: https://vixra.org/pdf/2409.0003v1.pdf

key words :

Diophantine equations; systems of équations

Diophantine equation of the second degree with two unknowns

1.0: Either to solve the following Diophantine equation, using the pseudo-differential calculation that I discovered

11: $P(x,y)=x^2-4xy+4y^2+5x-3y=0$

$$p\frac{\partial P(x,y)}{\partial x}=x-2y+5+\alpha y=0$$
$$p\frac{\partial P(x,y)}{\partial y}=-2x+4y-3-\alpha x=0$$

$\longrightarrow$

$$x+(\alpha-2)y+5=0$$
$$-(\alpha+2)x+4y-3=0$$

We put The determinant $\Delta=4+(\alpha-2)(\alpha+2)=1\Rightarrow\alpha=\pm1$

$if,\alpha=1$ $\quad x-y+5=0$, $\quad -3x+4y-3=0$ $\quad S_1=(-17,-12)$ first solution

1.2: For the second solution we make a change of variable

$$x=X-17$$
$$y=Y-12$$

$$P(X,Y)=X^2-4XY+4Y^2+19X-31Y=0$$

$$p\frac{\partial P(X,Y)}{\partial X}=X-2Y+19+\alpha Y=0$$
$$p\frac{\partial P(x,y)}{\partial y}=-2X+4Y-31-\alpha X=0$$

We take $\alpha=1$

$$X-Y+19=0$$
$$-3X+4Y-31=0$$

$X=-45;Y=-26\Rightarrow S_2=(-62;-38)$ Second solution

1.2: Important note: for the other solutions (there are an infinity), we make a change of variable each time.

$$x_{i+1}=X-x_i$$
$$y_{i+1}=Y-y_i$$

we replace these values in the polynomial (P)

and calculate the pseudo derivatives.

2.0: Either to solve the following Diophantine equation, using the pseudo-differential calculation that I discovered

$$P(x,y)=2x^2-3y^2+2xy+2x-5y=0$$
$$p\frac{\partial P(x,y)}{\partial x}=2x-3y+2+\alpha y=0$$
$$p\frac{\partial P(x,y)}{\partial y}=x-3y-5-\alpha x=0$$

$\begin{cases}2x-3y+2+\alpha y=0\\ x-3y-5-\alpha x=0\end{cases}$ $if\,\alpha=3\rightarrow S=(-7;3)$, the first solution

2.1: Let's check the other solutions, for this we make a change of variable

$x=X-7$
$y=Y+3$ we replace in $P(x)$

$$P(X;Y)=2X^2-3Y^2+2XY-20X-37Y$$
$$p\frac{\partial P(X;Y)}{\partial X}=2X+Y-20+\alpha Y=0$$
$$p\frac{\partial P(X;Y)}{\partial X}=X-3Y-37-\alpha X=0$$

$\begin{cases}2X+Y-20+\alpha Y=0\\ X-3Y-37-\alpha X=0\end{cases}$ $if\,\alpha=3\rightarrow,\ X=-108;Y=57\ \rightarrow S=(-111;60)$ the second solution

2.2: Let's make another variable change

$x=X-111$
$y=Y+60$ we replace in $P(x)$

$$P(X;Y)=2X^2-3Y^2+2XY-322X-587Y$$
$$\begin{cases}p\frac{\partial P(X;Y)}{\partial X}=2X+Y-322+\alpha Y=0\\ p\frac{\partial P(X;Y)}{\partial X}=X-3Y-587-\alpha X=0\end{cases}$$

$\begin{cases}2X+Y-322+\alpha Y=0\\ X-3Y-587-\alpha X=0\end{cases}$ $if\,\alpha=3\rightarrow$ $\begin{cases}2X+4Y-322=0\\ -2X-3Y-587=0\end{cases}$

