Radiative correction to electron scattering

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January 27, 2025

Abstract

The scattering process in the framework of the source theory is considered as the synergism of the elastic and the inelastic process. In this approach the infrared divergences never occur. The resulting differential cross section appears as a factor multiplying the lowest-order (Born) differential cross section plus a contribution referring to the magnetic form factor.

1 Introduction

The scattering process in the framework of the source theory is considered as the synergism of the elastic and the inelastic process. In this approach the infrared divergences never occur. The resulting differential cross section appears as a factor multiplying the lowest-order (Born) differential cross section plus a contribution referring to the magnetic form factor.

The problem of radiative corrections for particle scattering is presented in many articles and textbooks (Schwinger, 1949; Schwinger, 1973; Akchiezer et al., 1965) In this article we will solve this problem in the framework of the source theory and we will follow the articles of Lester de Raad et al. (1972) and Schwinger (1973).

At the previous papers there was an effort to separate the elastic process from the inelastic one. However, the realistic approach is to consider the scattering in its physical complexity i.e. to consider the elastic and inelastic scattering in a unified manner.

We will see that in this method of approach the infrared sensitivity never occurs. However, we will show, that there are also the infrared sensitive nonelastic processes that occur separately and have no associated elastic counterparts. The corresponding terms require to insert a photon mass, but the dependence on this mass vanishes after summing all such purely inelastic contributions.

Let us first remember the basic formalism. Consider a situation when an electron is moving in the time-independent electromagnetic field of the four-potential A_μ. The vacuum amplitude describing the propagation of electron from the source η_2 to source η_1 is as follows:

$$\langle 0_+|0_-\rangle = i\int (dx)(dx')\eta_1(x)\gamma^0 G_+(x-x')\eta_2(x') \quad +$$

$$i\int (dx)(dx')\psi_1(x)\gamma^0 Z(A,x,x')\psi_2(x') \tag{1}$$

where G_+ is the Green function of the free electron, ψ are fields associated with sources and $Z(A,x,x')$ is a functional of A. The first term describes the propagation of electron without interaction and the second term involves all interactions with the external field. The field ψ_2 is before any interaction and ψ_1 is after any interaction. The radiative corrections are obviously involved in $Z(A,x,x')$.

2 Radiative corrections to electron scattering

Our goal here is to determine the forward scattering amplitude $\langle p\sigma q|p\sigma q\rangle$, where the symbols refer to the momentum, spin and charge eigenvalues of the electron. It means that we must extract this amplitude from the vacuum amplitude $\langle 0_+|0_-\rangle$. The general treatment was described by textbooks of source theory (Schwinger, 1969; Schwinger; 1970; 1973; 1989) . In this article we briefly remark that the contribution of the first term in eq.(1) provides unity and we insert the following formulas into the second term of eq. (1):

$$\psi_1(x) \to (2md\omega_p)^{1/2}\, u^*_{p\sigma q}e^{-ipx} \tag{2a}$$

$$\psi_2(x) \to (2md\omega_p)^{1/2}\, u_{p\sigma q}e^{ipx} \tag{2b}$$

where $u_{p\sigma q}$ is the spinor with eigenvalues σ, q being the eigenvalues of the charge matrix q (Lester de Raad et al., 1972)

$$q = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{3}$$

and

$$(m+\gamma p)u_{p\sigma q} = 0 \tag{4}$$

$$u^*_{p\sigma q}\gamma^0 u_{p\sigma' q'} = \delta_{\sigma\sigma'}\delta_{qq'}. \tag{5}$$

The resulting formula after insertion is the forward-scattering probability amplitude

$$\langle p\sigma q|p\sigma q\rangle = 1 + i2md\omega_p \int (dx)(dx')e^{-ipx}u^*_{p\sigma q}\gamma^0 Z(A,x,x')u_{p\sigma q}e^{ipx'}. \tag{6}$$

which in turn yields

$$1 = |\langle p\sigma q|p\sigma q\rangle|^2 + 4md\omega_p \mathrm{Im}\left\{\int (dx)(dx')e^{-ipx}u^*_{p\sigma q}\gamma^0 Z(A,x,x')u_{p\sigma q}e^{ipx'}\right\} \tag{7}$$

The last formula is the sum of the probability that the initial electron goes to the state $\langle p\sigma q|$ and the probability that the initial electron goes to the state other than $\langle p\sigma q|$. From the later probability we can determine the total cross section which can be obtained by division by T and $2|\mathbf{p}|d\omega_p$, where T is the time during which the external field acts upon the electron and $2|\mathbf{p}|d\omega_p$ is the incident particle flux. The total cross section is then

$$\sigma = \frac{1}{T}\frac{2m}{|\mathbf{p}|}\mathrm{Im}\left\{\int (dx)(dx')e^{-ipx}u^*_{p\sigma q}\gamma^0 Z(A,x,x')u_{p\sigma q}e^{ipx'}\right\}. \tag{8}$$

Let us first consider the simple example of an electron scattering without radiative corrections in the electromagnetic field, expanded to the second order in this field. The basic task is to determine $Z(A,x,x')$. We determine is as follows.

