Relic Black Holes, in Terms of a Quantum Number n, linked to Torsion and Quantum Hair on Black Holes

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Abstract

Our idea is that a particular set of values of initial conditions for relic black holes will enable using the idea of torsion to formulate a cosmological constant and resultant dark energy. Relic Planck-sized black holes will allow for a spin-density term presenting an opportunity to cancel torsion. Meanwhile, speculation given by Corda replaces traditional firewalls in relic black holes with a quantum number, n. In addition, this idea can offer a solution to the incompleteness of hairless black holes.

Keywords: Inflation, Gravitational waves, Penrose CCC.

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1 Parameters of Black-Hole Physics and the Quantum Number n

We use BEC condensates, as indicated below, with a simple model for the loss of primordial black-hole mass. We use the substitutions outlined [1, 2, 4, 3] to reintroduce black-hole physics in terms of a quantum number, n. To begin, consider initial black-hole physics in the regime of Planckian physics to the onset of the big bang. The BEC condensate is given by Ref. [1, 2, 4, 3].

2 Introduction, Origins of the Black-Holes-Have-No-Hair Theorem and a Preview of Our Modifications

Our supposition, which will eventually end by challenging the idea that black holes have no hair, starts with a simple idea: the simple intuitive model for how a black hole of mass M could experience a loss of its essence. Here, M is a mass, T is temperature, and $\widetilde{a}$ is a proportionality term, which is assumed to be a constant.

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\widetilde{a} \cdot T^4 \tag{1}$$

The Hawking temperature, T, is related to primordial black-hole mass. As a first approximation, smaller black holes are hotter. We will make the following simple rule.

$$T = \frac{\hbar c^3}{8\pi k_B G M} \tag{2}$$

A Planck-mass black hole would approach the Planck temperature. If Eq. (1) is observed as far as black-hole mass loss over time, this leads to

$$M^5(\text{loss}) = \left(\frac{-5}{64^2} \cdot \widetilde{a}\right) \cdot \left(\frac{\hbar^4 c^{12}}{\pi^4 k_B^4 G^4}\right) \cdot t. \tag{3}$$

To parameterize this further in terms of our model as to how we can observe a violation of the black-holes-have-no-hair idea, we will need to do some parameterization of a mass M of black holes in terms of the following inputs for our article.

3 The Parameters of Black Hole Physics Used in this Essay, and How Torsion May Allow for Understanding New Bounds as to Black Hole Models, as Well as the Importance of a Quantum Number n

Following Ref. [1, 2], we do the following using the substitutions outlined to reintroduce black-hole physics in terms of a quantum number, n. To begin this, first look at the following for dynamic scaling as far as initial black-hole physics in the primordial moments in the Planckian regime of physics at the onset of the big bang. We then get

$$\sqrt{\Lambda} = \frac{k_B E}{\hbar c S}, \tag{4}$$

where Λ is an effective cosmological constant, $S = k_b N$ and N is the number of particles. Thus, with our current huge entropy value (say 10^{120}), the cosmological constant is negligible.

Then, reference the BEC condensate, m, as to scaling [2, 3].

$$\begin{aligned} m &\approx \frac{M_{\mathrm{P}}}{\sqrt{N_{\text{gravitons}}}} & M_{\mathrm{BH}} &\approx \sqrt{N_{\text{gravitons}}} \cdot M_{\mathrm{P}} \\ R_{\mathrm{BH}} &\approx \sqrt{N_{\text{gravitons}}} \cdot \ell_{\mathrm{P}} & S_{\mathrm{BH}} &\approx k_B \cdot N_{\text{gravitons}} \\ T_{\mathrm{BH}} &\approx \frac{T_{\mathrm{P}}}{\sqrt{N_{\text{gravitons}}}}. \end{aligned} \tag{5}$$

This is promising, but one more step will use the importance of Ref. [5] through the following energy expression. First, a time step:

$$\tau \approx \sqrt{GM\delta r}. \tag{6}$$

Using the simplest version of the HUP [6], *not* the version we finally use, we can use Eq. (7) for an energy for radiation of a particle pair from a black hole [5].

