Polynomials that generate prime numbers and series of prime numbers

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Authors: Ahcene Ait Saadi

Email: ait_saaadi@yahoo.fr

Abstract:

1) In this article, I study prime numbers from polynomials that generate prime numbers. I have put the list of these polynomials and I have applied this method to some polynomials.

2) I used an empirical method to find series of sum of numbers that give a sum of prime numbers. this is just the beginning, I hope that young researchers will deepen this work for the benefit of science and knowledge.

Keys words: Prime number, polynomial, series of prime numbers

Empirical method.

Polynomials that generated prime numbers

Introduction:

List of polynomials that generate prime numbers

$$1):n^2-21n+1;2):n^2-81n+1;3):2n^2+2n-71;4):3n^2+3n-79$$
$$4):2n^2-12n+47;1):n^2-n-19;5):n^2+19n-19;1):4n^2-17$$
$$6):6n^2+6n+17;7):n^2-n+17;7):n^2+7n-97;8):n^2+5n+17$$
$$8):1):n^2+81n+7;9):3n^2+3n-89;10):2n^2+2n-89;$$

1.1.0: Polynomial: $P(n)=6n^2+6n+17$,

1.1.1:Definition: $P(n)$ is prime number for all values of integers n,k,λ such as

$n\neq 6k^2+5k+16$ $\qquad$ $n\neq 17k$

$n\neq 6k^2+7k+17$ $\qquad$ $n\neq 17k-1$

$n\neq(6\lambda^2+6\lambda+17)k+\lambda$

$n\neq(6\lambda^2+6\lambda+17)k-\lambda-1$

1.1.2: Note: If P(n) is a prime number it takes the form $:a^2+b^2$, otherwise it takes the form $:a^2+b^2+c^2$