The single electron exchange between source η_2 and η_1 represents in the analogy with the non-interacting case the vacuum amplitude

$$\langle 0_+|0_-\rangle = i\int (dx)(dx')\eta_1(x)\gamma^0 G^A(x,x')\eta_2(x'), \tag{9}$$

where

$$G^A(x,x') = G_+(x-x') + \int (dy)G_+(x-y)eq\gamma A(y)G_+(y-x') \quad +$$

$$\int (dy)(dy')G_+(x-y)eq\gamma A(y)G_+(y-y')eq\gamma A(y')G_+(y'-x') + ... \tag{10}$$

The field is defined by the equation

$$\psi(x) = \int (dx')G_+(x-x')\eta(x'). \tag{11}$$

After insertion of eq. (10) into eq. (11) we get the contribution

$$Z(A,x,x') = eq\gamma A(x)\delta(x-x') + eq\gamma A(x)G_+(x-x')eq\gamma A(x') \tag{12}$$

and for σ we have

$$\sigma = \frac{2m}{|\mathbf{p}|}\,\mathrm{Im}\,\Big(u^*_{p\sigma q}\gamma^0 eq\gamma A(0)u_{p\sigma q} \quad +$$

$$\int \frac{(d\mathbf{p}')}{(2\pi)^3}u^*_{p\sigma q}\gamma^0 eq\gamma A(p-p')\frac{m-\gamma p'}{p'^2+m^2-i\varepsilon}eq\gamma A(p'-p)u_{p\sigma q}\Big) \quad =$$

$$\frac{m}{|\mathbf{p}|}\int\frac{(d\mathbf{p}')}{(2\pi)^2}\delta(p'^2+m^2)u^*_{p\sigma q}\gamma^0 eq\gamma A(p-p')(m-\gamma p')eq\gamma A(p'-p)u_{p\sigma q}, \quad (13)$$

where $p^0 = p'^0$ since the potential is time-independent and the three-dimensional Fourier transforms have been employed.

Using the identity

$$\int\frac{(d\mathbf{p}')}{(2\pi)^2}\delta(p'^2+m^2)=\int d\Omega\frac{|\mathbf{p}'|}{8\pi^2}, \quad (14)$$

we get

$$\frac{d\sigma}{d\Omega}=\frac{m}{8\pi^2}u^*_{p\sigma q}eq\gamma A(p-p')(m-\gamma p')eq\gamma A(p'-p)u_{p\sigma q}, \quad (15)$$

which is the Born formula for the differential cross section in the potential scattering, $\mathbf{p}'$ being the momentum of the scattered electron.

3 Calculation of the radiative corrections

There are two general types of radiative corrections to the lowest order. 1) The causal exchange of an electron between sources η_2 and η_1, 2) the exchange of an electron-photon pair.

The first process with a local external potential gives rise to a Born approximation as we have seen yet with with the absence of radiative processes. The second process involves radiative corrections.

The action involving the interaction of an electron with the electromagnetic field is

$$W_{int}=\frac{1}{2}\int(dx)\psi(x)\gamma^0 eq\gamma^\mu A_\mu(x)\psi(x) \quad (16)$$

and the three-field analogy is called the primitive interaction.

The vacuum amplitude corresponding to W_{int} is

$$\langle 0_+|0_-\rangle = i\int(dx)A_{1\mu}(x)\psi_1(x)\gamma^0 eq\gamma^\mu\psi_2(x) \quad (17)$$

as a reslut of insertion of $\psi=\psi_1+\psi_2$ into W_{int} and expansion of exp iW_{int}. In analogy with the principle of superposition for ψ-decomposition we write for sources

$$\eta=\eta_1+\eta_2, \quad (18)$$

where we take η_2 for the emission source of the time-like virtual particle excitation and η_1 is real particle detection source. The virtual particle decays into an electron and photon and the pair propagates without further interaction. the photon is detected by the photon source J_2. The situation can be graphically pictured. The source η_2 can be identified with so called effective electron-photon source ηJ because it emits through the virtual particle the electron and photon. The $\eta - J$ structure of this source can be determined after expansion of

$$\langle 0_+|0_-\rangle^{\eta J} = \langle 0_+|0_-\rangle^{\eta}\langle 0_+|0_-\rangle^{J} \tag{19}$$

and by the extraction from it the term

$$\langle 0_+|0_-\rangle = i\int (dx)(d\xi)A_{1\mu}(\xi)\psi_1(x)\gamma^0 i\eta_2(x)J_2^{\mu}(\xi) \tag{20}$$

and after comparison of eq. (20) with eq. (16). The result is

$$i\eta_2(x)J_2^{\mu}(\xi)|_{eff} = \delta(x-\xi)eq\gamma^{\mu}\psi_2(x). \tag{21}$$

In a similar manner, considering the situation with source η_1 as a detection source of the virtual particle, we get the effective electron-photon detection source of the form:

$$i\eta_1(x)\gamma^0 J_1^{\mu}(\xi)|_{eff} = \psi_1(x)\delta(x-\xi)\gamma^0 eq\gamma^{\mu}. \tag{22}$$

The process involving the both partial processes i.e. emission and absorption is synthetized.