$$|E| \approx \left(\sqrt{GM\delta r}\right)^{-1} \hbar \tag{7}$$

Here, we use some approximations. Namely, we assert that the range of applicability of the spatial variation goes as $\delta r \approx \ell_{\mathrm{P}}$. This is the order of a Plank length, and we assume in Eq. (7) a roughly Planck-sized black-hole mass.

$$M \approx \alpha M_{\mathrm{P}}. \tag{8}$$

If so, we transform Eq. (7) to be roughly of the form for, say, an electron–positron pair radiating to form a black hole [5].

$$|E| \approx \left(\sqrt{G(\alpha M_{\mathrm{P}})\delta r}\right)^{-1} \hbar \tag{9}$$

We argue that black holes on the order of a Planck mass release intense radiation. So, we approximate this in Eq. (9) as roughly equivalent to the effective mass of a relic black hole, so, up to a point, we use the Carlip energy expression as roughly equivalent to the mass of a microsized black hole. Now, consider the following normalization of Planck units [7, 6].

$$c = 4\pi G = \hbar = \varepsilon_0 = k_B = 1 \tag{10}$$

Also, the initial treatment of energy, E, for a black hole is [8]

$$E_{\mathrm{BH}} = -\frac{n_{\text{quantum}}}{2}. \tag{11}$$

We can provisionally use the following scaling for a black hole.

$$|E| \approx \frac{\hbar}{\sqrt{G\alpha M_{\mathrm{P}}\delta r}} \xrightarrow[G=M_{\mathrm{P}}=\hbar=k_B=\ell_{\mathrm{P}}=c=1]{} \left(\frac{1}{M_{\mathrm{BH}}}\right)^{\frac{1}{2}} \approx \frac{n_{\text{quantum}}}{2} \tag{12}$$

We can then reference Eq. (5) to observe the following.

$$M_{\mathrm{BH}} \approx \sqrt{N_{\text{gravitons}}} M_{\mathrm{P}} \Rightarrow \left(\frac{1}{M_{\mathrm{BH}}}\right)^{\frac{1}{2}} \approx \frac{n_{\text{quantum}}}{2} \approx \frac{1}{(N_{\text{gravitons}})^{\frac{1}{4}}} \Rightarrow n_{\text{quantum}} \approx \frac{2}{(N_{\text{gravitons}})^{\frac{1}{4}}} \tag{13}$$

Black hole temperature increases dramatically with smaller and smaller black holes. Does this imply that corresponding increases in quantum number, per black hole, n, are commensurate with increasing temperature? Obviously, this is a preliminary result, but it ties in with what we can say about table 1. Table 1 is used for the following modification of dark energy and the cosmological constant [1, 2, 4, 3, 9]. To begin this, look at Ref. [1], which purports to show a global cancellation of a vacuum energy term. This is akin, as we will discuss, to Eq. (14) [1, 4].

$$\rho_\Lambda c^2 = \int_0^{E_{\text{Planck}}/c} \frac{4\pi p^2\,\mathrm{d}p}{(2\pi\hbar)^3}\cdot\left(\frac{1}{2}\cdot\sqrt{p^2c^2+m^2c^4}\right) \approx \frac{(3\times10^{19}\,\text{GeV})^4}{(2\pi\hbar)^3} \xrightarrow[E_{\text{Planck}}/c\to10^{-30}]{} \frac{(2.5\times10^{-11}\,\text{GeV})^4}{(2\pi\hbar)^3} \tag{14}$$

The first line of the table is the vacuum energy [4] and is completely cancelled in their formulation of torsion. In our article, we argue for the second line. Our reduction to the second line of Eq. (25) confirms the following change in the Planck-energy term [4].

