Exact $\pi(n)$ via Function of Sets Related to Combinatorial Divisory Set

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In memory of Srinivasa Ramanujan (1887-1920).

0. Abstract:
I show in this paper some relations of functions which following the logic of the definition of all of them, it shows a concise perspective of the function $\pi(n)$, namely the number of primes less than a number n.

1.Functions Description:
We define in first place $\pi(n)$ as:

$$\pi(n) = \Sigma \rho_\tau(x) \tag{1}$$

As you can see we defined $\pi(n)$ as the Sum of a function $\rho_\tau(x)$, which is defined as follows:

$$\rho_\tau(x) := 1 \quad if \quad \forall \quad elements \quad of \quad \overset{2}{\underset{i=x\leq n}{c\Delta a_i}} \quad \notin \mathbb{N} \tag{2}$$

And:

$$\rho_\tau(x) := 0 \quad if \quad any \quad element \quad of \quad \overset{2}{\underset{i=x\leq n}{c\Delta a_i}} \quad \in \mathbb{N} \tag{3}$$

And:

$$\rho_\tau(2) := 1 \tag{4}$$

And:

$$\rho_\tau(1) := 0 \tag{5}$$

For $x > 2$ and $x \in \mathbb{N}$ and where $\overset{2}{\underset{i=x}{c\Delta a_i}}$ is the Combinatorial Divisory Set

(CDS) and defines a set with this properties:

$$\begin{matrix} 2 \\ c\Delta a_i \\ i = x \end{matrix} = \{a_i \div a_{i-1}, a_i \div a_{i-2}, ..., a_i \div 2\} \tag{6}$$

So, every division of two numbers is a quantity of the set.

2. Example

For example we are going to analyze $\pi(7) = \Sigma\rho_\tau(x)$ the quantity of prime numbers less than 7, so, and knowing the value of the function in x=1 and x=2, we should first do the CDS of x in 7, 6, 5, 4 and 3:

$$\begin{matrix} 2 \\ c\Delta a_i \\ i = 7 \end{matrix} = \{7 \div 6, 7 \div 5, 7 \div 4, 7 \div 3, 7 \div 2\} = \{7/6, 7/5, 7/4, 7/3, 7/2\} \tag{7}$$

all elements $\notin \mathbb{N}$ so $\rho_\tau(7) := 1$,

$$\begin{matrix} 2 \\ c\Delta a_i \\ i = 6 \end{matrix} = \{6 \div 5, 6 \div 4, 6 \div 3, 6 \div 2\} = \{6/5, 6/4, 2, 3\} \tag{8}$$

any elements $\in \mathbb{N}$ so $\rho_\tau(6) := 0$,

$$\begin{matrix} 2 \\ c\Delta a_i \\ i = 5 \end{matrix} = \{5 \div 4, 5 \div 3, 5 \div 2\} = \{5/4, 5/3, 5/2\} \tag{9}$$

all elements $\notin \mathbb{N}$ so $\rho_\tau(5) := 1$,

$$\begin{matrix} 2 \\ c\Delta a_i \\ i = 4 \end{matrix} = \{4 \div 3, 4 \div 2\} = \{4/3, 2\} \tag{10}$$

any elements $\in \mathbb{N}$ so $\rho_\tau(4) := 0$,

$$\begin{matrix} 2 \\ c\Delta a_i \\ i = 3 \end{matrix} = \{3 \div 2\} = \{3/2\} \tag{11}$$

all elements $\notin \mathbb{N}$ so $\rho_\tau(3) := 1$, so we have analyzed enough to do the Sum, and we have in this case: $\pi(7) = \Sigma\rho_\tau(x) = \rho_\tau(7) + \rho_\tau(6) + \rho_\tau(5) + \rho_\tau(4) + \rho_\tau(3) + \rho_\tau(2) + \rho_\tau(1) = 1 + 0 + 1 + 0 + 1 + 1 + 0 = 4$. Which is the correct answer, there are four prime number less or equal to 7. Following the logic of this example we can compute it to any finite x.

3. Conclusions

For any $\pi(n)$ with any $n \in \mathbb{N}$ we will know the exact number of primes if we follow the defined function $\rho_\tau(x)$. This is a logical answer to the exact value of $\pi(n)$. It is not a fast computational method, but in any case can be seen as sort kind of opposite of Eratosthenes sieve.