The corresponding vacuum amplitude describing the exchange of a noninteractig electron-positron pair is extracted from

$$\langle 0_+|0_-\rangle^{\eta J} = \langle 0_+|0_-\rangle^{\eta}\langle 0_+|0_-\rangle^{J} \tag{23}$$

or, from

$$\langle 0_+|0_-\rangle^{\eta J} = \int (dx)(dx')\eta_1(x)\gamma^0 G_+(x-x')\eta_2(x')i\int (d\xi)(d\xi')J_1^{\mu}(\xi)D_+(\xi-\xi')J_{2\mu}(\xi'). \tag{24}$$

Then, we insert eqs.(21) and (22) into (24) in order to get

$$\langle 0_+|0_-\rangle^{\eta J} = e^2\int (dx)(dx')\psi_1(x)\gamma^0\gamma^{\mu}G_+(x-x')D_+(x-x')\gamma_{\mu}\psi_2(x') \tag{25}$$

Since we are interested in the electron-photon process in the presence of the external electromagnetic field, we replace the electron fields ψ and the propagation function G_+ by ψ^A and G_+^A corresponding to situation with the presence of electromagnetic field. Then, we have:

$$\langle 0_+|0_-\rangle^{\eta J}|_{A\neq 0} = e^2\int (dx)(dx')\psi_1^A(x)\gamma^0\gamma^{\mu}G^A(x,x')D_+(x-x')\gamma_{\mu}\psi_2^A(x'). \tag{26}$$

The validity of eq. (26) is restricted to $x^0 > x'^0$ as a consequence of the causal situation i.e. the detection source is later than the emission source. The extension to the general situation is postulated by the space-time extrapolation.

When G^A in eq. (26) is replaced by expansion

$$G^A \approx G_+ + G_+ eq\gamma A G_+ + G_+ eq\gamma A G_+ eq\gamma A G_+, \tag{27}$$

which is a sufficient approximation for the propagation function of electron in the external field, we get three types of processes. 1) $G^A \to G_+$ implies the electron propagator modification, 2) $G^A \to G_+ eq\gamma A G_+$ is the linear term contribution , 3) $G^A \to G_+ eq\gamma A G_+ eq\gamma A G_+$ is the double scattering contribution.

Only the vacuum amplitude terms quadratic in A are retained and they are sufficient for an approximation. The diagram corresponding to the linear interaction can be graphically pictured.

Now, we approach to the discussion of of the contribution of vacuum polarization and double scattering.

4 Vacuum polarization calculation

After performing the Fourier transformation, the total external potential is written as

$$A^\mu(q) = D_+(q^2) J^\mu(q) \tag{28}$$

where J^μ is the associate source. We know that the vacuum polarization leads to the following modification of the photon Green function

$$\tilde{D}_+(q^2) = \frac{1}{q^2} + \int_{4m^2}^{\infty} dM^2 \frac{a(M^2)}{q^2 + M^2} \tag{29}$$

where

$$a(M^2) = \frac{\alpha}{3\pi} \frac{1}{M^2} \left(1 + \frac{2m^2}{M^2}\right) \left(1 - \frac{4m^2}{M^2}\right)^{1/2} \tag{30}$$

Using. eq. (28) it means that the Born cross section is multiplied by factor $(\tilde{D}_+/D_+)^2 \approx D_+^2(1 + 2\varepsilon/D_+)$, and it determines the parameter δ_1 in the corrected cross section

$$\frac{d\sigma}{d\Omega} = (1 - \delta_1) \left(\frac{d\sigma}{d\Omega}\right)_{Born} \tag{31}$$

is of the form

$$\delta_1 = -2q^2 \int_{4m^2}^{\infty} dM^2 \frac{a(M^2)}{q^2 + M^2}, \tag{32}$$

which can be expressed by change of variables

$$M^2 = \frac{4m^2}{1 - v^2} \tag{33}$$

as

$$\delta_1 = -\frac{\alpha}{6\pi}\frac{q^2}{m^2}\int_0^1 dv \frac{v^2(3-v^2)}{1-\frac{q^2}{4m^2}(1-v^2)}. \tag{34}$$

5 Propagator modifications

The vacuum amplitude corresponding to the transformation $G^A \to G_+$ in eq. (26) is

$$\langle 0_+|0_-\rangle = e^2 \int (dx)(dx')\psi_1^A(x)\gamma^0\gamma^\mu G_+(x,x')D_+(x-x')\gamma_\mu\psi_2^A(x') \tag{35}$$

where we use for G_+ and D_+ the causal representation

$$G_+(x-x') = \left(m - \frac{1}{i}\gamma^\mu\partial_\mu\right)\Delta_+(x-x') \tag{36}$$

with

$$\Delta_+(x-x') = i\int d\omega_p e^{ip(x-x')}; \quad x^0 > x'^0 \tag{37}$$

$$D_+(x-x') = \Delta_+(x-x'; m^2 = 0). \tag{38}$$

Using the Fourier transformation and the identity

$$1 = \frac{1}{2\pi}\int d\omega_P dM^2 (2\pi)^4 \delta(P-p-k) \tag{39}$$

with $-P^2 = M^2$, we get with eq. (36), (37) and (38):

$$\langle 0_+|0_-\rangle = \frac{ie^2}{2\pi}\int i d\omega_P dM^2 \psi_1^A(-P)\gamma^0 \int d\omega_p d\omega_k (2\pi)^4\delta(P-p-k)\gamma^\mu(m-\gamma p)\gamma_\mu\psi_2^A(P), \tag{40}$$

where we supposed the mas of photon is $\mu \ll m$.