We can then reference Eq. (5) to observe the following.

$$\frac{\Delta E}{c} = 10^{18}\,\text{GeV} - \frac{n_{\text{quantum}}}{2c} \approx 10^{-12}\,\text{GeV}, \tag{15}$$

where n comes from a Corda-derived expression of the energy level of relic black holes [8]. We argue that our application will be commensurate with Eq. (14), which uses the value given in Ref. [1, 4]. That is, relic black holes will contribute to the generation of a cut off of the energy of the integral given in Eq. (14), whereas what is done in Eq. (14) by Ref. [1, 4] is restricted to a different venue reproduced below: cancellation of the following by Torsion.

$$\rho_\Lambda c^2 = \int_0^{E_{\text{Planck}}/c} \frac{4\pi p^2\,\mathrm{d}p}{(2\pi\hbar)^3}\cdot\left(\frac{1}{2}\cdot\sqrt{p^2c^2+m^2c^4}\right) \approx \frac{(3\times10^{19}\,\text{GeV})^4}{(2\pi\hbar)^3} \tag{16}$$

Furthermore, Ref. [4] claims that there is no cosmological constant. That is, torsion always cancels Eq. (16), which we view as incommensurate with table 1 [4]. We claim that the influence of torsion will aid in the decomposition of what is given in table 1 and will furthermore lead to the influx of primordial black holes, which we claim is responsible for the behavior of Eq. (16).

4 Black Hole Physics Useful for Modeling Dark Energy: Inputs into the Torsion Spin-Density Term

Consider the spin-density term [1, 4]. We insert black hole physics into this to form a spin-density term for primordial black holes.

$$\sigma_{\text{Planck}} = n_{\text{Planck}}\hbar \approx 10^{71}. \tag{17}$$

5 Statement of the Torsion Problem [1, 2, 4]

We are very aware of quack science, purported torsion-physics presentations, and wishes to state that the torsion problem is not linked to anything other than disruption of the initial

Table 1: Assuming Penrose Recycling of the Universe

Time Frame	Mass	Count
End of prior universe: super massive end-of-time black holes at galactic centers	10^{41}–10^{44} g	10^6–10^9
Planck-era black-hole formation: micro-black-hole pairs merging assuming little matter-energy destruction from pre-Planck to Planck conditions	10^{-5}–10^{-4} g (≈ Planck mass)	10^{40}–10^{45}
Post-Planck black holes using Eq. (5) and Eq. (6) to have 10^{10} gravitons/s released as black-hole pairs collapse	10 g–10^6 g	10^{20}–10^{25}

Source: Ref. [4].

expansion of the universe and cosmology. These are more in the spirit of Ref. [4] and nothing else. Hence, we wish to delve into what was given in Ref. [4] with a subsequent follow up and modification: To do this, note that, in Ref. [4], the vacuum energy density is

$$\rho_{\mathrm{vac}} = \frac{\Lambda_{\mathrm{eff}} c^4}{8\pi G}. \tag{18}$$

Finally, in the case of massless particles with torsion present, we have a space–time metric:

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 + a^2(\tau)\,\mathrm{d}^2\Omega_3, \tag{19}$$

where $\mathrm{d}^2\Omega_3$ is the metric of S^3. Then, in this torsion application, the Einstein field equations reduce to (massless particles) as

$$\left(\frac{\mathrm{d}a}{\mathrm{d}\tau}\right)^2 = \left[1 - \left(\frac{r_{\mathrm{min}}^4}{a^4}\right)\right] \tag{20}$$

with, if S is the so-called spin scalar and identified as the basic $\hbar$ unit of spin,

$$r_{\mathrm{min}}^4 = \frac{3G^2S^2}{8c^4}. \tag{21}$$

6 How to Modify Eq. (20) in the Presence of Matter via Yang–Mills Fields, $F_{\mu\nu}^{\beta}$

Eventually, we have a redo of Eq. (20) to

$$\left(\frac{\mathrm{d}a}{\mathrm{d}\tau}\right)^2 = \left[1 - \left(\frac{\beta_1}{a^2}\right) - \left(\frac{\beta_2}{a^4}\right)\right]. \tag{22}$$

