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PARADOXES OF GRAVITY: Newton's Law Does Not Work at Large Distances, but... it's too early to throw it in the scrapheap. It's still needed.

Abstract. *The dominant force in the Universe is gravity. For over 300 years, the gravitational force was represented by a single simple and mathematically perfect formula - Newton's law of gravity* ***F_N = GmM/r^2****. However, the boundaries of its applicability are limited to the solar system. The revealed gravitational anomalies in the dynamics of stars show that at large distances, Newton's law is not satisfied and has significant discrepancies with observations. The real law of gravity is more complex than Newton's law. For large distances and large masses, the gravitational force dominates in the Universe, which Newton's law "does not see". A new law of gravity* ***F_{cos} = (mc^2)√Λ*** *was obtained, which shows the cosmological force not taken into account by Newton's law. Two laws of gravity (Newton's law + the law of cosmological force) provide a complete description of the gravitational interaction in the Universe.*

Keywords: *cosmology; dark matter; Newton's law of gravity; MOND theory; Augmented Newtonian Dynamics (AuND); Galaxy rotation curve; Pioneer effect; pioneer anomaly; cosmological force; law of cosmological force; cosmological constant Λ; law of universal gravitation.*

1. Introduction

For a long time, it was believed that Newtonian dynamics is a complete description of all types of motion occurring in the Universe. Newton's law of gravitation **F_N=GmM/r^2** was a real breakthrough in science. Newton's law of gravitation captivates with its simplicity and mathematical perfection. Gravitational interaction is the fourth fundamental interaction. Newton's law of gravitation allows us to explain and predict with great accuracy the motion of celestial bodies within the solar system. The attractiveness of Newton's law of gravitation is the simple dependence of force on the parameters of interacting bodies.

At the same time, Newton's law of gravitation, simple and perfect in mathematical representation, has limitations and boundaries of applicability. Newton's law of gravitation describes the gravitational interaction of two bodies, but does not take into account the gravitational interaction of bodies with the mass of the Universe. The revealed gravitational anomalies in the dynamics of stars show that at large distances Newton's law is not fulfilled and has significant discrepancies with observations [1 - 8].

In 1859, U. Le Verrier discovered an anomalous shift of Mercury's perihelion. Mercury's perihelion shift did not obey Newton's law of universal gravitation [3]. Newcomb discovered a perihelion shift not only for Mercury, but for other planets as well [9]. This means that some additional force acts on the planets. Newton's formula "does not see" this additional force. Newton's law of gravitational interaction between two bodies is based on the assumption of elliptical orbits of the planets. The shift of the perihelion of the planets shows that the real trajectories of the planets are not elliptical and are not closed. An unknown additional force distorts elliptical trajectories and deviates the planets from an ideal elliptical trajectory. Kepler did not know this. Newton did not know this. Kepler's third law is valid for ideal elliptical orbits. Newton's law of gravitation is also valid for ideal elliptical orbits. Both Newton's law and Kepler's third law are idealized, approximate models of gravity.

The limits of applicability of Newton's law of gravity became especially acute in the study of spiral galaxies. Galaxy rotation curve showed a significant discrepancy with Newton's law [4, 10]. Serious difficulties with Newton's model of gravity indicate that Newtonian dynamics is unsuitable for solving cosmological problems. Simple and perfect in mathematical representation, Newton's law of gravity has limitations and limits of applicability.

Attempts have been made repeatedly to modify Newton's law and make it applicable on the scale of the Universe. In 1745, Alexis Clairaut [11] proposed a modification of Newton's law, in which the inverse square law was changed. In 1894, Hall A. [12] proposed replacing the square of the distance with a slightly higher power. Hugo von Seeliger and Carl Gottfried Neumann proposed a modification of the law with a faster decrease in gravity with distance than Newton's [13]. Milgrom developed the MOND theory, which provides for a modification of Newtonian dynamics at low accelerations [1, 2].

No refinements or cosmetic edits to Newton's law of gravity made it applicable on the scale of the Universe. The simple and mathematically perfect formula of Newton's law turned out to be unsuitable for the Universe. Several factors interfere with this: the inverse square law, the law of gravitational interaction of two bodies, point masses.

