Viola or Guitar Models Based on Split-Sphere-Volume Geometry

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Abstract

Recently, *Wilhelm Brückner*, Germany's oldest active viola maker, past away with 92 years in Erfurt. He created an own felicitous viola model. This is the opportunity to present own thoughts about possible guitar models based on the Split-Sphere-Volume approach. Its fundamental importance for life, physics and cosmos was already emphasized by the present author.

Keywords: Viola Model, Guitar Model, Split-Sphere-Volume Geometry, Golden Mean, Quartic Polynomial and Golden Mean.

Introduction

Some geometrical similarities between models of violas or guitars and the below outlined geometrical approach between spheres exist [1]. So it is our intuitive idea to present new viola or guitar models based on this geometrical concept, where the golden mean as one of nature's leading principles is involved. Golden mean and harmony are sisters. It was the true motivation behind our ideas. Now we present first the Split-Sphere Volume approach and derive from this the concept of new viola or guitar models. However, the author doesn't know the many secrets of viola makers based on their centuries-long experiences, beginning in the sixteenth century in Spain. However, the development will never come to an end as master *Brückner* had shown [2]. Music is an essential element of a peaceful world.

Split-Sphere-Volume Approach

Following the concept of paired entities, we will split the volume of a parent sphere with unit radius into two smaller but equal spheres. Assuming constant density, the volume is proportional to mass and also to energy. Following **Figure 1**, interesting geometrical relations can be confirmed showing signature of the golden mean respectively its fifth power.
The starting sphere volume is denoted as V_0 and the half volume as V_1. Then we get the trivial results

$$V_0 = 2 \cdot V_1 \tag{1}$$

$$V_0 = \frac{4}{3}\pi r_0^3 \tag{2}$$

$$V_1 = \frac{V_0}{2} = \frac{4}{3}\pi r_1^3 = \frac{4}{3}\pi\left(\frac{r_0}{\sqrt[3]{2}}\right)^3 \tag{3}$$

$$\cos(\alpha_{1)} = \frac{r_0}{2r_1} \tag{4}$$

$$\alpha_1 = 50.9527898° \tag{5}$$

$$\frac{\alpha_1}{360} = 0.141535527 \approx \pi - 3 = 0.141592653 \tag{6}$$

$$\frac{\alpha_1}{180} = 0.28307105 \approx \pi \cdot \varphi^5 = 0.283277231 \quad (7)$$

$$h_1 = 0.61640938 \cdot r_0 \approx \varphi \cdot r_0 \quad (8)$$

where $\varphi = \frac{\sqrt{5}-1}{2} = 0.6180339887$ is the golden mean. If we change only marginally the sphere radius ratio, then exact golden mean solution for instance to relation (7) can be obtained using $V_0 = 1.99521 \cdot V_1$ instead of $V_0 = 2 \cdot V_1$. We note that the following relation holds [3]

$$\arctan(2\varphi) = 51.026552° \quad (9)$$

The reader may become aware that $\pi \cdot \varphi^5$ represents the ratio of the in-sphere volume to the pyramid volume of the Great Pyramid at Giza [4] [5].

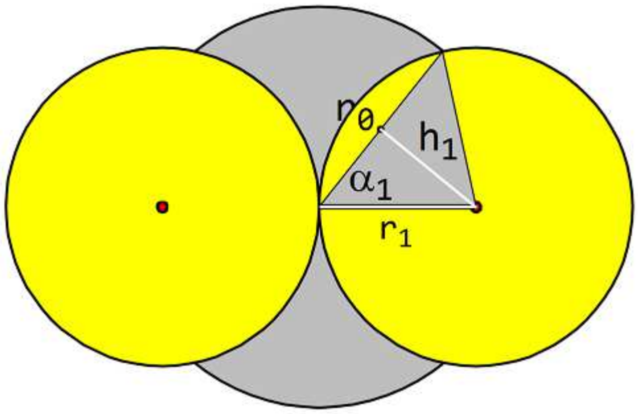

Figure 1. Splitting of a parent sphere (red) into two spheres each with half volume (yellow)