If $g = \hbar c$, we have $\beta_1 = r_{\mathrm{min}}^2$, $\beta_2 = r_{\mathrm{min}}^4$, and the minimum radius is identified with a Planck radius. So then

$$\left(\frac{\mathrm{d}a}{\mathrm{d}\tau}\right)^2 = \left[1 - \left(\frac{\beta_1 = \ell_{\mathrm{P}}^2}{a^2}\right) - \left(\frac{\beta_2 = \ell_{\mathrm{P}}^4}{a^4}\right)\right]. \tag{23}$$

Eventually, in the case of an unpolarized spinning fluid in the immediate aftermath of the big bang, we would see a Roberson–Walker universe given as, if σ is a torsion spin term added due to Ref. [4],

$$\left(\frac{\dot{\widetilde{R}}}{\widetilde{R}}\right)^2 = \left(\frac{8\pi G}{3}\right)\cdot\left[\rho - \frac{2\pi G\sigma^2}{3c^4}\right] + \frac{\Lambda c^2}{3} - \frac{\widetilde{k}c^2}{\widetilde{R}^2}. \tag{24}$$

7 What Ref. [4] Does for Eq. (24) versus What We Would Do and Why

In the case of Ref. [1], we would see σ identified as due to torsion so that Eq. (24) reduces to

$$\left(\frac{\dot{\widetilde{R}}}{\widetilde{R}}\right)^2 = \left(\frac{8\pi G}{3}\right)\cdot[\rho] - \frac{\widetilde{k}c^2}{\widetilde{R}^2} \tag{25}$$

This is due to spinning particles [4] and remains invariant so the cosmological vacuum energy, or cosmological constant is always cancelled. Our approach instead will yield [4]

$$\left(\frac{\dot{\widetilde{R}}}{\widetilde{R}}\right)^2 = \left(\frac{8\pi G}{3}\right)\cdot[\rho] + \frac{\Lambda_{\mathrm{observed}} c^2}{3} - \frac{\widetilde{k}c^2}{\widetilde{R}^2}. \tag{26}$$

That is, the observed cosmological constant is 10^{-122} times smaller than the initial vacuum energy. The main reason for the difference between Eq. (25) and Eq. (26) is in the following observation. From table 1, σ^2 is due to the dynamics of spinning black holes in the precursor to the big bang, the Planckian regime of space time, whereas in the aftermath of the big bang, we would have a vanishing of the torsion spin term: The dynamics in the aftermath of the Planckian regime of space–time would largely eliminate the σ^2 term.

8 Collapse of the Cosmological Term versus Preservation via numerical values

First look at the numbers provided by Ref. [4] as inputs. These are very revealing.

$$\Lambda_{\mathrm{P}} c^2 \approx 10^{87} \tag{27}$$

This is the vacuum energy and is 10^{122} times larger than the observed cosmological constant. Torsion physics, as given by Ref. [4], simply removes this giant number. To remove it, Ref. [1] and Ref. [4] make the following identification.

$$\left(\frac{8\pi G}{3}\right) \cdot \left[-\frac{2\pi G \sigma^2}{3c^4}\right] + \frac{\Lambda c^2}{3} = 0 \tag{28}$$

We argue, instead,

$$\left(\frac{8\pi G}{3}\right) \cdot \left[-\frac{2\pi G \sigma^2}{3c^4}\right] + \frac{\Lambda c^2}{3} \approx 10^{-122} \cdot \frac{\Lambda_{\mathrm{P}} c^2}{3}. \tag{29}$$

Our timing for Eq. (27) is to unleash a Planck time interval t, about 10^{-43} s. For Eq. (28) versus Eq. (29), the creation of the torsion term is due to a presumed particle density of

$$n_{\mathrm{P}} \approx 10^{98}\,\mathrm{cm}^{-3}. \tag{30}$$

Finally, we have a spin density term of Eq. (17), $\sigma_{\mathrm{P}} = n_{\mathrm{P}}\hbar \approx 10^{71}$, due to innumerable black holes initially.