$S=(-1768;969)$ The third solution

2.3: Let's make another variable change

$$x = X - 1768$$
$$y = Y + 969$$ we replace in $P(x)$

$$P(X;Y) = 2X^2 - 3Y^2 + 2XY - 5132X - 9355Y$$

$$\begin{cases} p\dfrac{\partial P(X;Y)}{\partial X} = 2X + Y - 5132 + \alpha Y = 0 \\ p\dfrac{\partial P(X;Y)}{\partial X} = X - 3Y - 9355 - \alpha X = 0 \end{cases}$$

$$\begin{cases} 2X + Y - 5132 + \alpha Y = 0 \\ X - 3Y - 9355 - \alpha X = 0 \end{cases} \quad if\, \alpha = 3 \rightarrow \begin{cases} 2X + 4Y - 5132 = 0 \\ -2X - 3Y - 9355 = 0 \end{cases} \quad S = (-28176;15456)$$

2.4: Fundamental theorem:

Any polynomial of the form: $ax^2 + by^2 + 2cxy + dx + ay = 0$

If is a particular solution $S = (x_i; y_i)$, then the polynomial:

$$a(X - x_i)^2 + b(Y - y_i)^2 + 2c(X - x_i)(Y - y_i) + d(X - x_i) + a(Y - y_i) = 0$$

admits an infinity of solutions.

3.0: Either to solve the following Diophantine equation, using the pseudo-differential calculation that I discovered

$$P(x, y) = 11y^3 - 2xy - 15x + 7y - 23 = 0$$

$$x = X + k$$
$$y = Y + \lambda \quad k, \lambda$$ integers.

$$P(X,Y) = 11Y^3 + 33\lambda Y^2 + 33\lambda^2 Y + 11\lambda^3 + 2XY + 2\lambda X + 2kY + 2k\lambda - 15X$$
$$-15K + 7Y + 7\lambda - 23 = 0$$

Let's take: $11\lambda^3 + 7\lambda - 23 + (2\lambda - 15)k = 0$

if $\lambda = 7 \Rightarrow k = 3799$

$$P(X,Y)=11Y^3+231Y^2+5425Y+2XY-X=0$$

$$\begin{cases} p\dfrac{\partial P(X,Y)}{\partial X}=Y-1=0 \\ p\dfrac{\partial P(X,Y)}{\partial Y}=11Y^2+231Y+9222+X=0 \end{cases}$$

$$\begin{cases} Y-1=0 \\ 11Y^2+231Y+9222+X=0 \end{cases} \quad Y=1; X=-9464 \Rightarrow S=(-5665;8)$$

4.0: Either to solve the following Diophantine equation, using the pseudo-differential calculation that I discovered

$$P(x,y)=x^2-y^2+4xy-3x+9y-4=0$$

$$\begin{cases} p\dfrac{\partial P(x,y)}{\partial x}=x+2y-3+\alpha y=0 \\ p\dfrac{\partial P(x,y)}{\partial y}=2x-y+9-\dfrac{4}{y}-\alpha x=0 \rightarrow we.take;(\alpha=3) \end{cases}$$

$$\begin{cases} x+5y-3=0 \Rightarrow x=-5y+3 \\ -x-y+9-\dfrac{4}{y}:0 \Rightarrow 4y+6-\dfrac{4}{y}=0 \Rightarrow 4y^2+6y-4=0 \Rightarrow y=-2 \Rightarrow S_1=(13,-2) \end{cases}$$

4.1: Let's check the other solutions, for this we make a change of variable

$$x=X+13$$
$$y=Y-2 \quad \text{we obtain}$$

$$X^2-Y^2+4XY+15X+65Y=0$$

$$\begin{cases} p\dfrac{\partial P(X,Y)}{\partial X}=X+2Y+15+\alpha Y=0 \\ p\dfrac{\partial P(X,Y)}{\partial Y}=2X-Y+65-\alpha X=0 \rightarrow we.take;(\alpha=3) \end{cases}$$

$$\begin{cases} X+3Y+15=0 \\ -X-Y+65=0 \rightarrow X=85; Y=-20 \Rightarrow S_2=(98;-22) \end{cases}$$

By making the successive variable changes, we find all the solutions that correspond to: α=3.

$$x_{n+1}=X+x_n$$
$$y_{n+1}=Y+y_n$$

5.0: Either to solve the following Diophantine equation, using the pseudo-differential calculation that I discovered

$4x^2-5y^2+20xy-13x+5=0$; we make a change of variable: $y=Y+1$

$4x^2-5Y^2+20xY+7x-10Y=0$

$$\begin{cases} p\dfrac{\partial P(x,Y)}{\partial x}=4x+(10+\alpha)Y+7=0 \\ p\dfrac{\partial P(x,Y)}{\partial Y}=(10-\alpha)x-5Y-10=0 \end{cases}$$

We calculate the determinant $\Delta=\alpha^2-120=1\Rightarrow\alpha=11$

$4x+21Y+7=0$
$-x-5Y-10=0\Rightarrow x=-175, Y=33 \quad S=(-175;34)$ ....etc

6.0: Solving the Diophantine equation: $ax+by+c=0$ without making a Euclidean algorithm.