The $p-k$ phase-space integral it is suitable to calculate in the P rest frame. Then after application of the space-time extrapolation, we have

$$\langle 0_+|0_-\rangle = i\frac{\alpha}{4\pi}\int \frac{(dP)}{(2\pi)^4}\int_{(m+\mu)^2}^{\infty}\frac{dM^2}{M^2}\left[(M^2-m^2)-4M^2\mu^2\right]^{1/2} \times$$

$$\psi_1^A(-P)\left\{\left(-4m^2 - \frac{M^2+m^2}{M^2}\gamma P\right)\frac{1}{P^2+M^2-i\varepsilon} + C.T.\right\}\psi_2^A(P), \tag{41}$$

where the space-time extrapolation was realized as

$$d\omega_P \to \frac{dP}{(2\pi)^4}\frac{1}{P^2+M^2-i\varepsilon}. \tag{42}$$

The contact term C.T. is introduced here as a necessity, because the causal process does not inform us about behavior of vacuum amplitude for $x^0 \approx x'^0$. This term is consequently proportional to $\delta(x-x')$ or derivatives thereof. When $\psi^A \to \psi$, the factor $\{...\}$ in eq. (41) and its first derivative with respect to γP must vanish for $\gamma P = -m$. This requirement has consequence to determine C.T. in such a way that

$$\{./.\} = \frac{(\gamma P + m)^2 \omega}{P^2 + M^2 - i\varepsilon}, \tag{43}$$

where

$$\omega = \frac{1}{2M^2}\left[\left(1 - \frac{2Mm}{(M-m)^2}\right)(M - \gamma P) + \left(1 + \frac{2Mm}{(M+m)^2}\right)(-M - \gamma P)\right]. \tag{44}$$

If we further retain only the infrared singular part of ω and appropriate ψ^A by

$$(\gamma P + m)\psi^A(P) = \int (dx) e^{-iPx} eq\gamma A(x) \equiv (eq\gamma A\psi)(P), \tag{45}$$

we get instead of (41)

$$\langle 0_+|0_-\rangle = i\frac{\alpha}{4\pi}\int \frac{(dP)}{(2\pi)^4}\int \frac{dM^2}{M^2}\left[(M^2 - m^2) - 4M^2\mu^2\right]^{1/2} (\psi_1\gamma^0 eq\gamma A)(-P) \quad \times$$
$$\left(-\frac{m}{M}\right)\frac{1}{(M-m)^2}\frac{m - \gamma P}{P^2 + M^2 - i\varepsilon}(eq\gamma A\psi_2)(P), \tag{46}$$

from which we extract the factor δ_2 of the correction to the Born cross section

$$\delta_2 = -\frac{\alpha}{4\pi}\int_{(m+\mu)^2}^{(m+\delta M)^2} \frac{dM^2}{M^2}\left[(M^2 - m^2) - 4M^2\mu^2\right]^{1/2}\left(-\frac{m}{M}\right)\frac{1}{(M-m)^2} \quad \approx$$
$$\frac{\alpha}{\pi}\int_{\mu}^{\delta M} d(M-m)\left[(M-m)^2 - \mu^2\right]^{1/2}(M-m)^{-2} \quad =$$
$$\frac{\alpha}{\pi}\left[\ln\frac{2\delta M}{\mu} - 1\right]. \tag{47}$$

where $m^2 \leq -p'^2 \leq (m+\delta M)^2$, $\delta M \ll m$.

6 Double scattering

The vacuum amplitude for double scattering is obtained by transformation

$$G^A \to G_+ A G_+ A G_+ \tag{48}$$

in the vacuum amplitude

$$\langle 0_+|0_-\rangle^{\eta J}_{A\neq 0} = e^2 \int (dx)(dx')\psi_1^A(x)\gamma^0\gamma^\mu G^A(x,x')D_+(x-x')\gamma_\mu\psi_2^A(x'). \tag{49}$$

The resulting amplitude for double scattering can be expressed after necessary calculations as

$$\langle 0_+|0_-\rangle = ie^2 \int \frac{(dP_1)}{(2\pi)^4} \int \frac{(dP_2)}{(2\pi)^4} \quad \times$$

$$\psi_1(-P_1)\gamma^0\gamma^\mu \frac{m-\gamma P_1+\gamma k}{(P_1-k)^2+m^2} eq\gamma A(P_1-P) dM^2 d\omega_P d\omega_k \quad \times$$

$$\delta((P-k)^2+m^2)(m-\gamma P+\gamma k) eq\gamma A(P-P_2)\frac{m-\gamma P_2+\gamma k}{(P_2-k)^2+m^2}\gamma_\mu\psi_2(P_2). \tag{50}$$

where the momentum of the exchanged electron has been substituted according to $p = P - k$

$$d\omega_P = \frac{(dP)}{(2\pi)^3}\delta((P-k)^2+m^2) = dM^2 d\omega_P \delta((P-k)^2+m^2) \tag{51}$$

Equation (50) leads to purely inelastic contribution and it means that only its infrared singular part need be retained, which means the γk factor in the numerator may be dropped. Upon rearrangement and operation of the projection on the fields, the vacuum amplitude reduces to

$$\langle 0_+|0_-\rangle = ie^2 \int \frac{(dP_1)}{(2\pi)^4} \int \frac{(dP_2)}{(2\pi)^4} i d\omega_P \int_{(m+\mu)^2}^{\infty} dM^2 \psi_1(-P)\gamma^0 eq\gamma A(P_1-P) \quad \times$$

$$\left(\int d\omega_k \delta((P-k)^2+m^2)\frac{P_2^2}{(P_2k)^2}(m-\gamma P)\right) eq\gamma A(P-P_2)\psi_2(P_2). \tag{52}$$