1. In the real Universe, gravity is stronger than Newton's law of gravity suggests. Observations show that gravity weakens at large distances less than Newton claims. This means that the inverse square law does not apply at large distances.

2. In the real Universe, more than two bodies and even the entire Universe participate in gravitational interaction.

3. There are no point masses in the real Universe.

4. Newton's law of gravity does not include measurable parameters of the Universe.

The real law of gravity is more complex than Newton's law. Newton's law $\mathbf{F_N = GmM/r^2}$ gives the value of the gravitational force for small distances and small masses with high accuracy. For large distances and large masses, the gravitational force prevails in the Universe, which Newton's law "does not see". Obviously, a different law of gravity applies to the Universe, different from Newton's law of gravity.

2. Gravitational interaction of bodies with the mass of the Universe.

Newton's law, being the law of interaction of two bodies, "does not see" the gravitational interaction of bodies with the Universe. At the same time, in the real Universe, more than two bodies and even the entire Universe participate in gravitational interaction. This is not taken into account in Newton's formula. This cannot be taken into account by the formula $\mathbf{F_N=GmM/r^2}$, which does not include the parameters of the Universe. A complete and final mathematical model of gravity should contain not only the parameters of two bodies, but also the parameters of the Universe accessible for measurement. Newtonian dynamics does not provide a solution to this problem. A new law of gravitational interaction is needed. A new law of gravitational interaction should be sought beyond

Newtonian dynamics. A new law of gravitational interaction should be applicable to large distances. This should be a law of gravitation applicable to the entire Universe.

An attempt to extend Newton's law to the Universe encounters known difficulties. It is known that the gravitational problem of N bodies has no analytical solution. In the case of the Universe, there is no way to specify the value of N. It is completely impossible to determine. I propose replacing the differential N-body problem with its integral equivalent. The integral N-body problem has a solution. The integral N-body problem leads to the total gravitational force if a parameter that is an integral characteristic of the N-body system is used for the solution. In the case of the Universe, such an integral characteristic for the N-body system is the cosmological constant Λ.

3. Parameters of the Universe for the New Law of Gravity.

To obtain a new law of gravity, it is necessary to know the parameters of the Universe. The parameters of the Universe can be obtained from the following system of cosmological equations of the Universe [14] (Fig. 2).

$$\begin{cases} G\hbar / r_e^3 A_0 = c \\ 1/T_U^2 \Lambda = c^2 \\ M_U A_0 G = c^4 \\ M_U R_U A_0 G / T_U = c^5 \\ M_U R_U A_0^2 G = c^6 \end{cases}$$

Fig. 1. System of cosmological equations for calculating the parameters of the observed Universe. Where: $\mathbf{M_U}$ is the mass of the observable Universe, **ħ** is Planck's constant, **G** is the Newtonian gravitational constant, **Λ** is the cosmological constant, $\mathbf{R_U}$ is the radius of the observable Universe, $\mathbf{T_U}$ is the time of the Universe, $\mathbf{A_0}$ is the cosmological acceleration, $\mathbf{r_e}$ is the classical radius of the electron; **c** is the speed of light in vacuum.

The solution of the system of cosmological equations gives the following values of the parameters of the Universe:

$$M_U = 1.15348... \bullet 10^{53} kg$$
$$R_U = 0.856594... \bullet 10^{26} m$$
$$T_U = 2.85729 ... \bullet 10^{17} s$$
$$\Lambda = 1.36285 ... \bullet 10^{-52} m^{-2}$$
$$A_0 = 10.4922 ... \bullet 10^{-10} m / s^2$$

Fig. 2. Parameters of the Universe.

Among the parameters of the Universe, the cosmological constant Λ is a measurable parameter. We will use the cosmological constant Λ to obtain the formula for the law of gravitational interaction of bodies with the Universe.

4. Law of cosmological force

The formula for the law of cosmological force is [15]:

$$F_{Cos} = mc^2 \sqrt{\Lambda}$$

Fig. 3. The law of cosmological force. Where: F_{Cos} is the cosmological force, m is the mass of the body, c is the speed of light in vacuum, Λ is the cosmological constant.