6.1: Introduction:

Let be the polynomial: $P(x;y)=k(x,y)*q(x,y)=(2x-5y+7)(x+2y-10)=0$

$P(x,y)=2x^2-xy-10y^2-13x+64y-70=0$

I make the pseudo derivatives without taking into account the constant(-70). But the pseudo derivatives with respect to (x) and (y), I divide them by 2. the solutions of this system are the same as those of polynomials k(x,y) and q(x,y)

$$p\frac{\partial P(x,Y)}{\partial x}=2x-\frac{y}{2}-\frac{13}{2}=0 \qquad \begin{cases} 2x-\dfrac{y}{2}-\dfrac{13}{2}=0 \\ -\dfrac{x}{2}-10y+\dfrac{64}{2}=0 \end{cases} \quad S=(4;3)$$
$$p\frac{\partial P(x,Y)}{\partial Y}=-\frac{x}{2}-10y+\frac{64}{2}=0$$

We will use this method to solve the Diophantine equations of the form:

Let’s be the polynomial: ; we proceed as follows

$(4x+3y+3)(x+ay+b)=4x^2+(4a+3)xy+3ay^2+(4b+3)x+(3a+3b)y+3b=0$

$$\begin{cases} p\dfrac{\partial P(x,Y)}{\partial x}=4x+\dfrac{(4a+3)}{2}y+\dfrac{4b+3}{2}=0 \\ p\dfrac{\partial P(x,Y)}{\partial Y}=\dfrac{(4a+3)}{2}x+3ay+\dfrac{3(a+b)}{2}=0 \end{cases}$$

$$4x+\frac{(4a+3)}{2}y+\frac{4b+3}{2}=0$$

$$\frac{(4a+3)}{2}x+3ay+\frac{3(a+b)}{2}=0$$

the determinant $\Delta=-\dfrac{(4a-3)^2}{4}=-1\Rightarrow a=\dfrac{5}{4}$

$$y=\frac{3}{2}-2b \quad x=\frac{3b}{2}-\frac{15}{8} \qquad b=\frac{k}{4}$$

$$y=\frac{3-k}{2};x=\frac{3k-15}{8}\Rightarrow S=(\frac{3k-15}{8};\frac{3-k}{2})$$

$$if,k=-3\Rightarrow S_1(-3;3) \qquad if,k=-11\Rightarrow S_2=(-6;7)........etc$$

6.2: General solution

Let us solve the following Diophantine equation $ax+by+c=0$

We multiply this equation by: $x+my+p$

$$(ax+by+c)(x+my+p)=ax^2+(am+b)xy+bmy^2+(ap+c)x+(bp+cm)y+cp=0$$

$$\frac{\partial Q(x,y)}{\partial x}=ax+\frac{(am+b)}{2}y+\frac{ap+c}{2}=0$$

$$\frac{\partial Q(x,y)}{\partial y}=\frac{(am+b)}{2}x+bmy+\frac{bp+cm}{2}=0$$

$$\begin{cases} ax+\dfrac{(am+b)}{2}y+\dfrac{ap+c}{2}=0 \\ \dfrac{(am+b)}{2}x+bmy+\dfrac{bp+cm}{2}=0 \end{cases}$$

