Research on the control of time-varying systems using lattice matrix operators and integral equations

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ABSTRACT

In this paper, we propose a response optimization method for time-varying or non linear systems using lattice matrix operators.

In order to compare the effectiveness and accuracy of this method against previou s nonlinear optimal control methods, simulation results for nonlinear plants with ti me-varying and gap nonlinearity are presented in this paper.

This method overcomes the drawbacks of previous controller design methods that have been complicated by the characteristics of the plant and allows easy and gen eral development of controllers by matrix algebraic equations for objects with tim e-varying or nonlinear or uncertain parameters, which have strong robustness to v ariations in disturbances, environmental noise and parameters.

keywords: integral equation, uncertain systems, time-dependent system.

Declarations of interest: A new proposal for easy solving controller design for uncertain and nonstationary plants in algebraic methods.

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1. Introduction

So far, many methods have been proposed to solve the optimization problems of time-varying and nonlinear systems, such as approximation methods, analytical m ethods and parametric transfer function methods.[1~3]

A parametric transfer function approach limited to the study of individual compo nents of the system could not be used for system synthesis.

In addition to these methods, there are equilibrium Operator methods, generative function methods, delta Operator methods, adaptive control, and robust control.

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However, those methods have problems in practices or have many search algorithms.[4~6]

For example, adaptive control is discussed only for linear objects.

For systems with simultaneous time-varying, nonlinear, and gray-level properties (TVNLG), only stability analysis methods are considered in the time domain, and the optimal controller design method is not fully established.[7,8]

In this paper, we will establish a general optimization scheme that meets the control requirements of TVNLG system and overcomes the effects of noise and disturbance.

In general, among the advantages of various advanced control schemes, feature points are real-time control methods such as system configuration and fuzzy control of feedback mode.

In other words, when the controller is updated to reduce the error in real time, it can fully achieve its own performance.

However, so far, in most system designs, only a fixed controller according to the specific characteristics of the plant has been used.

Therefore, the controller can be complex or difficult to implement depending on the characteristics of the object.

On the other hand, all control laws must be robust according to the changes of the object and environment.

So the current trend of system design is numerical methods [9, 10].

Some papers have proposed a system synthesis method by pseudo-spectral method to solve the optimal control problem of time-varying systems [11-17], but these methods require many solutions to boundary value problems.

To avoid this, some papers have presented a new method for harmonic observation of hysteresis systems by nonlinear smoothing operators [18,19].

In this paper, we propose an algebraic optimization method for time-varying nonlinear systems by lattice matrix operators.

In the course of the integral calculation, the quadrature method is used to transform the integral equation into matrix form and consider the total motion of the system by the lattice matrix operator.

In this paper, as an application example, the response optimization of nonlinear objects with gap nonlinearity and objects with time-varying characteristics is prese nted.

We have demonstrated that this method is effective in overcoming noise and noi se, especially.

We believe that the proposed method has the advantage of being simpler, general, and optimal than previous control methods.

It is also believed that this method will be an obvious advance in optimal contr ol methods for non-linear objects as well as time-varying objects.

Using this method, the controller can be easily and generally developed with ma trix algebraic equations.

The paper is organized as follows.

In Section 2, we introduce lattice matrix operators.

Representing the lattice matrix operator of the basic elements by the quadrature formula is discussed in Section 3.

The controller synthesis method of TVNLG system with lattice matrix Operator i s presented in Section 4.

Section 5 presents the equivalent lattice matrix operator of time-varying nonlinear elements.

Section 6 shows the simulation results applied to the control of nonlinear elemen t with backlash characteristics.

Conclusions are given in Section 7.

2. lattice matrix operator

The lattice matrix operator means mapping the product of a linear space directly to another linear space.

It has the form of a matrix and is called the lattice matrix operator.

That is, the lattice matrix operator is to map a linear space to another linear space.

The matrix operator is classified as lattice matrix Operator and projection matrix Operator.

The quadrature formulas used to study lattice matrix operators include rectangular, trapezoidal, and Simpson's rules.

The integral equation has an operational kernel and the lattice matrix operator is used in the process of solving the integral equation.