Instead of the gravitational constant G, the law of cosmological force contains the cosmological constant Λ. The new law of gravitation shows that any body of mass m is acted upon by a cosmological force proportional to the mass of the body and the cosmological constant Λ. The law of cosmological force operates beyond the applicability of Newton's law of gravitation. It is applicable to the gravitational interaction of bodies with the mass of the Universe. The cosmological force has a linear dependence on the mass of the body and does not obey the inverse square law. This is an additional gravitational force to the Newtonian force. It acts on both small and large scales of the Universe. The law of cosmological force has no limitations associated with point masses. The law of cosmological force shows that the entire Universe participates in gravitational interaction. The additional cosmological force remained unnoticed for a long time. The reason is the small value of the cosmological constant Λ ($\Lambda = 1.36285...\text{x } 10^{-52}\ \text{m}^{-2}$). On small scales, the additional cosmological force is much smaller than the Newtonian force. On the scale of the Universe, the cosmological force is enormous. At large distances, it exceeds the Newtonian force. At large distances, the main part of the force of universal gravitation is the cosmological force F_{Cos}.

The study of the equation of the new law of the cosmological force shows that the value of the cosmological force in the limit is equal to the Planck force:

$$\lim_{m \to M_U} F_{Cos} = \lim_{m \to M_U} mc^2 \sqrt{\Lambda} = 1.21027 \bullet 10^{44}\, N = \frac{c^4}{G} \qquad (1)$$

The theoretical limit of the cosmological force at $m \to M_U$ reaches the enormous value c^4/G = 1.21027 x 10^{44} N.

The law of cosmological force is presented by a simple formula, not inferior in simplicity and perfection to the formula of Newton's law of gravitation. The law of cosmological force does not contain constant G. Instead of constant G, it includes the cosmological constant Λ. The law shows with what force the body interacts with the mass of the Universe distributed in space.

The main feature of the cosmological force is that it has a linear dependence on the mass of the body and does not obey the inverse square law. The inverse square law was formulated in 1645 by Ismail Bullialdus [13]. The inverse square law turned out to be very productive for solving the problem of gravitational interaction of two bodies. This same inverse square law became an insurmountable obstacle when trying to extend the action of Newton's law of gravitation to the Universe. The law of cosmological force shows that gravity weakens at large distances less than follows from Newton's law.

5. Newton's law together with the law of cosmological force explain the Galaxy rotation curve without involving the concept of dark matter.

The cosmological force has a linear dependence on the mass of the body (Fig. 4) and does not obey the law of inverse squares. Fig. 4 conventionally shows the contribution of the cosmological force to the Galaxy rotation curve.

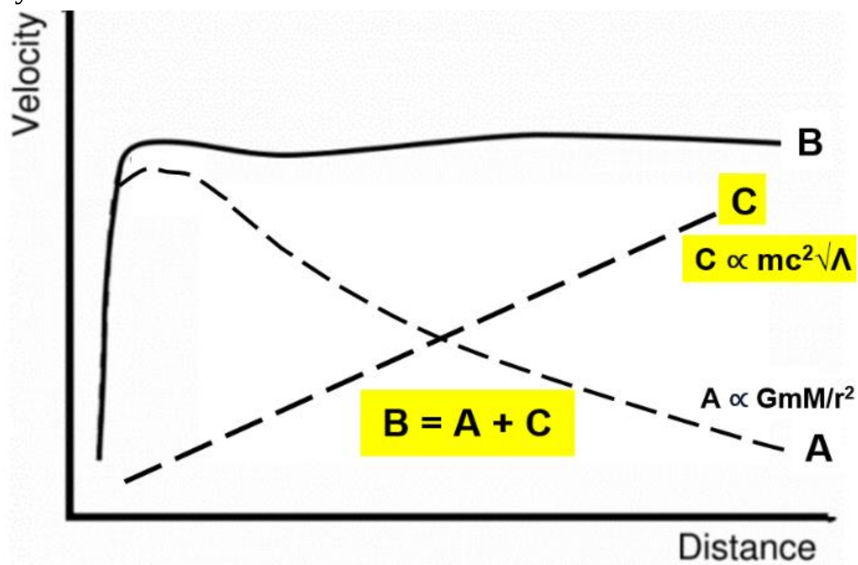

Fig. 4. Galaxy rotation curve (B) as a result of two forces: the contribution of the Newtonian force (A) and the contribution of the cosmological force (C).