9 Future Work for this First Section

We will assume for the moment that Eq. (28) and Eq. (29) share in common Eq. (30). This appears to be trivial, a mere round off, but I can assure you the difference is anything but trivial. This is where table 1 really plays a role in terms of why there is a torsion term to begin with. It will make the following determination. The term of 'spin density' in Eq. (27) by Eq. (30) is defined to be an ad hoc creation, as to Ref. [1]. No description as to its origins is really offered.

First, a future task will be to derive a coherent expression for $\left(\frac{8\pi G}{3}\right) \cdot \left[-\frac{2\pi G \sigma^2}{3c^4}\right]$ arising as a result of the dynamics of table 1.

Second, we state that the term $\left(\frac{8\pi G}{3}\right) \cdot \left[-\frac{2\pi G \sigma^2}{3c^4}\right]$ is due to initial micro black holes, which create the cosmological term.

In the case of Pre-Planckian space–time, the idea is to do the following [4]. If we have an inflaton field [4, 10, 11, 12, 13, 14, 15, 16, 17, 18]

$$\begin{aligned} |\,\mathrm{d}p_\alpha\,\mathrm{d}x^\alpha| \approx \frac{L}{l} \cdot \frac{h}{c} \cdot \left[\frac{\mathrm{d}l}{l}\right]^2 \xrightarrow[\alpha=0]{} |\,\mathrm{d}p_0\,\mathrm{d}x^0| \simeq |\Delta E \Delta t| \approx \left(\frac{h}{a_{\mathrm{init}}^2}\phi(t)\right) \\ \Rightarrow \frac{L}{l} \cdot \frac{h}{c} \cdot \left[\frac{\mathrm{d}l}{l}\right]^2 \approx \left(\frac{h}{a_{\mathrm{init}}^2}\phi(t_{\mathrm{init}})\right). \end{aligned} \tag{31}$$

Making use of all this leads to making sense of the quantum number [10] n as given by reference to black holes [8], $E_{\text{BH}} = -\frac{n_{\text{quantum}}}{2}$.

Third, the conclusion of Ref. [1] states that Eq. (31) would remain invariant for the evolution of the universe. We make no such assumption. We assume that, as will be followed up later, Eq. (29) is due to relic black holes with the suppression of the initially gigantic cosmological vacuum energy. The details of what follows after this initial period of inflation remain a task to be completed in full generality, but we are still assuming as a given the following inputs [4, 14]. A possible future endeavor can also make sense of Ref. [15] as well.

9.1 Torsion and Black-Hole Physics in the Early Universe

First, this formulation puts a premium on table 1 as of Ref. [4]. Second, it means use of Eq. (26), which accounts for the black-hole energy equation given by Corda [8]. It also freely uses the spin-density term, Eq. (17). We refer to black-hole creation as given by torsion this way as a correction to Ref. [1], largely due to the insufficiency of primordial black-hole theory as given in Ref. [16]. We cite their admonition on the insufficiency of current theory (p 366)

> Black holes of masses significantly smaller than a solar mass cannot be formed by gravitational collapse of a star; such miniholes can only form in the early stages of the universe from fluctuations in the very dense primordial matter.

Our torsion argument is directly due to this acknowledgement and to the sterility of theoretical thinking, as well as Corda's tremendously important Eq. 12 [8]. Furthermore, to obtain more detail on Eq. (12) being used for black holes, we state that a quantum state of the early universe will use Ref. [17] and its discussion, page 184, as to how Feynman visualized the quantization of the gravitational field: Eq. 9.121 and 9.122 of Ref. [17] for an early wave-function path-integral treatment for quantized gravity and its use for black holes.

In addition, we outlined the stunning result of Eq. (14) as far as a more-than-an-inverse relationship between graviton number per generated black hole (presumably primordial) and quantum number n, attached to a black hole as due to Ref. [8]. What we see is that if we have small black holes, with BEC characteristics with small numbers of gravitons per primordial black hole, the quantum number n climbs dramatically. We need to obtain the complete dynamics of this relationship as it pertains to how very small black holes have high quantum number n, which we presume is commensurate with initially high temperatures. The details of this development as well as its tie into the dynamics of table 1 as given and torsion must be fine tuned.