Consider the differential equation of the system as follows:

$$\sum_{k=0}^{n} a_k(t) y^{(k)}(t) = \sum_{k=0}^{m} b_k(t) u^{(k)}(t), n > m \qquad (1)$$

Where $u(t)$ - input , $y(t)$ - output

This differential equation is represented by the Volterra integral equation of the second kind.

Using the quadrature method, the integral equation with the operational kernel is approximated as follows:

$$y(t_i) - \int_0^{t_i} k_y(t_i,\tau) y(\tau) d\tau = \int_0^{t_i} k_u(t_i,\tau) u(\tau) d\tau \qquad (2)$$

where

$$k_y(t,\tau) = -\sum_{k=0}^{n-1} \frac{(-1)^k}{(n-1)!} \frac{d^k}{d\tau^k} \left[a_k(\tau)(t-\tau)^{n-1} \right] \qquad (3)$$

$$k_u(t,\tau) = \sum_{k=0}^{m} \frac{(-1)^k}{(n-1)!} \frac{d^k}{d\tau^k} \left[b_k(\tau)(t-\tau)^{n-1} \right] \qquad (4)$$

$$y(t_i) - \sum_{j=1}^{i} H_j k_y(t_i,\tau_j) y(\tau_j) = \sum_{j=1}^{i} H_j k_u(t_i,\tau_j) u(\tau_j) + R_i(y) \qquad (5)$$

where H is the weight of quadrature.

$R_i(y)$ is the error, which has been pointed out in [2].

Now let us introduce the following notation

$$u(t_i) = u_i \,,\, y(t_i) = y_i \,,\, k_y(t_i,\tau_j) = \tilde{k}_{ij} \,,\, k_u(t_i,\tau_j) = k_{ij}^u$$

Disregarding $R_i(y)$, formula (5) is as follows.

$$y_i - \sum_{j=1}^{i} H_j \tilde{k}_{ij} y_j = \sum_{j=1}^{i} H_j k_{ij}^u u_j \qquad (i = \overline{1,N}) \qquad (6)$$

Equation (6) is rewritten in matrix form

$$\begin{bmatrix} 1-H_1\tilde{k}_{11} & 0 & \cdots & 0 \\ -H_1\tilde{k}_{21} & 1-H_2\tilde{k}_{22} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ -H_1\tilde{k}_{N1} & -H_2\tilde{k}_{N2} & \cdots & 1-H_N\tilde{k}_{NN} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} =$$

$$= \begin{bmatrix} H_1 k_{11}^u & 0 & \cdots & 0 \\ H_1 k_{21}^u & H_2 k_{22}^u & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ H_1 k_{N1}^u & H_2 k_{N2}^u & \cdots & H_N k_{NN}^u \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix} \tag{7}$$

Representing the formula (7) to form of vector

$$Y_N - A_N^Y Y_N = A_N^u U_N$$

Therefore

$$Y_N = \left[I - A_N^y\right]^{-1} A_N^u U_N \tag{8}$$

From formula (8), we can write as follow.

$$A_N Y_N = A_N^u U_N \tag{9}$$

Therefore

$$Y_N = A_N^{-1} A_N^u U_N \tag{10}$$

where

$$A_N = \begin{bmatrix} 1 - H_1 \tilde{k}_{11} & 0 & \cdots & 0 \\ -H_1 \tilde{k}_{21} & 1 - H_2 \tilde{k}_{22} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ -H_1 \tilde{k}_{N1} & -H_2 \tilde{k}_{N2} & \cdots & 1 - H_N \tilde{k}_{NN} \end{bmatrix}$$

$$A_N^u = \begin{bmatrix} H_1 k_{11}^u & 0 & \cdots & 0 \\ H_1 k_{21}^u & H_2 k_{22}^u & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ H_1 k_{N1}^u & H_2 k_{N2}^u & \cdots & H_N k_{NN}^u \end{bmatrix}$$

From formula (8) and (10),

$$\left[I - A_N^y\right]^{-1} A_N^u = A_N^{-1} A_N^u = A_c \tag{11}$$

[Definition] in (11) , A_c is defined as lattice matrix operator of the system.