The Concept of New Viola or Guitar Models

On the basis of this approach we propose a new music instrument model despite the centuries-long experience of master instrument makers. The construction uses two different circles of radius quotient

$$\boldsymbol{\frac{r_0}{r_1} = \sqrt[3]{2} = 1.25992105} \quad (10)$$

If the small circle has unit radius and its circumference is placed at the midpoint of the large one (**Figure 2**), then the length of the sound body of the instrument resulted in

$$\mathrm{l} = \mathrm{r}_0 + 2\mathrm{r}_1 = 3.25992105 \cdot r_1 \quad (11)$$

The circle areas are proportional to the volume of the sound body. We get

$$r_0^2 = \sqrt[3]{4} = 1.587401 \qquad r_1^2 = 1 \quad (12)$$

$$\boldsymbol{\frac{r_9^2}{r_0^2+r_1^2}} = 0.613512 \approx \varphi = 0.6180339887 \quad (13)$$

However, when we use for r_0^2

$$r_0^2 = \Phi = \varphi^{-1} = \varphi + 1 = 1.6180339887 \tag{14}$$

then the volume of the two circular sound bodies indicate a perfect golden mean ratio

$$\frac{\boldsymbol{r_9^2}}{\boldsymbol{r_0^2+r_1^2}} = \varphi \qquad\qquad \frac{\boldsymbol{r_1^2}}{\boldsymbol{r_0^2+r_1^2}} = \varphi^2 = 0.381966012 \tag{15}$$

However, they are penetrating each other, which can be calculated and separately considered. The small circle to big circle area ratio would then approximate the value of 0.3128 to 0.6873 (see **Appendix**). Now we need only to connect the circles by generating the waist of the instrument with two tighter curves inward and place the sound hole. In this way common viola or guitar models can be well replicated. The waist can be constructed by means of an asymmetric double-well potential curve (full quartic polynomial) including minima that touch the circle minima and the saddle point as the narrowest point of the instrument. The waist of the instrument shifts the area (volume) ratio again towards a golden mean one.

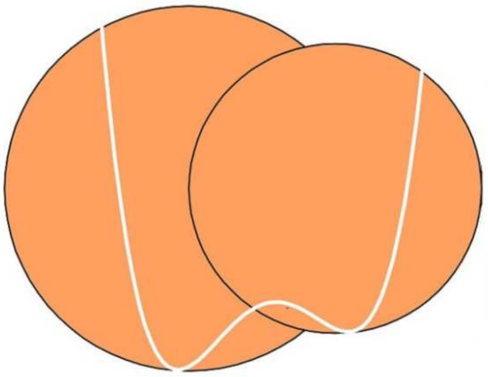

Figure 2. Overlapping circles with radius quotient of :$\frac{r_0}{r_1}$ = 1.26 gives the basis for a common guitar model. The waist of the instrument can be made by connecting the half-circles with two tighter curves inward represented by an asymmetric double-well potential curve in white (see **Appendix**).

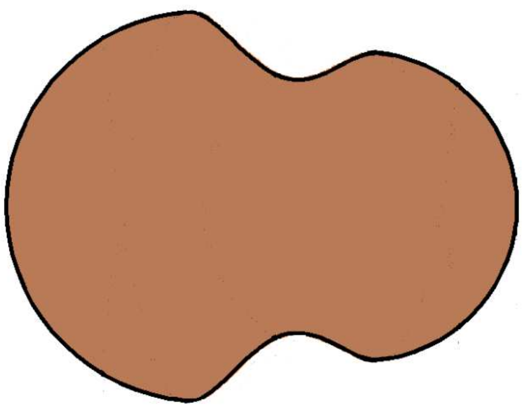

Figure 3. Sketch of the developed model.