After space-time extrapolation we get (the contact terms are not inserted since they are not physically required) for the correction δ_3:

$$\delta_3 = -e^2 \int_{(m+\mu)^2}^{(m+\delta M)^2} dM^2 d\omega_k \delta((P-k)^2+m^2)\frac{P_2^2}{(P_2k)^2} \approx$$

$$\frac{\alpha}{\pi}\int_\mu^{\delta M} d(M-m)\left[(M-m)^2-\mu^2\right]^{1/2}\left[(M-m)^2+\mu^2\frac{q^2}{m^2}\left(1+\frac{q^2}{4m^2}\right)\right]^{-1} =$$

$$\frac{\alpha}{\pi}\left(\ln\frac{2\delta M}{\mu} - 1 - \frac{q^2}{4m^2}\int_0^1 dv\, \frac{1+v^2}{1+\frac{q^2}{4m^2}(1-v^2)}\right). \tag{53}$$

where $q - P - P_2$ is the momentum transfer. The k-integral was evaluated in the P rest frame and using relations:

$$P_2^0 = -\frac{PP_2}{M} = \frac{1}{2M}(M^2 + m^2 + q^2) \tag{54}$$

and

$$|\mathbf{P}_2|^2 = \frac{1}{4M^2}(M^2 + m^2 + q^2)^2 - m^2 \approx q^2 \left(1 + \frac{q^2}{4m^2}\right). \tag{55}$$

7 The linear term. The electric part

The diagram corresponding to the linear term $G^A \to G_+ A G_+$ follows from the formula

$$\langle 0_+|0_-\rangle = e^2 \int (dx)(dx')\psi_1^A(x)\gamma^0\gamma^\mu G_+^A(x,x')D_+(x-x')\gamma_\mu\psi_2^A(x') \tag{56}$$

Upon transformation of the linearized amplitude (56) into momentum space we get

$$\langle 0_+|0_-\rangle = -\int \frac{(dP_1)}{(2\pi)^4}\int \frac{(dP_2)}{(2\pi)^4}\psi_1(-P_1)\gamma^0 eqI^\mu A_\mu(q)\psi_2(P_2) \tag{57}$$

with

$$q = P_1 - P_2 \tag{58}$$

and

$$I^\mu = ie^2 \int d\omega_k d\omega_p d\omega_{p'}(2\pi)^4\delta(p+k-P_2)(2\pi)^4\delta(p'+k-P_1)\gamma^\nu(m-\gamma p')\gamma^\mu(m-\gamma p)\gamma_\nu \tag{59}$$

which can be expressed in the general form as

$$I^\mu = i\alpha\pi\gamma^\mu f(M_1^2, M_2^2, q^2) + \frac{\alpha\pi}{2m}\sigma^{\mu\nu}q_\nu g(M_1^2, M_2^2, q^2), \tag{60}$$

where $-P_1^2 = M_1^2$ and functions f and g are to be determined. The eq.(57) contains no q^μ because we work in the Lorentz gauge. Upon contraction of I^μ with appropriate vector the functions f and g are isolated and expressed in terms of the known kinematic factors as follows (de Raad et al., 1972):

$$f = -2q^2\Delta^{-5/2}\left\{q^8 + q^6\left[3(M_1^2 + M_2^2) + 4m^2\right] \quad +\right.$$

$$q^4\left[3M_1^4 + 3M_1^2M_2^2 + 3M_2^2 + 9m^2(M_1^2 + M_2^2) + 5m^4\right] \quad +$$

$$q^2\left[M_1^6 - 2M_1^4M_2^2 - 2M_1^2M_4^2 + M_6^2 + 6m^2M_1^4 + 2m^2M_1^2M_2^2\right] \quad +$$

$$q^2\left[6m^2M_2^4 + 13m^4(M_1^2 + M_2^2) - 6m^6\right] \quad -$$

$$(M_1^2-M_2^2)^2\left[2M_1^2M_2^2-m^2(M_1^2+M_2^2)-8m^4\right]\Big\} \tag{61}$$

and

$$g=-4m^2q^2\Delta^{-3/2}\times$$

$$\Big\{6\left[q^2(M_1^2-m^2)(M_2^2-m^2)-m^2(M_1^2-M_2^2)^2\right]\Delta^{-1}-(M_1^2+M_2^2-2m^2)\Big\} \tag{62}$$

with

$$\Delta=(q^2+M_1^2+M_2^2)^2-4M_1^2M_2^2. \tag{63}$$

Quantities M_1^2 and M_2^2 satisfy the relation:

$$q^2(M_1^2-m^2)(M_2^2-m^2)\geq m^2(M_1^2-M_2^2)^2. \tag{64}$$

After space-time extrapolation, the vacuum amplitude of vertex is as

$$\langle 0_+|0_-\rangle=\frac{i\alpha}{4\pi}\int\frac{(dP_1)}{(2\pi)^4}\int\frac{(dP_2)}{(2\pi)^4}\quad\times$$