From (10)

$$A_N = I - A_N^y$$

As above (7), Matrix A_N^y, A_N^u are conform *to* the shape of triangular matrix.

On the function $y(t)$, it's time interval divide into equal parts, and representing ea ch part as h.

Using trapezoid formula, I take as follows.

$$H_1 = H_N = \frac{h}{2}$$

$$H_2 = H_3 = \cdots = H_{N-1} = h$$

usually, $$h = \frac{T}{N-1}$$

3. Representation of basic elements of control system by lattice matrix oper ator

3.1 Lattice Matrix Operator of Integral Elements

On the integral element

$$y(t) = \int_0^t u(\tau)d\tau$$

where $y(t)$ is output, $u(t)$ is input.

On the each discrete interval t_i,

$$y(t_i) = \int_0^{t_i} u(\tau)d\tau = \sum_{j=1}^{i} H_j u(t_j), i = \overline{1, N} \tag{12}$$

Where coefficients H_j is selected from quadrature formula

Representing $y(t_1), y(t_2), \cdots y(t_N)$ to $y_1, y_2, \cdots y_n$, and $u(t_1), u(t_2), \cdots u(t_N)$ to $u_1, u_2, \cdots u_n$, above formula written as following form of matrix.

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} H_1 & 0 & 0 & \cdots & 0 \\ H_1 & H_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ H_1 & H_2 & H_3 & \cdots & H_N \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}$$

Representing above formula to the form of vector,

$$Y_N = A_i U_N \tag{13}$$

Where A_i is integral lattice matrix operator.

Example of A_i

$$A_i = \begin{bmatrix} 0.05 & 0 & 0 & \cdots & 0 \\ 0.05 & 0.1 & 0 & \cdots & 0 \\ 0.05 & 0.1 & 0.1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0.05 & 0.1 & 0.1 & \cdots & 0.05 \end{bmatrix}$$

3.2 Differential lattice matrix operator

Differential lattice matrix operator A_d is defined as follows.

$$A_d = A_i^{-1}$$

3.3 lattice matrix operator multiplying on the function

a(t) is a given function, and let us y(t)=a(t)u(t).

Representing $y(t_1), y(t_2), \cdots y(t_N)$ to $y_1, y_2, \cdots y_n$, and $u(t_1), u(t_2), \cdots u(t_N)$ to $u_1, u_2, \cdots u_n$

Follow formula is concluded.

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} a(t_1) & 0 & 0 & \cdots & 0 \\ 0 & a(t_2) & 0 & \cdots & 0 \\ 0 & 0 & a(t_3) & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & a(t_N) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}$$

Consequently,

$$Y_N = A_u(a) U_N \tag{14}$$

where $A_u(a)$ is Product lattice matrix operator on the function $a(t)$.

4. The controller Synthesis method of the time-dependent system by lattice matrix operator

Let us consider following time-dependent system.

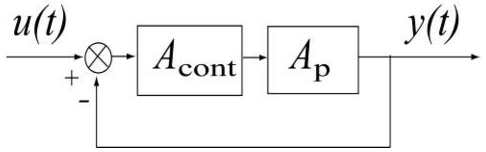


Fig 1. Block diagram of time-dependent system

The operator of closed system is as follows.

$$A_{\mathrm{C}} = A_p A_{\mathrm{cont}} (I + A_p A_{\mathrm{cont}})^{-1} \quad (15)$$

where A_p is lattice matrix operator of the plant and A_{cont} is as follows.

$$A_{\mathrm{cont}} = \left[(I - A_{\mathrm{C}})A_p\right]^{-1} A_{\mathrm{C}} = A_p^{-1}(I - A_{\mathrm{C}})^{-1} A_{\mathrm{C}}$$

Representing the operator of the standard system (required system) as A_{R}, our demand is realization of the formula $A_{\mathrm{C}} = A_{\mathrm{R}}$.

Therefore

$$A_{\mathrm{cont}} = A_p^{-1}(I - A_{\mathrm{R}})^{-1} A_{\mathrm{R}} \quad (16)$$

Consequently, the block diagram of synthesized system is as follows.