The adjustment of the different curves can certainly be designed somewhat smoother. My own guitar made by a Spanish master (*Almansa*, **Figure 4**) has the following approximated measures:

$$l = 48.4 \text{ cm} \qquad r_0 = 18.6\ cm \qquad r_1 = 14.1\ cm,$$

with the sound hole posited at 15 cm from the top of the sound body having a diameter of 8.9 cm.

Surprisingly, if we use the formulas (10) and (11), we can determine similar values for our model

$$l = 48.0 \text{ cm} \qquad r_0 = \frac{1.26}{3.26} \cdot l = 18.6 \text{ cm} \qquad r_1 = \frac{l}{3.26} = 14.7\ cm$$

Figure 4. Almansa **c**oncert guitar

We show in the **Appendix** that $\sqrt[3]{4}$ is connected with the fifth power of the golden mean φ^5. The type and storage history of matured wood used as well as its cutting with respect to the fiber direction are essential prerequisites for fullness of sound. The reader may learn about helical fibers of wood in references [6] and [7]. In addition, the wood may be impregnated with a Li silica gel (pore-filling silicification). But the optimum geometry of the sound body makes the difference between common instruments and master creations. It must be experimentally verified whether an exact adaption of golden mean geometry according to relation (15) can further gain an exceptional sound and the fidelity of the crafted instrument. The positioning of the sound hole and its diameter should be selected by fine-tuning of the resulting sound. About golden mean and music take a look at *Olsen*'s monograph [8].

The Soul of a Music Instrument

When we speak about the soul of a music instrument, we are confronted with the question whether besides the sound wave spectrum another energy fields are involved and can be detected by exceptionally sensitive human beings. Golden mean geometry connects the entire cosmos up to superluminal graviton effects [9] [10]. I believe, we are only at the beginning to understand all these mysteries. The design of the sound body of the presented instrument may especially stimulate the spectrum of overtones. The angle defined by relation (4) and (5) has cosmic importance in combination with the golden mean, where m_{Hi} respectively m_p and m_e are the masses of the *Higgs* boson, the proton and the electron [11]

$$\frac{\arccos(\frac{r_0}{2r_1})}{\varphi^2} = 133.3961 \approx \frac{m_{Hi}}{m_p + m_e} = 133.3959$$

By connecting both circles of the sound body by means of a curve determined by a quartic polynomial, we always introduced golden mean geometry, because every quartic in golden [12].

Conclusion

The Split-Sphere-Volume approach with its proximity to golden mean geometry can be applied to construct guitar or viola instruments. However, this new geometric interpretation delivers no groundbreaking new results. Obviously, the old instrument makes had always found out centuries before the best measures of the sound body and the connection between geometry and harmony. However, could we improve again this class of music instruments by stimulating especially overtones? If we design the waist of the instrument by a quartic polynomial curve, we should remember that every quartic polynomial is golden. Young inventors are invited to realize the new master piece.

Conflicts of Interest

The author declares no conflict of interests regarding the publication of this paper.

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Appendix

Quartic Polynomial

For laymen I give the full quartic polynomial of the white curve in **Figure 3**, where the radius of the small circle is chosen as $r_1 = 1$

$$f(x) = 2.227 \cdot x^4 - 4.764 \cdot x^3 + 2.240 \cdot x^2 + 0.556 \cdot x - 1.23 \tag{16}$$

One can generate this polynomial by a rational curve through 5 points ***P***(i) given below with the chosen saddle point *P*(3). In order to design a smooth connection via this polynomial curve, the *y*-coordinates of the points *P*(2) and *P*(4) were chosen somewhat smaller than the circle coordinates at minimum.