On small scales, the additional cosmological force is much smaller than the Newtonian force. On the scale of the Universe, the cosmological force is enormous. For large distances it exceeds the Newtonian force and has a theoretical limit equal to the Planck force $F_P = c^4/G = 1.21027 \cdot 10^{44}$ N. The total force of universal gravitation, which acts on a body, consists of two forces. This is the Newtonian force of gravitational interaction of two bodies and the additional cosmological force of gravitational interaction of a body with the mass of the Universe. Accordingly, the Galaxy rotation curve (B) (Fig. 4) is represented by the sum B = A + C. Combining the two laws of forces F_N and F_{Cos} gives the law of universal gravitation. At small distances, the main share of the force of universal gravitation is represented by the Newtonian force F_N. At large distances, the cosmological force F_{Cos} represents the major fraction of the universal gravitational force. As a result of the two forces, the velocity in the graph (Fig. 4) is represented by curve (B). There is no need to invoke the dark matter hypothesis to explain the galaxy rotation curve. Instead of a dark matter halo, the gravitational force of the "light matter" of the visible Universe is at work.

The assumption about the influence of the rest of the visible Universe on the rotation curves of galaxies was first made by Philip D. Mannheim [16].

6. The share of the cosmological force FCos = (mc^2)√Λ in the gravitational interaction.

The share of the cosmological force F_{Cos} in the total gravitational force depends on the scale of the gravitational interaction. On small scales, the additional cosmological force F_{Cos} is much smaller than the Newtonian force. For example, on Earth, the Newtonian force is 9.346 x 10^9 times greater than the cosmological force F_{Cos}. As we can see, within the solar system, Newton's law of gravitation is highly accurate. At small distances, the Newtonian force makes up the main part of the universal gravitational force. With an increase in the mass of interacting bodies and the distance, the share of the Newtonian force in the law of universal gravitation decreases, and the share of the cosmological force increases.

On the scale of the Universe, the cosmological force is enormous. At large distances, it exceeds the Newtonian force. The cosmological force F_{Cos} has a theoretical limit equal to the Planck force F_P = c^4/G = 1.21027 • 10^44 N. At large distances, the main part of the universal gravitational force is the cosmological force F_{Cos}.

At the scale of galaxies, the cosmological force Fcos becomes comparable with the Newtonian force. This occurs at the corresponding value of the ratio M/r^2

$$\mathbf{M/r^2 = (c^2\sqrt{\Lambda})/G = 15.720...\ kg/m^2} \qquad (2)$$

At the value M/r^2 = 15.720... kg/m^2, the cosmological force (F_{Cos} = (mc^2)√Λ) is equal to the Newtonian force (F_N = GMm/r^2). It is noteworthy that the same constant (15.7202... kg/m^2), but with much greater accuracy, is given by the following combination of fundamental physical constants of the electron:

$$\mathbf{m_e/\alpha r_e^2 = 15{,}7202729...\ kg/m^2} \qquad (3)$$

Where: α is the fine structure constant, r_e is the classical radius of the electron; m_e is the electron mass.

The constant 15.7202729... kg/m^2 corresponds to a mass density of $18.3520 \cdot 10^{-26}$ kg/m^3.

7. Tully–Fisher relation for the Universe.

The empirical baryonic Tully–Fisher relation [17] is known, which gives the relationship between the baryon mass and the asymptotic rotation velocity in the form: M ∝ V4. Here we will show that the Tully–Fisher relation is satisfied for the Universe. In the MOND theory [1, 2], the equation is obtained:

$$\mathbf{V = (GMa_0)^{\wedge}1/4} \qquad (4)$$

Where: M_U is the mass of the observable Universe, G is the Newtonian gravitational constant, V is the speed, a_0 is the acceleration.

The Tully–Fisher relation is universal. It is valid not only for galaxies, but also for the Universe as a whole. For $\mathbf{M \rightarrow M_U}$ the velocity $\mathbf{V \rightarrow c}$:

$$\mathbf{V^4 = GM_UA_0 = c^4} \qquad (5)$$

Where: V is the speed, M_U is the mass of the observable Universe, G is the Newtonian gravitational constant, A_0 is the cosmological acceleration, c is the speed of light in vacuum.