$$\psi_1(-P_1)\gamma^0\left(eq\gamma^\mu A_\mu(q)F(P_1,P_2)+\frac{eq}{2m}\,\frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu}(q)G(P_1,P_2)\right)\psi_2(P_2) \tag{65}$$

with

$$F(P_1,P_2)=\int dM_1^2dM_2^2\frac{f(M_1^2,M_2^2,q^2)}{(P_1^2+M_1^2-i\varepsilon)(P_2^2+M_2^2-i\varepsilon)} \tag{66}$$

and

$$G(P_1,P_2)=\int dM_1^2dM_2^2\frac{g(M_1^2,M_2^2,q^2)}{(P_1^2+M_1^2-i\varepsilon)(P_2^2+M_2^2-i\varepsilon)}. \tag{67}$$

where the region of integration is determined by eq. (64).

Using variables x and v, defined by

$$\frac{1}{2}(M_1^2+M_2^2)=m^2+(m^2+\frac{1}{4}q^2)2x \tag{68}$$

$$(M_1^2-M_2^2)=\left[q^2(m^2+\frac{1}{4}q^2)\right]^{1/2}2xv \tag{69}$$

where $x\in(0,\infty)$ and $v\in(-1,1)$, we have:

$$\frac{dM_1^2dM_2^2}{\Delta^{1/2}}=\frac{dv}{2}\,\frac{xdx}{\beta^{1/2}}(q^2+4m^2) \tag{70}$$

$$\Delta^{1/2} f = -\frac{1}{2\beta^2}\left[q^2(4+12x+9x^2+3x^2v^2-x^3+5x^3v^2-2x^4v^2+2x^4v^4) \quad + \right.$$

$$\left. 4m^2(2+6x+2x^2+2x^2v^2-x^3-x^3v^2-2x^4v^2)\right] \tag{71}$$

and

$$\Delta^{1/2} g = -\frac{1}{2}(q^2+4m^2)m^2 x\left[3x(1-v^2)\beta^{-2}-2\beta^{-1}\right], \tag{72}$$

where

$$\beta = 1+2x+x^2v^2. \tag{73}$$

Now, the vacuum amplitude is of the form:

$$\langle 0_+|0_-\rangle = \frac{i\alpha}{4\pi}\int\frac{(dP_1)}{(2\pi)^4}\int\frac{(dP_2)}{(2\pi)^4}\int_{-1}^{1}\frac{1}{2}dv \quad \times$$

$$\int_{x_0}^{\infty}\frac{xdx}{\beta^{1/2}}\psi_1^A(-P_1)\gamma^0 eqM^\mu A_\mu(q)\psi_2^A(P_2), \tag{74}$$

where

$$x_0 = \mu(m^2+\frac{1}{4}q^2)^{-1/2}(1-v^2)^{-1/2} \tag{75}$$

with

$$M^\mu = \gamma^\mu\left[\frac{q^2(q^2+4m^2)}{(P_1^2+M_1^2-i\varepsilon)(P_2^2+M_2^2-i\varepsilon)}f_1+\frac{q^2/4m^2}{1+\frac{q^2}{4m^2}(1-v^2)} \quad + \right.$$

$$\left.\frac{4f_3}{x^2}\left(\frac{(M_1^2-m^2)(M_2^2-m^2)}{(P_1^2+M_1^2-i\varepsilon)(P_2^2+M_2^2-i\varepsilon)}-1\right)\right] \quad +$$

$$\{2mf_4\left[(m+\gamma P_1)\gamma^\mu+\gamma^\mu(m+\gamma P_2)\right]+f_5(m+\gamma P_1)\gamma^\mu(m+\gamma P_2)\} \quad \times$$

$$\frac{1}{(P_1^2+M_1^2-i\varepsilon)(P_2^2+M_2^2-i\varepsilon)}. \tag{76}$$

where f_1 nd f_3 are extracted form eq. (71). To find the extraction, we substitute the equality

$$4m^2 = \left[4m^2+q^2(1-v^2)\right]-q^2(1-v^2) \quad =$$

$$\frac{4}{x^2}(M_1^2-m^2)(M_2^2-m^2)(q^2-4m^2)^{-1}-q^2(1-v^2) \tag{77}$$

into eq. (71). Then f_1 is identified as the coefficient of q^2 in eq. (71) and f_3 as the coefficient of

$$\frac{4}{x^2}(M_1^2 - m^2)(M_2^2 - m^2)(q^2 - 4m^2)^{-1}. \tag{78}$$

Then,

$$f_1 = -(1+x)(1+v^2)\beta^{-1} - \frac{3}{2}x^2(1-v^2)(1+xv^2)\beta^{-2} \tag{79}$$

$$f_3 = -(1+x-x^2)\beta^{-1} - \frac{3}{2}x^3(1-v^2)\beta^{-2}. \tag{80}$$

Till this moment we do not discuss the contact terms. They are determined as it is known by the special physical conditions. Here the contact terms are $-4f_3/x^2$ and f_2, f_3 in the expression for M^μ. The former was determined by the physical situation of non external electromagnetic influence, i.e. $J = 0$ and zero vacuum amplitude (74). The contact term f_2 is determined by te requirement that for real external electrons $\gamma P_1 = \gamma P_2 = -m^2$, eq. (74) with $\psi^A \to \psi$ reproduces the ordinary electric form factor (f_3, f_4, f_5 terms vanish). The contact term f_3 was derived from identification of eq. (41) with eq. (43) with $P \to P - eqA$, by the identification of their linear parts in A. The explicit form of f_2 is found to be