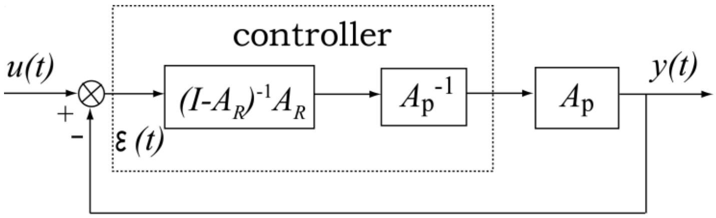


Fig 2. Block diagram of synthesized system

If the servo system is composed with controller and plant is worked perfectly, the output signal $y_{\mathrm{R}}(t)$ is

$$y(t) = A_p A_{\mathrm{cont}}(P_1, P_2, \cdots, P_r)\left[I + A_p A_{\mathrm{cont}}(P_1, P_2, \cdots P_r)\right]^{-1} u(t) \quad (17)$$

where $P_1, P_2, \cdots, P_r$ - parameter of controller

If the actual output signal of the system is most nearly allied to typical output signal $y_{\mathrm{R}}(t)$, formula (17) is

$$y(t) \cong y_{\mathrm{R}}(t) \quad (18)$$

Where, we consider

$$E(t, P_1, P_2, \cdots P_r) = u(t) - y(t) \quad (19)$$

Then, controller synthesis problem is defined.

$$J(P_1, P_2, \cdots P_r) = \max_{0<t<T} |E(t, P_1, P_2, \cdots P_r)| \quad : \quad \min_{P_i} (i = 1, \cdots, r)$$

or

$$J(P_1, P_2, \cdots P_r) = \int_0^T E^2(t, P_1, P_2, \cdots, P_r) dt \quad : \quad \min_{P_i} (i = 1, \cdots, r) \quad : \qquad (20)$$

5. Equivalent lattice matrix operator of time-dependent nonlinear element

Equivalent lattice matrix operator of time-dependent nonlinear element is determined through iterative computation and simulated test.

Input-output relation of nonlinear element is denoted as following nonlinear function. $y_{nl}(t) = F[u_{nl}(t)]$.

[Theorem] Computational algorithm of equal lattice matrix operator A_{nl} of nonlinear element is as follows.

Step 1: Computation of zero approximation $A_{nl}(0)$ of lattice matrix operator and

$u_{nl}(t_0)$, $y_{nl}(t_0)$

$u_{nl}(t_0), y_{nl}(t_0)$ are obtained using input signal $u_{nl}(t)$ and static characteristic of nonlinear element, and then $A_{nl}(0)$ is determined.

Step 2: Computation of first approximation $A_{nl}(1)$ of lattice matrix operator A_{nl}

$$A_{nl}(1) = A_{amp}(t_1) = y_{nl}(t_1) / u_{nl}(t_1)$$

where, $A_{nl}(1)$ is equivalent lattice matrix operator of nonlinear element.

$y_{nl}(t_1) / u_{nl}(t_1)$ is obtained from static characteristic of nonlinear element and denotes the gain coefficient $A_{amp}(t_1)$ of nonlinear element when $t = t_1$.

Step 3: Computation of second approximation $A_{nl}(2)$ of nonlinear element

Input signal $u_{nl}(t_2)$ of nonlinear element is determined using curve of $u_{nl}(t)$ (This curve is determined through measurement or multiple simulated test) in instant of $t = t_2$.

Then, $y_{nl}(t_2) / u_{nl}(t_2)$ is determined using static characteristic of nonlinear element.

As a result, $A_{nl}(2) = A_{amp}(t_2) = y_{nl}(t_2)/u_{nl}(t_2)$ is determined.

Step 4: If not $\|y_{nl}(t_2) - y_{nl}(t_1)\| < \varepsilon$, $y_{nl}(t_3)/u_{nl}(t_3)$ is computed according upper step.