P(i)	***x***(i)	***y***(i)
1	-0.20	-1.21
2	0	-1.23
3	0.6	-0.83
4	1	-0.97
5	1.2	-0.95

Specific Coordinates of the Two Penetrating Circles

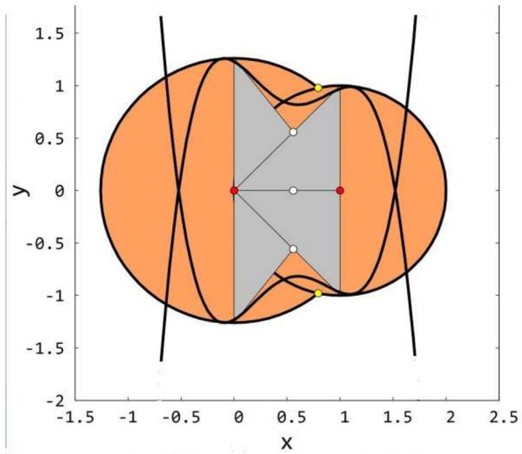

Figure 4. Center of gravity of the two penetrating circles represented by small white circles

The coordinates of the commom points of the two circles (yellow points) are (**Figure 4**)

$$x_c = \sqrt{\frac{r_0}{2}} = \frac{1}{r_0} = 0.7937,\ \mathrm{y_c} = \pm\, r_0 \sqrt{1 - (\frac{r_0}{2})^2} = \sqrt{\frac{2}{r_0} - \frac{r_0}{2}} = 0.9785 \qquad (17)$$

The $\mathrm{y_c}$ coordinate shows a remarkable reciprocitiy relation.

The x_g coordinate of the center of gravity (white points) yields

$$x_g = y_g = \frac{r_0}{1+r_0} = 0.5577 \qquad (18)$$

How is the fifth power of the golden mean $\boldsymbol{\varphi^5}$ connected with $\boldsymbol{\sqrt[3]{4}}$?

$$\sqrt[3]{4} + \frac{1}{\sqrt[3]{4}} = 2.217361576 = \frac{1}{0.450986438} \approx \frac{1}{5 \cdot \varphi^5} = \frac{1}{0.450849935} \qquad (19)$$

The quadratic equation $$x^2 - \left(\sqrt[3]{4} + \frac{1}{\sqrt[3]{4}}\right) + 1 = 0 \qquad (20)$$

has solutions $$x_1 = \sqrt[3]{4}, \quad x_2 = 1/\sqrt[3]{4} \qquad (21)$$

Correction of the Circle Areas by Cutting the Overlapping Circle Sections

We use the following formula for the estimation of the corrected relative area A

$$\frac{A_i}{\pi} = r_i^2 (1 - \frac{\mu_i}{360} + \frac{\sin(\mu_i)}{\pi}) \qquad (22)$$

with the section opening angles

$$\mu_0 = 2 \cdot rccos\left(\frac{r_0}{2r_1}\right) = 101.90558° \qquad \mu_1 = 360° - 2 \cdot \mu_0 = 156.18884° \qquad (23)$$

Then we get $$\frac{A_0}{\pi} = 1.385262 \qquad \frac{A_1}{\pi} = 0.63039 \qquad (24)$$

and $$\frac{A_0}{A_0+A_1} = 0.68725 \qquad \frac{A_1}{A_0+A_1} = 0.31275 \qquad (25)$$

However, taking into account the different areas changed by the worked out constriction, we again can obtain a ratio near the golden one.

The areas can be obtained by the following relation, which combined the half-circle with twice the integral under the polynomial $P(\mathrm{x})$ with integration limits between 0 and 0.6 respectively 0.6 and 1

$$A_i = \frac{\pi}{2} r_i^2 + 2 \int P(x)\, dx \qquad (26)$$

We get the following result for the relative area amounts adapted to the golden mean φ

$$\frac{A_0}{A_\Sigma} = 0.61785 \qquad \frac{A_1}{A_\Sigma} = 0.38215 \qquad (27)$$

By a very slight shift of the saddle point x-coordinate one would get an exact golden ratio.