Equation (5) is one of the cosmological equations in the system of equations of the Universe in Fig. 1. The Tully–Fisher baryon relation for the Universe has a power law index of exactly 4..

From this cosmological equation (5) follows the well-known formula of the Planck force:

$$\mathbf{M_U A_0 = c^4/G = 1.21027 \bullet 10^{44}\ N} \qquad (6)$$

From this cosmological equation (5) follows the well-known formula of A. E. H. Bleksley [18]:

$$\mathbf{G = c^2 R_U / M_U} \qquad (7)$$

From this cosmological equation (5) follows the Kepler constant for the Universe:

$$\mathbf{GM_U = R_U^3/T_U^2 = c^2 R_U = c^4/A_0 = 7.69868 \bullet 10^{42}\ m^3/s^2} \qquad (8)$$

Where: M_U is the mass of the observable Universe, G is the Newtonian gravitational constant, R_U is the radius of the observable Universe, T_U is the time of the Universe, A_0 is the cosmological acceleration, c is the speed of light in vacuum.

The Baryonic Tully–Fisher relation for a Universe with power-law index exactly 4 is a direct consequence of the law of cosmological force.

8. Cosmological acceleration and the Pioneer anomaly.

The effect of the cosmological force is manifested both at small distances and on the scale of the Universe. An unknown cosmological force was experimentally discovered in the Pioneer effect [19, 20, 21]. The Pioneer effect still has no convincing explanation. The new force, which follows from the law of cosmological force, surprisingly turned out to be close to the Pioneer anomaly.

The value of cosmological acceleration, which follows from the law of cosmological force:

$$c^2\sqrt{\Lambda} = 10.4922... \bullet 10^{-10}\, m/s^2 \qquad (9)$$

Cosmological force value:

$$F_{Cos} = m \bullet (10.4922 \bullet 10^{-10})N \qquad (10)$$

Unknown force value detected in the pioneer effect:

$$F_{Pioneer} = m \bullet (8.74 \pm 1.33) \bullet 10^{-10}\, N \qquad (11)$$

In addition to the Pioneer-10 and Pioneer-11 experiment, there is anomalous acceleration data from Galileo and Ulysses [22 - 25].

Unknown force value for Galileo:

$$F_{Galileo} = m \bullet (8 \pm 3) \bullet 10^{-10}\, N \qquad (12)$$

Unknown force value for Ulysses:

$$F_{Ulysses} = m \bullet (12 \pm 3) \bullet 10^{-10}\, N \qquad (13)$$

The law of cosmological force gives a value of force very close to both the Pioneer anomaly and the values for Galileo and Ulysses. The value of force F = m(10.4922... × 10^{-10}) N is very close to the experimental values F=m((8.74 ± 1.33) x 10^{-10}) N, F=m((8±3) x 10^{-10}) N, F=m((12±3)x10^{-10})N.

9. New law of universal gravitation.

The reason for the discrepancies with observations is that Newton's law of gravitation gives only a part of the total force of universal gravitation. Therefore, calling the law of gravitational interaction of two bodies the law of universal gravitation is an exaggeration.

Newton's law alone is not enough to fully describe gravity. The law of cosmological force alone is not enough to fully describe gravity. The real law of gravity includes two laws: Newton's law and the law of cosmological force. Newton's law together with the new law of gravity provide a complete and consistent description of gravitational interaction both on the scale of the solar system and on the scale of Galaxies and the Universe. Newton's law of gravity is included as a component in the new law of universal gravitation.

A consistent description of gravitational interaction is given by two laws of gravity: Newton's law and the law of cosmological force:

$$\begin{cases} F_N = G\dfrac{Mm}{r^2} \\ F_{Cos} = mc^2\sqrt{\Lambda} \end{cases}$$

Fig. 5. Law of universal gravitation.

10. Newton's law of gravitation + law of cosmological force = law of universal gravitation

The law of cosmological force ($F_{Cos}=(mc^2)\sqrt{\Lambda}$) leads to a new mathematical model of gravitation. This allows us to present the law of universal gravitation in a new form. The law of universal gravitation includes two laws: Newton's law of gravitation and the law of cosmological force (Fig. 6).