The determinant is: $\Delta=\dfrac{-(am-b)^2}{4}=$

$$y=\frac{-a^2mp+acm+abp-bc}{(am-b)^2}=\frac{ap(b-am)-c(b-am)}{(am-b)^2}=\frac{(b-am)(ap-c)}{(am-b)^2}$$

$$y=\frac{(b-am)(ap-c)}{(am-b)^2}=-\frac{ap-c}{am-b}$$

$$x=\frac{abmp-acm^2+bcm-b^2p}{(am-b)^2}=\frac{bp(am-b)-cm(am-b)}{(am-b)^2}=\frac{(am-b)(bp-cm}{(am-b)}$$

$$x=\frac{(am-b)(bp-cm)}{(am-b)^2}=\frac{bp-cm}{(am-b)}$$

let's take: $am-b=1\Rightarrow b=am-1 \Rightarrow S=(bp-cm;-ap+c)$

theorem: Any Diophantine equations of the form. ax+(am-1)y+c=0,
has for solutions: $\Rightarrow S=(bp-cm;-ap+c)$

Applications:

1) $13x+38y+47=0\Rightarrow S_1=(-141;47);S_2=(-103;34)....$

2) $131x+654y-1041=0\Rightarrow S_1=(5205,-1041),S_2=(5859;-1172)....$

7.0: Diophantine equation of the first degree with three unknown

$ux+vy+wz=\lambda$

We solve it by associating it with an equations of degree 2 to 3 unknown

$$P(x,y,z)=ax^2+by^2+cz^2+2xy+2kxz+2yz+ux+vy+wz=0......(A)$$

7.1: Example: solve: $u-2v+3w=2\Rightarrow x=1;y=-2;z=3$

$$p\frac{\partial P}{\partial x}=ax+y+kz+u=0$$
$$p\frac{\partial P}{\partial x}=by+x+z+v=0$$
$$p\frac{\partial P}{\partial x}=cz+kx+y+w=0$$

$$\begin{cases} ax+y+kz+u=0......(1) \\ by+x+z+v=0.........(2) \\ cz+kx+y+w=0.......(3) \end{cases}$$

in (A) I replace x, y , z with their values, we obtain: $a+4b+9c=14-6k$

We take $c=1;b=k\Rightarrow a=5-10k$

From (1): $a-2+3k+u=0\Rightarrow 5-10k-2+3k+u=0\Rightarrow u=-3+7k$

From (2): $-2b+1+3+v=0\Rightarrow -2k+4+v=0\Rightarrow v=2k-4$

From (3): $3+k-2+w=0\Rightarrow w=-1-k \quad S=(-3+7k;-4+2k;-1-k)$

7.2:Example: solve: $5u-7v+11w=9$

We solve it by associating it with an equation of degree 2 to 3 unknown

$$P(x,y,z)=ax^2+by^2+cz^2+2xy+2kxz+2yz+ux+vy+wz=0......(B)$$

$$p\frac{\partial P}{\partial x}=ax+y+kz+u=0$$

$$p\frac{\partial P}{\partial x}=by+x+z+v=0$$

$$p\frac{\partial P}{\partial x}=cz+kx+y+w=0$$

$$\begin{cases} ax+y+kz+u=0......(1) \\ by+x+z+v=0.........(2) \\ cz+kx+y+w=0.......(3) \end{cases}$$

In (A) I replace x, y , z with their values, we obtain: $25a+49b+121c+110k=215$

We take $c=-10; b=10k \Rightarrow a=-24k+57$

$u=109k-278; v=70k-16; w=-5k+117$

$S=(109k-278; 70k-16; -5k+117)$

7.3:General solution of a Diophantine equation of the form

$xu+yv+zw=m \quad (x,y,z)$ Integers $(u;v;w;\text{var}\,iables)$

We solve it by associating it with an equation of degree 2 to 3 unknown

$$P(x,y,z)=ax^2+by^2+cz^2+2xy+2kxz+2yz+dx+ey+fz+ux+vy+wz=0......(C)$$

$$ax^2+by^2+cz^2+2xy+2kxz+2yz+dx+ey+fz=m......(C_1)$$

$$p\frac{\partial P}{\partial x}=ax+y+kz+d+u=0$$

$$p\frac{\partial P}{\partial x}=by+x+z+e+v=0$$

$$p\frac{\partial P}{\partial x}=cz+kx+y+f+w=0$$

$$\begin{cases} ax+y+kz+d+u=0 \\ by+x+z+e+v=0...........(C_2) \\ cz+kx+y+f+w=0 \end{cases}$$

we give a value for b as a function of k, we give values to, c,d,e,f, we find the value of (a) as a function of k. We return to the system (C_2) ,we find the values of u,v,w as a function o