1.1.3: Examples

$P(0)=17=1^2+4^2\rightarrow prime$

$P(1)=29=2^2+5^2\rightarrow prime$

$P(2)=53=2^2+7^2\rightarrow prime$

$P(3)=89=5^2+8^2\rightarrow prime$

$P(4)=137=4^2+11^2\rightarrow prime$

$P(5)=197=1^2+16^2\rightarrow prime$

$P(6)=269=10^2+13^2\rightarrow prime$

$P(7)=353=8^2+17^2\rightarrow prime$

$P(8)=449=7^2+20^2\rightarrow prime$

$P(9)=557=14^2+19^2\rightarrow prime$

$P(10)=677=1^2+26^2\rightarrow prime$

$P(11)=809=5^2+28^2\rightarrow prime$

$P(12)=953=13^2+28^2\rightarrow prime$

$P(13)=1109=22^2+25^2 \to prime$

$P(14)=1277=11^2+34^2 \to prime$

$P(15)=1457=31*47 \neq a^2+b^2=2^2+3^2+38^2 \to not.prime; see(1.1.0)$

$P(16)=1649=7^2+40^2 \to prime$

$P(17)=1853=17*109=2^2+43^2 \to not.prime \to n=17k$

$P(18)=2069=25^2+38^2 \to prime$

$P(19)=2297=19^2+44^2 \to prime$

$P(20)=2537=43*59 \neq a^2+b^2=1^2+6^2+50^2 \to not.prime; see(1.1.0)$

$P(21)=2789=17^2+50^2 \to prime$

$P(22)=3053=43*71 \neq a^2+b^2=4^2+11^2+54^2 \to not.prime; see(1.1.0)$

$P(23)=3329=25^2+52^2 \to prime$

$P(24)=3617=41^2+44^2 \to prime$

$P(25)=3917=14^2+61^2 \to prime$

$P(26)=4229=2^2+65^2 \to prime$

$P(27)=4553=29*157=8^2+67^2=1^2+14^2+66^2 \to not.prime; see(1.1.0)$

$P(28)=4889=20^2+67^2 \to prime$

$P(29)=5237=14^2+71^2 \to prime$

$P(30)=5597=29*193=11^2+74^2=11^2+24^2+70^2 \to not.prime; see(1.1.0)$

$P(31)=5969=47*127 \neq a^2+b^2=5^2+19^2+72^2 \to not.prime; see(1.1.0)$

$P(32)=6353=32^2+73^2 \to prime$

$P(33)=6749=17*397=25^2+82^2=3^2+4^2+82^2 \to not.prime; see(1.1.0)$

$P(34)=7157=17*421=41^2+74^2=1^2+10^2+84^2 \to not.prime; see(1.1.0)$

$P(35)=7577=59^2+64^2 \to prime$

$P(36)=8009=28^2+85^2 \to prime$

$P(37)=8453=79*107 \neq a^2+b^2=8^2+17^2+90^2 \to not.prime; see(1.1.0)$

$P(38)=8909=59*151 \neq a^2+b^2=2^2+16^2+93^2 \to not.prime; see(1.1.0)$

$P(39)=9377=56^2+79^2 \to prime$

$P(43)=11369=37^2+100^2 \to prime$

$P(44)=11897=4^2+109^2 \to prime$

$P(46)=12989=31*419=2^2+41^2+112^2 \to not.prime; see(1.1.0)$

$P(48)=14129=71*199=3^2+14^2+118^2 \to not.prime; see(1.1.0)$

$P(49) = 14717 = 59^2 + 106^2 \rightarrow prime$

$P(54) = 17837 = 26^2 + 131^2 \rightarrow prime$

$P(64) = 24977 = 104^2 + 119^2 \rightarrow prime$

$P(65) = 25757 = 43 * 599 = 6^2 + 11^2 + 160^2 \rightarrow not.prime; see(1.1.0)$

$P(66) = 26549 = 139 * 191 = 4^2 + 17^2 + 162^2 \rightarrow not.prime; see(1.1.0)$

$P(69) = 28997 = 107 * 271 = 4^2 + 9^2 + 170^2 \rightarrow not.prime; see(1.1.0)$

$P(70) = 29837 = 109^2 + 134^2 \rightarrow prime$

$P(74) = 33317 = 86^2 + 161^2 \rightarrow prime$

$P(75) = 34217 = 19^2 + 184^2 \rightarrow prime$

$P(76) = 35129 = 115^2 + 148^2 \rightarrow prime$

<u>Spécial cases :</u>

$P(80) = 38897 = 97 * 401 = 9^2 + 20^2 + 196^2 = 14^2 + 26^2 + 195^2 = 71^2 + 184^2 \rightarrow not.prime$

$P(81) = 39869 = 70^2 + 187^2 = 6^2 + 32^2 + 197^2 \rightarrow not.prime$

$P(670) = 2697437 = 469^2 + 1574^2 \rightarrow prime$

1.2.0: Polynomial: $P(n) = n^2 + 19n - 19$

$P(2) = 23 \rightarrow prime$

$P(3) = 47 \rightarrow prime$

$P(4) = 73 \rightarrow prime$

$P(5) = 101 \rightarrow prime$

$P(6) = 131 \rightarrow prime$

$P(7) = 163 \rightarrow prime$

$P(8) = 197 \rightarrow prime$

$P(9) = 233 \rightarrow prime$

$P(11) = 311 \rightarrow prime$

$P(12) = 353 \rightarrow prime$

$P(13) = 397 \rightarrow prime$

$P(14) = 443 \rightarrow prime$

$P(15) = 491 \rightarrow prime$

$P(16) = 541 \rightarrow prime$

$P(17) = 593 \rightarrow prime$

$P(18) = 647 \rightarrow prime$

$P(10)=271 \rightarrow prime$ $\qquad P(21)=821 \rightarrow prime$

1.3.0: Polynomial: $P(n)=2n^2-12n+47$

1.3.1: Definition: $P(n)$ is prime number for all values of integers n,k,λ such as

$1.3.1: n \neq 47k$
$1.3.2: n \neq 47k+6$
$1.3.3: n \neq (2\lambda^2-12\lambda+47)k+\lambda$
$1.3.4: n \neq (2\lambda^2-12\lambda+47)k-\lambda+6$
$1.3.5: n \neq (2\lambda^2+11\lambda+47)k$
$1.3.5: n \neq 2k^2+5k+47$
$1.3.6: n \neq 2k^2+7k+53$

1.3.2: If the number n is odd number; $n=2k+1$; $k,$ integer , then the number P(n) takes the form: a^2+b^2

Examples:

1.3.6:

$P(1)=37=1^2+6^2 \rightarrow prime$
$P(3)=29=2^2+5^2 \rightarrow prime$
$P(5)=37=1^2+6^2 \rightarrow prime$
$P(7)=61=5^2+6^2 \rightarrow prime$
$P(9)=101=1^2+10^2 \rightarrow prime$
$P(11)=157=6^2+11^2 \rightarrow prime$
$P(13)=229=2^2+15^2 \rightarrow prime$
$p(17)=421=14^2+15^2 \rightarrow prime$
$P(19)=541=10^2+21^2 \rightarrow prime$
$P(21)=677=1^2+26^2 \rightarrow prime$
$P(23)=829=10^2+27^2 \rightarrow prime$
$P(25)=997=6^2+31^2 \rightarrow prime$
$P(27)=1181=5^2+34^2 \rightarrow prime$
$P(29)=1381=15^2+34^2 \rightarrow prime$
$P(31)=1597=21^2+34^2 \rightarrow prime$
$P(33)=1829=31*59 \neq a^2+b^2 \rightarrow not.prime; see, 1.3.1$

$P(35)=2077=31*67\neq a^2+b^2\rightarrow not.prime; see, 1.3.1$

$P(37)=2341=15^2+46^2\rightarrow prime$

$P(39)=2621=11^2+50^2\rightarrow prime$

$P(41)=2917=1^2+54^2\rightarrow prime$

$P(43)=27^2+50^2\rightarrow prime$

$P(49)=4261=6^2+65^2\rightarrow prime$

$P(51)=4637=34^2+59^2\rightarrow prime$

$P(53)=5029=47*107\neq a^2+b^2\rightarrow not.prime; see, 1.3.1$

$P(55)=5437=26^2+69^2\rightarrow prime$

$P(57)=5861=31^2+70^2\rightarrow prime$

$P(59)=6301=26^2+75^2\rightarrow prime$

$P(61)=6757=29*233=14^2+81^2\rightarrow not.prime; see, 1.3.1$

$P(63)=7229=2^2+85^2\rightarrow prime$

$P65)=7717=34^2+81^2\rightarrow prime$

$P(67)=8221=11^2+90^2\rightarrow prime$

$P(69)=8741=50^2+79^2\rightarrow prime$

$P(71)=9277=21^2+94^2\rightarrow prime$

$P(73)=9829=15^2+98^2\rightarrow prime$

$P(75)=10937=37*81\rightarrow not.prime; see, 1.3.1$

$P(77)=10981=79*139\rightarrow not.prime; see, 1.3.1$

$P(79)=11581=37*313\rightarrow not.prime; see, 1.3.1$

$P(81)=12197=31^2+106^2\rightarrow prime$

$P(83)=12829=27^2+110^2\rightarrow prime$

$P(85)=13477=34^2+111^2\rightarrow prime$

$P(87)=14141=79*179\rightarrow not.prime; see, 1.3.1$

$P(89)=14821=50^2+111^2\rightarrow prime$

$P(91)=15517=59*263\rightarrow not.prime; see, 1.3.1$

$P(93)=16229=10^2+127^2\rightarrow prime$

$P(95)=16957=31*547\rightarrow not.prime; see, 1.3.1$

$P(97)=17701=31*571\rightarrow not.prime; see, 1.3.1$

$P(99)=18461=85^2+106^2\rightarrow prime$

1.4.0: If the number n is even number; $n=2k$; $k,$ integer , then the number P(n) , takes the form: $a^2+b^2+c^2+d^2$

1.4.1: Examples

$$P(0) = 47 = 1^2 + 1^2 + 3^2 + 6^2$$
$$P(2) = 71 = 1^2 + 3^2 + 5^2 + 6^2$$
$$P(4) = 31 = 1^2 + 1^2 + 2^2 + 5^2$$
$$P(6) = 47 = 1^2 + 1^2 + 3^2 + 6^2$$
$$P(8) = 79 = 1^2 + 2^2 + 5^2 + 7^2$$
$$P(10) = 127 = 1^2 + 1^2 + 5^2 + 10^2$$
$$P(12) = 191 = 2^2 + 3^2 + 3^2 + 13^2$$
$$P(14) = 271 = 1^2 + 3^2 + 6^2 + 15^2$$
$$P(16) = 367 = 2^2 + 5^2 + 7^2 + 17^2$$
$$P(18) = 479 = 1^2 + 1^2 + 6^2 + 21^2$$
$$P(20) = 607 = 2^2 + 5^2 + 7^2 + 23^2$$
$$P(22) = 751 = 1^2 + 5^2 + 10^2 + 25^2$$

$$P(24) = 911 = 1^2 + 1^2 + 3^2 + 30^2$$
$$P(26) = 1087 = 1^2 + 5^2 + 10^2 + 31^2$$
$$P(28) = 1279 = 1^2 + 1^2 + 11^2 + 34^2$$
$$P(30) = 1487 = 3^2 + 3^2 + 5^2 + 38^2$$