LAW OF UNIVERSAL GRAVITATION $= \boxed{F_N = G\dfrac{Mm}{r^2}} + \boxed{F_{Cos} = mc^2\sqrt{\Lambda}}$

Fig. 6. Law of universal gravitation = Newton's law of gravity + law of cosmological force.

The force of universal gravitation is represented by the vector sum of two forces: the Newtonian gravitational force F_N and the cosmological gravitational force F_{Cos}:

$$\vec{F}_U = \vec{F}_N + \vec{F}_{Cos} \qquad (14)$$

The value of the resulting force of universal gravitation is in the range of values from **$F_U = GmM/r^2 - (mc^2)\sqrt{\Lambda}$ to $F_U = GmM/r^2 + (mc^2)\sqrt{\Lambda}$** (Fig. 7).

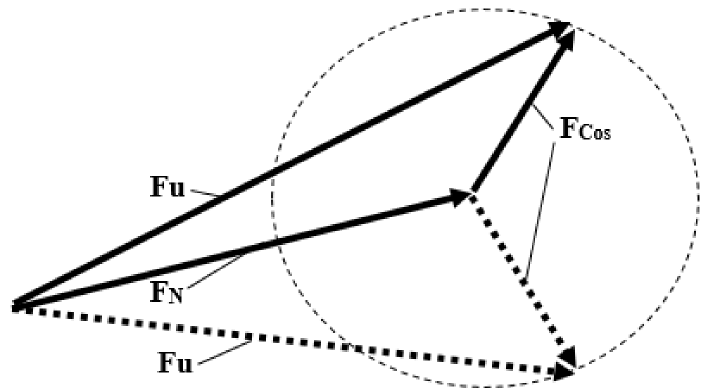

Fig. 7. The universal gravitational force F_U as a vector sum of two forces: F_N and F_{Cos}.

Newton's gravity model does not reflect the real picture of gravitational interaction. Newton's law alone is not enough to fully describe gravity. Newton's law together with the new law of gravity provide a complete and consistent description of gravitational interaction both on the scale of the solar system and on the scale of galaxies and the Universe. Accordingly, the law of universal gravitation is represented by two laws of gravity Fig. 8.

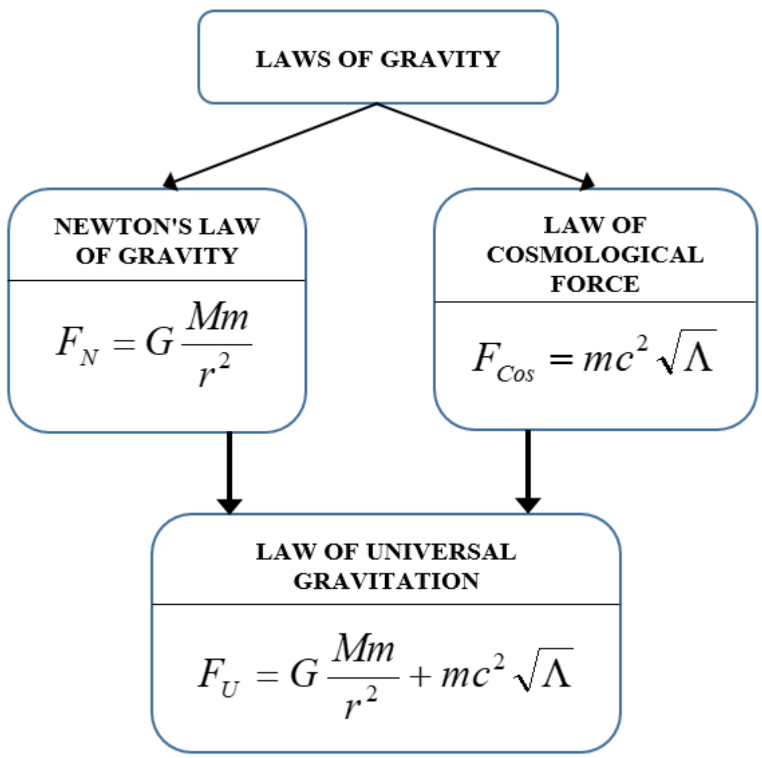

Fig. 8. Laws of Gravity.