$$f_2 = \frac{-6(1+x)v^2}{x\beta}. \tag{81}$$

The basic structure, which appears in the imaginary part calculation of the cross section is M^μ multiplied by the propagator $(m - \gamma p)(p^2 + m^2 - i\varepsilon)^{-1}$, where p is P_2 in one vacuum term and P_1 in the other term. Then,

$$\begin{aligned}
\frac{1}{\pi}\text{Im}\,(G_+ M^\mu) \quad &= \\
\frac{m - \gamma P_1}{(M_1^2 - m^2)(M_2^2 - m^2)}\gamma^\mu q^2(q^2+4m^2) f_1 \left[\delta(P_1 + m^2) - \delta(P_1^2 + M^2)\right] \quad &+ \\
(m - \gamma P_1)\gamma^\mu \frac{q^2}{4m^2} f_2 \left(1 + \frac{q^2}{4m^2}(1-v^2)\right)^{-1} \delta(P_1^2 + m^2) \quad &- \\
(m - \gamma P_1)\gamma^\mu 4 f_3 x^{-2} \delta(P_1^2 + M_1^2) \quad &+ \\
2m f_4 \gamma^\mu (M_2 - m^2)^{-1} \delta(P_1^2 + M_1^2) + (P_1 \leftrightarrow P_2, M_1 \leftrightarrow M_2) &
\end{aligned} \tag{82}$$

The f_5 term in M^μ did not enter in (82) because of the projection factors associated with it. The f_4 term is not infrared singular, so, as a purely inelastic contribution, it may be dropped. The major contribution comes from f_1 which is unification of elastic-inelastic structure and it is not infrared sensitive.

The x-integration limits for the inelastic contribution are given by $(x_0 \le x \le x_1)$

$$x_0 = \frac{\mu}{m}\left[(1-v^2)\left(1+\frac{q^2}{4m^2}\right)\right]^{-1/2} \tag{83}$$

$$x_1 = \frac{\delta M}{m}\left\{1+\frac{q^2}{4m^2} - v\left[\frac{q^2}{4m^2}\left(1+\frac{q^2}{4m^2}\right)\right]^{1/2}\right\}^{-1}. \tag{84}$$

So, finally , if we express the contribution to the cross section by δ_4 in analogy with the previous text, we have:

$$\delta_4 = -\frac{\alpha}{2\pi}\int_{-1}^{1}\frac{dv}{2}\left(\int_{x_1}^{\infty}\frac{4q^2 f_1}{x^2[4m^2+q^2(1-v^2)]} \quad + \right.$$

$$\left.\int_0^{\infty}\frac{(q^2/4m^2)f_2}{1+\frac{q^2}{4m^2}(1-v^2)} - \int_{x_0}^{x_1}\frac{4f_3}{x^2}\right)\frac{xdx}{\beta^{1/2}} \quad =$$

$$-\frac{2\alpha}{\pi}\left\{\ln\frac{2\delta M}{\mu} - 1 + \frac{q^2}{4m^2}\int_0^1 dv\frac{1}{1+\frac{q^2}{4m^2}(1-v^2)} \quad \times\right.$$

$$\left.\left[(1+v^2)\ln\left(\frac{\delta M}{4m}\,\frac{(1-v^2)^{3/2}}{v^2}\right)+\frac{1}{2}+v^2\right]\right\} \tag{85}$$

where we have used the identity

$$\int_{-1}^{1}\frac{dv}{2}\frac{1+v^2}{1+\frac{q^2}{4m^2}(1-v^2)} \quad \times$$

$$\ln\left\{1+\frac{q^2}{4m^2} - v\left[\frac{q^2}{4m^2}\left(1+\frac{q^2}{4m^2}\right)\right]^{1/2}\right\} \quad =$$

$$\int_0^1 dv\frac{1}{1+\frac{q^2}{4m^2}(1-v^2)}\left((1+v^2)\ln\frac{2v^2(1+v)}{(1-v)^{3/2}} - v - v^2\right) \tag{86}$$

in the derivation of δ_4 and this identity obtained from (4-5.104) and (4-12.42) in text by Schwinger (1973).

8 The linear term. The magnetic part

The corresponding vertex correction in this case is the magnetic part of vacuum amplitude (65). The magnetic vertex is not infrared singular and therefore to one order of approximation in δM, the inelastic contributions may be neglected. The correction reduces to the ordinary magnetic form factor and the vacuum amplitude is of the form:

$$\langle 0_+|0_-\rangle = \frac{i\alpha}{2\pi}\int (dx)(dx')(dx'')$$

$$\Big\{\psi_1(x)\gamma^0 eq\gamma A(x)G_+(x-x')\frac{eq}{2m}\frac{1}{2}\,\sigma^{\mu\nu}\psi_2(x')F_2(x'-x'')F_{\mu\nu}(x'') \quad +$$

$$F_{\mu\nu}(x'')F_2(x''-x)\psi_1(x)\gamma^0\frac{eq}{2m}\frac{1}{2}\,\sigma^{\mu\nu}G_+(x-x')eq\gamma A(x')\psi_2(x')\Big\}\,, \tag{87}$$