Computation is performed until satisfying follow condition.

$$\|y_{nl}(t_n) - y_{nl}(t_{n-1})\| < \varepsilon,$$

Then, $A_{amp}(t)$ is function of time so that is denoted as function related to time and calculated with multiplication lattice matrix operator related to time.

$$A_{nl}(n) = A_{amp}(t_n) = y_{nl}(t_n)/u_{nl}(t_n)$$

Hence, equivalent lattice matrix operator of time-dependent or nonlinear element is defined as transfer coefficient on each time interval.

Equivalent lattice matrix operators of nonlinear elements such as backlash, saturation, non-sensitivity, magnetic hysteresis and delay are defined as above.

6. Simulation results

Example 1: Let's consider rolling control plant of the missile (or DC motor velocity servo system) as time-dependent plant.

$$W(s) = \frac{K}{Ts+1}$$

Then, let's take following mark.

$$\begin{cases} C_{11} = -\dfrac{M_x^{\omega_x}}{J_x}(1/s) \\ C_{13} = -\dfrac{M_x^{\delta_x}}{J_x}(1/s^2) \end{cases} \tag{21}$$

Now, we can rewrite K,T as follows by C_{11} and C_{13}

$$\begin{aligned} K &= -\frac{C_{13}}{C_{11}}(1/s) \\ T &= \frac{1}{C_{11}}(s) \end{aligned} \tag{22}$$

Block diagram of this element is as follows.

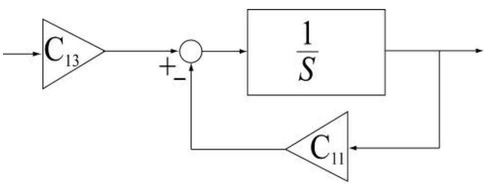


Fig 3. Block diagram of inertial element.

It is assumed that C_{11}, C_{13} are being changed in 45 seconds, and measurement value

is being changing, too.

Namely, this plant is the time-dependent plant.

C_{11}, C_{13} are configured by Look-up table.

The lattice matrix operator of integral element is as follows by Equation (13) when sampling time is 0.2ms

$$A_1 = 10^{-3} * \begin{bmatrix} 0.1 & 0 & 0 & \cdots & 0 \\ 0.1 & 0.2 & 0 & \cdots & 0 \\ 0.1 & 0.2 & 0.2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0.1 & 0.2 & 0.2 & \cdots & 0.1 \end{bmatrix}$$

The controller can be configured from Equation (16). Ideal output signal is obtained from design requirement. For an example, if transient time must be 3ms, sampling with 0.2ms, the following column vector for equation (17) is obtained.

$$y_R(t) = [0.06 \quad 0.12 \quad 0.18 \quad 0.2 \quad 0.26 \quad 0.33 \quad 0.4 \quad 0.46 \quad 0.53 \quad 0.6 \quad 0.66 \quad 0.73 \quad 0.8 \quad 0.86 \quad 1]$$

The simulation block diagram is as follows.

