The Neutron Coincidence

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Abstract

A previously unknown relationship involving the masses of the neutron, proton, and electron is reported. Expressed simply, it states $\frac{m_n}{m_e} \approx \frac{2\pi}{\alpha}(\gamma - \frac{1}{\gamma})$, where γ is defined as $(m_n - m_p)/m_e$, and $\alpha = e^2/(2hc\epsilon_0)$ represents the fine structure constant. The accuracy of this approximation is $2.0 \cdot 10^{-4}$, which is outside the experimental error of $4.0 \cdot 10^{-6}$. Additionally, the coincidence $\gamma \approx \log 4\pi$ holds, with an even closer match of $2.0 \cdot 10^{-5}$, yet still outside the experimental precision of $1.0 \cdot 10^{-6}$. It is not claimed that these coincidences have a physical meaning.

Motivation

Inspired by the nuclear electron hypothesis Stuever (1983), one may consider the neutron as a compound state of a proton and an electron. It is well established that the mass difference between the neutron and the proton corresponds to approximately 2.53 electron masses. Thus it is a natural idea to interprete this number as a relativistic factor $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, which implies an approximate velocity of $v \approx 0.918c$ for the electron. Assuming a hypothetical scenario where this relativistic electron orbits the proton (contrary to Bohr's quantization rule of angular momentum), equating the centripetal force to Coulomb's force results in:

$$\frac{e^2}{4\pi\epsilon_0 r^2} = \frac{\gamma m_e v^2}{r}. \tag{1}$$

Solving for r yields

$$r = \frac{e^2}{4\pi\epsilon_0 \gamma m_e v^2}, \tag{2}$$

which, after inserting the latest CODATA values, astonishingly closely matches $(1.31933 \cdot 10^{-15} m)$ the Compton wavelength of the neutron:

$$\lambda_C = \frac{h}{m_n c} = 1.31959 \cdot 10^{-15} m. \tag{3}$$

However, without invoking the Compton wavelength, the coincidence may be formulated in an even more simple way. If we again interprete $\gamma := (m_n - m_p)/m_e$ as relativistic factor, then we can rewrite (1) with the substitution

$$v^2 = c^2(1 - \frac{1}{\gamma^2}), \tag{4}$$

which after canceling r and γ leads to

$$c^2 \frac{\gamma^2 - 1}{\gamma} = \frac{e^2}{4\pi m_e \epsilon_0 r}. \tag{5}$$

With the additional substitution $r = \lambda_C = h/(m_n c)$ we derive

$$\frac{1}{r} = \frac{4\pi m_e \epsilon_0 c^2}{e^2} \frac{\gamma^2 - 1}{\gamma} = \frac{1}{\lambda_C} = \frac{m_n c}{h} \tag{6}$$

Using the definition of the fine structure constant $\alpha = \frac{e^2}{2\epsilon_0 hc}$ and simplifying, this can be transformed to

$$\frac{m_n}{m_e} \approx \frac{2\pi}{\alpha}(\gamma - \frac{1}{\gamma}). \tag{7}$$

Despite these observations, the precision of current CODATA values rules out this approximation. Correction for the proton's motion in the hypothesized Kepler problem further diminishes the match (the calculated electron-proton distance would shrink to $1.31752 \cdot 10^{-15} m$), contradicting this simplistic approach. Moreover, the model of an electron orbiting the proton at relativistic speeds does not account for potential energy, which in the hydrogen atom is double the kinetic energy with an opposite sign (due to the virial theorem). Thus, the original motivating hypothesis cannot be physically substantiated, though it presents a numerically intriguing result worth discussing for potential new insights into the masses of the neutron, proton, and electron.

The second conicidence to report is very simple:

$$\gamma \approx \log 4\pi \tag{8}$$

There seems to be no physical model whatsoever, similar to concidences such as $\frac{m_p}{m_e} \approx 6\pi^6$. Though even more accurate, it will presumably remain without explanation.

Outlook

Coincidences such as the ones mentioned are often dismissed as 'numerology'. However, history reminds us that the search for such numerical relationships led Johann Jakob Balmer in 1885 to discover the Rydberg constant, a pivotal moment in atomic physics. Similarly, the discovery of the relationship $\epsilon_0 \mu_0 = 1/c^2$ by Kirchhoff and Weber revolutionized our understanding and application of electromagnetic theory, cfr. see Unzicker (2020, 2021). Therefore, these coincidences should be documented. However, establishing credibility in their physical meaning requires a corresponding theory that quantitatively justifies these observations.

References

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