……..etc

2.0: Empirical method allowing me to find series of prime numbers, using the polynomials that I discovered

$79^2+73^2+43^2+11^2+47^2=15749 \to prime$

$73^2+61^2+137^2+11^2+191^2+551^2+641^2=778903 \to prime$

$37^2+79^2+241^2+379^2+457^2=418181 \to prime$

$37^2+79^2+241^2+379^2+457^2+541^2+631^2+79^2+73^2+11^2+83^2=1127603 \to prime$

$137^2+79^2+191^2+947^2+1061^2+1871^2+1721^2+251^2+317^2+$
$37^2+1^2=8711363 \to prime$

$19^2+43^2+61^2+191^2+251^2+1721^2+1307^2=4775503 \to prime$

$1307^2+947^2+11^2+79^2+37^2=2612789 \to prime$

$61^2+19^2+89^2+947^2+839^2=1612733 \to prime$

$19^2+47^2+251^2+389^2+641^2+839^2+1061^2+1439^2+1721^2+2027^2+2531^2+$
$2711^2+3491^2=37541269 \to prime$

$89^3+71^3+29^3+37^3+181^3=7067663 \to prime$

$1^3+79^3+181^3+307^3+457^3=130801217 \to prime$

$83^3+53^3+29^3+1^3+79^3+457^3=96657697 \to prime$

$43^3+19^3+1^3=87697 \to prime$

$71^3+61^3+29^3+19^3+11^3=617471 \to prime$

$47^3+37^3+89^3+79^3+137^3=3923837 \to prime$

$191^3+181^3+251^3+241^3+317^3=74563397 \to prime$

$19^3+73^3+191^3=7363747 \to prime$

$3917^3+73^3+19^3+191^3+187^3=66655159271 \to prime$

$37^3+127^3+241^3+379^3+457^3+47^3+137^3+191^3+251^{33}=191436787 \to prime$

$\frac{13^2+1373^2+1528223^2}{3}=778489141009 \to prime$

$\frac{11^4+47^4+89^4}{3}=22545521 \to prime$

$\frac{79^8+61^8}{2}=854408061451921 \to prime$

$\frac{191^8+317^8}{2}=887566937228959201 \to prime$

$\frac{389^3+467^3+643^3}{3}=141362051 \to prime$

2.1: Still using an empirical method, here are other series of prime numbers

$$\frac{11^4+47^4+89^4}{3};\frac{79^8+61^8}{2};\frac{181^8+307^8+11^8}{3};\frac{119^8+317^8}{3}$$

$$\frac{127^8+241^8+641^8}{3};\frac{457^8+631^8+251^8}{3};\frac{29^8+37^8+47^8}{3}$$

$$\frac{29^8+37^8+839^8}{3^2};\frac{457^8+631^8+947^8}{3^2};\frac{37^4+79^4+127^4}{3^2}$$

$$\frac{307^4+379^4+457^4}{3^3};\frac{11^5+47^5+89^5}{3^3};\frac{79^5+37^5+127^5}{3}$$

$$\frac{79^5+181^5+307^5}{3};\frac{379^5+457^5+727^5}{3^2};\frac{19^6+47^6+89^5}{3}$$

$$\frac{47^6+89^6+191^6}{3};\frac{191^6+251^6+389^6}{3};\frac{79^6+73^6+43^6}{3}$$

$$\frac{947^6+1061^6+1307^6}{3};\frac{1161^6+1181^6+1439^6}{3};\frac{79^6+127^6+241^6}{3}$$

$$\frac{79^8+61^8}{2}=854408061451921$$

$$\frac{89^8+83^8}{2}=3094440518920561$$