Newton's law of gravitation is a component of the new law of universal gravitation. For this reason, it does not lose its fundamental status in gravitation.

11. Conclusion

Observations show that gravity weakens at large distances less than Newton claims. In the real Universe, gravity is stronger than Newton's law of gravity suggests. To calculate the force of gravitational interaction, Newton's law of gravity F_N=GmM/r^2 is not enough. Newton's law of gravity is only part of the law of universal gravitation. The law of universal gravitation turned out to be much more complex than Newton claimed. Newton's law of gravity "sees" the force of gravitational interaction between two bodies, but "does not see" the force of gravitational interaction between bodies and the mass of the Universe. The force not taken into account by Newton's law tends to increase linearly with increasing distance. At large distances, it exceeds the Newtonian force.

The new law of gravity F_{Cos}=√Λ(mc^2) gives the value of the force not taken into account by Newton's law. The description of gravity without the law of gravitational interaction between bodies and the mass of the entire Universe is incomplete. The cosmological force acts on both small and large scales of the Universe. It does not obey the inverse square law. The constant in the new law of gravitation is the cosmological constant Λ. The law of cosmological force F_{Cos}=√Λ(mc^2) complements Newton's law of gravitation F_N=GmM/r^2. Two laws of gravitation (Newton's law + the law of cosmological force) provide a complete description of gravitational interaction in the Universe. For small distances and small masses, the cosmological force is much smaller than the Newtonian force. At small distances, the main part of the force of universal gravitation is the Newtonian force. At large distances, the main part of the force of universal gravitation is the cosmological force F_{Cos}. The law of universal gravitation is represented by two laws: Newton's law and the law of cosmological force.

12. Conclusions

1. In the real Universe, gravity is stronger than Newton's law of gravity suggests. Gravity weakens at large distances less than Newton's law of gravity suggests.

2. Newton's law gives the value of the force of gravitational interaction between two bodies, but it does not take into account the gravitational interaction of bodies with the Universe.

3. A new law of gravity is proposed: F_{Cos}=√Λ(mc^2), which shows the cosmological force not taken into account by Newton's law.

4. The cosmological force acts on both small and large scales of the Universe. It does not obey the inverse square law. The constant in the new law of gravity is the cosmological constant Λ. The new law of gravity shows that the cosmological force increases linearly with distance.

5. The complete law of gravity includes two laws: Newton's law and the law of cosmological force. Newton's law together with the new law of gravity provide a complete and consistent description of gravitational interaction both on the scale of the solar system and on the scale of galaxies and the Universe.

6. An attempt to represent all gravity by Newtonian gravity alone led to the false concept of dark matter.

7. Two laws of gravity (Newton's law and the law of cosmological force) explain the Galaxy rotation curve without involving the concept of dark matter.

8. A new parameter of the Universe was introduced - the cosmological acceleration A_0. The value Ao = √Λc^2 = 10.4922•10^-10 m/s^2 was obtained as a solution to a system of cosmological equations.

9. The value Ao = √Λc^2 = 10.4922•10^-10 m/s^2 is very close to the pioneer anomaly ((8.74 ± 1.33)•10^-10 m/s^2) and to the acceleration (1.2•10^-10 m/s^2) obtained in the MOND theory.

10. The Tully–Fisher relation is universal. This relation is valid not only for galaxies, but for the Universe as a whole ($V^4 = GM_U A_0 = c^4$).

11. From the Tully–Fisher relation for the Universe ($V^4 = GM_U A_0 = c^4$) follows the value of cosmological acceleration Ao = 10.4922•10^-10 m/s^2.

12. The law of universal gravitation is presented in a new form. The law of universal gravitation takes into account both the gravitational interaction of two bodies and the gravitational interaction of bodies with the mass of the Universe. Newton's law of gravitation is an integral part of the new law of universal gravitation.

13. Newton's law of gravitation does not provide a complete description of gravitational interaction. The law of universal gravitation turned out to be much more complex than Newton claimed. At the same time, Newton's law does not lose its fundamental status in gravity, since it is an integral part of the new law of universal gravitation.

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