where the magnetic form factor is given by de Raad et al.(1972)

$$F_2(q) = \int_0^1 dv \frac{1}{1+\frac{q^2}{4m^2}(1-v^2)}. \tag{88}$$

From the amplitude (87) then can be deduced the cross section of the form

$$\left(\frac{d\sigma}{d\Omega}\right)_{mag} = \frac{\alpha m}{32\pi^3}F_2(q)\Big\{u^*_{p\sigma q}\gamma^0\frac{e}{2m}\sigma^{\mu\nu}F_{\mu\nu}(-q)(m-\gamma p')eq\gamma A(q)u_{p\sigma q} \quad +$$

$$u^*_{p\sigma q}\gamma^0 eq\gamma A(-q)(m-\gamma p')\frac{e}{2m}\sigma_{\mu\nu}F_{\mu\nu}(q)u_{p\sigma q}\Big\} \tag{89}$$

9 Discussion

We have presented here some new methods for calculating the radiative corrections for the scattering of an electron by an external electromagnetic field. This work differs from previous efforts' on the subject because it is formulated within Schwinger's source theory. But an additional important difference is that the conventional separation of elastic and inelastic (electron plus soft photon) processes is avoided. In such a way we obtained a reduction of calculation, relative to the conventional approach, and some standard infrared divergences are absent.

Our method combines a given elastic contribution with a corresponding inelastic contribution, and in such sums infrared sensitivity never occurs.

But there are also infrared-sensitive inelastic contributions that occur separately. In these terms we must insert a photon mass, then vanishing when all such purely inelastic contributions are explicitly summed.

Let us summarize the final results. We have yet mentioned that the complete formula of the cross section is of the form

$$\frac{d\sigma}{d\Omega} = (1-\delta)\left(\frac{d\sigma}{d\Omega}\right)_{Born} + \left(\frac{d\sigma}{d\Omega}\right)_{mag}, \tag{90}$$

where $(d\sigma/d\Omega)_{mag}$ is given by eq. (84) and δ is the sum of $\delta_1, \delta_2, \delta_3$ and δ_4, and it is explicitly equal to

$$\delta = \frac{\alpha}{2\pi}\frac{q^2}{m^2}\int_0^1 dv \frac{1}{1+\frac{q^2}{4m^2}(1-v^2)} \quad \times$$

$$\left[(1+v^2)\ln\left(\frac{4m}{\delta M}\,\frac{v^2}{(1-v^2)^{3/2}}\right)-1-\frac{5}{2}v^2+\frac{1}{3}v^4\right]. \tag{91}$$

After evaluation of the v-integral we get (Schwinger, 1973).

$$\delta=-\frac{2\alpha}{\pi}\left\{-\frac{19}{18}+\frac{4}{3}\frac{m^2}{q^2}+\ln\frac{m}{2\delta M}\quad-\right.$$

$$\frac{2m^2}{q^2}\zeta\left[-\frac{4}{3}\frac{m^2}{q^2}+\frac{11}{6}+\frac{19}{6}\frac{q^2}{4m^2}-\left(1+\frac{q^2}{2m^2}\right)\ln\frac{m}{2\delta M}\right]\ln\frac{1-\zeta}{1+\zeta}\quad+$$

$$\left.\frac{3}{2}\left(\frac{2m^2}{q^2+1}\right)\zeta\left[-f\left(\frac{2\zeta}{1-\zeta}\right)+f\left(-\frac{2\zeta}{1-\zeta}\right)+\frac{4}{3}f(\zeta)-\frac{4}{3}f(-\zeta)\right]\right\}, \tag{92}$$

where

$$\zeta^2=\frac{q^2}{4m^2+q^2} \tag{93}$$

and $f(x)$ is the Spence function (Berestetskii et al, 1982) defined as

$$f(x)=-\int_0^x\frac{dt}{t}\ln|1-t| \tag{94}$$

The non relativistic and ultra-relativistic asymptotic forms are

$$\delta_{nonrel.}=-\frac{\alpha}{2\pi}\frac{q^2}{m^2}\left(\frac{4}{3}\ln\frac{2\delta M}{m}-\frac{31}{90}\right);\quad\frac{q^2}{m^2}\ll 1 \tag{95}$$

and

$$\delta_{ultra-rel.}=\frac{2\alpha}{\pi}\left[\left(\ln\frac{q^2}{m^2}-1\right)\left(\ln\frac{4m}{\delta M}-\frac{19}{12}\right)-\frac{19}{36}\quad+\right.$$

$$\left.\frac{3}{4}\left(\ln\frac{q^2}{4m^2}\right)^2+3(\ln 2)(1-\ln 2)\right];\quad\frac{q^2}{m^2}\gg 1. \tag{96}$$

In the non-relativistic limit the magnetic cross section reduces to the Born cross section multiplied by a factor

$$\delta+\delta_{mag}=-\frac{2\alpha}{3\pi}\frac{q^2}{m^2}\left(\ln\frac{2\delta M}{m}-\frac{19}{30}\right), \tag{97}$$

where

$$\delta_{mag}=\frac{\alpha}{4\pi}\frac{q^2}{m^2}. \tag{98}$$

In this limit $\delta M=\delta E$. Let us remark that during calculation we have not considered such effect as recoil which requires together other effect more further work.

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