9.2 Modification of Black Holes and Hair

Reference [18] offers the essential black-holes-have-no-hair theorem:

> The idea is that beyond mass, charge and spin, black holes don't have distinguishing features—no hairstyle, cut or color to tell them apart.

How do we get this? Note that Ref. [19] has a pseudo extension we can chalk up to Hawking before he died. However, to apply an even more direct treatment, we go to Eq. (65) of Ref. [20]. This will give a variation of the radius of a black hole, over the radius, according to a quantum number n *again*. Before we get there, we will do some initial work up to that quantum number, n. Using our Eq. (13) for N and also the Planck scale normalization as given by $\hbar = k_B = c = G = M_{\mathrm{P}} = \ell_{\mathrm{P}} = 1$, and if we take $\widetilde{a}$ approximately scaled to 1 as well, we have

$$|N| \approx |N_{\text{gravitons}}| \approx \left(\frac{5t}{64^2\pi^4}\right)^{\frac{2}{5}}. \tag{32}$$

Using Ref. [3],

$$M \approx \sqrt{N} M_{\mathrm{P}}. \tag{33}$$

M here is linked to the mass of a BEC black hole. Also, use Eq. (7) for the loss of mass from a black hole over time.

$$|N_{\text{gravitons}}| \cdot (M_{\text{P}} \equiv 1)^{\frac{5}{2}} \approx \left(\frac{5t}{64^2\pi^4}\right)^{\frac{2}{5}} \tag{34}$$

Then, the last equation of Eq. (13) yields a quantum number associated with a graviton just outside a BEC primordial black hole.

$$n_{\text{graviton quantum number}} \equiv n_{\text{gravitons}} \approx \frac{2 \cdot 64^{\frac{1}{10}} \pi^{\frac{1}{5}}}{5^{\frac{1}{20}} \cdot t^{\frac{1}{20}}} \approx \frac{2.16245415907}{t^{\frac{1}{20}}} \tag{35}$$

Assuming a Planck time scale, or close to it, and renormalization to have Planck time set to 1. This means then that the quantum number, n, associated with a graviton with respect to a Planck-sized black hole would be close to 2, initially. If so, then, and this is for primordial black holes, we then associate this graviton number, n, for a graviton as linked to the following from Ref. [20]. Therefore, we have the radius of a BEC black hole deformed by quantum number n:

$$\frac{\Delta R_n}{R_n} \equiv \frac{\sqrt{n^2+2}}{3n}. \tag{36}$$

If we use $n = 2.16245415907$ for a graviton *quantum number* at roughly normalized Planck time, scaled to about one, and we have according to Ref. [20] an ADM mass variance of M. So, then, there is, due to gravitons, a rough change in initial Planck-sized black holes:

$$\Delta R_n = \frac{\sqrt{n^2+2}}{3n} \cdot R_n \approx \left.\frac{\sqrt{n^2+2}}{3n}\right|_{n \equiv 2.16245415907} \cdot R_n, \tag{37}$$

where $n \geq (1-\epsilon) \cdot \left(\frac{M}{M_{\text{P}}}\right)^2$ and we can compare our value of R, as given in Eq. (5) with Ref. [20] having a different scale for R, as given in their Eq. (60).

Needless to say, graviton number n due to the processes within the primordial black hole would lead to a violation of the black-holes-have-no-hair theorem [19]. We assert that this value of n for gravitons would be the following as to the Corda result on Eq. (12).

$$n_{\text{BH}} = N_{\text{gravitons}} \times n_{\text{quantum}} \tag{38}$$

The left hand side of Eq. (38) would be fully commensurate with Eq. (12) of Corda's black-hole quantum number. The right hand side of Eq. (38) would be commensurate with n being for a quantum number per graviton associated per black hole. Many gravitons existing, associated with a primordial black hole, would be associated with a very high initial quantum number, n (black holes), yielding Corda's great result [8].

In the future, we wish to obtain data set analysis to confirm these suppositions and the essence of this presentation as a working model.

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