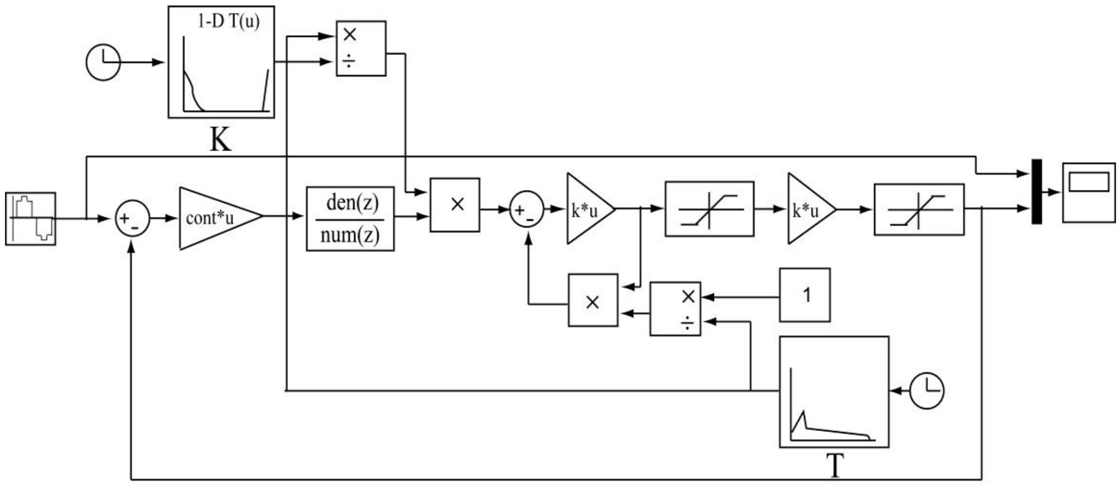


Fig 4. Block diagram for simulation.

Simulation result is as follows.

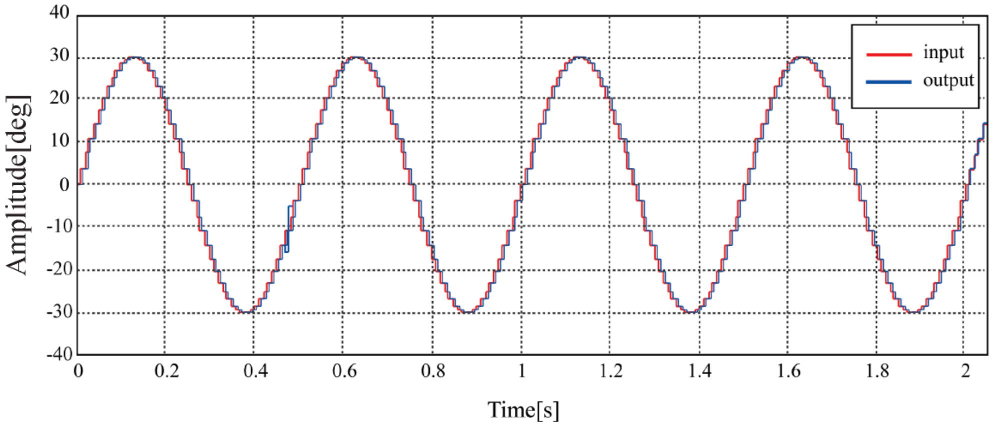
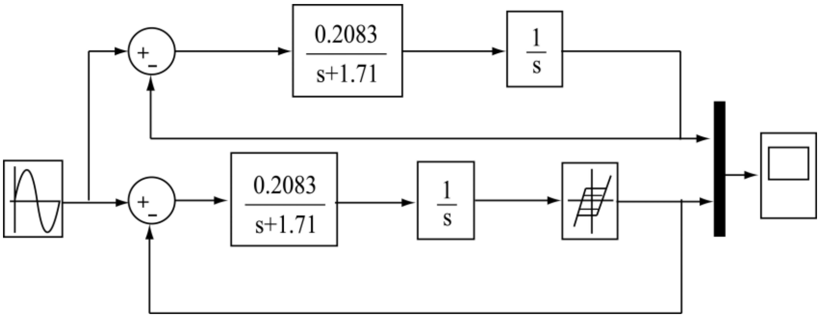


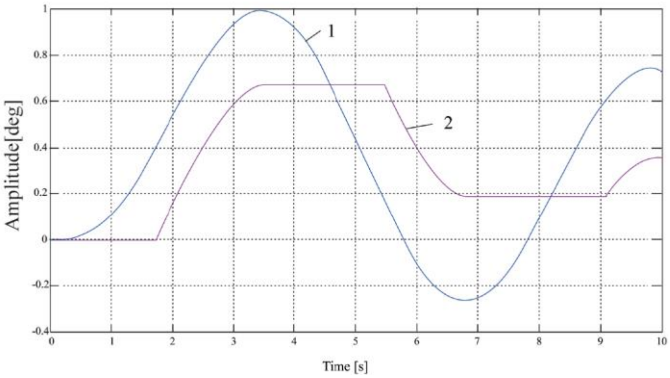
Fig 5. Simulation result

Here, when noise or dynamical disturbance be acted to the system and para meters of the system are changed in the range of 20% , there is no difference with above result.

Example 2: Let's consider the following non-linear control system with backlash p roperty.



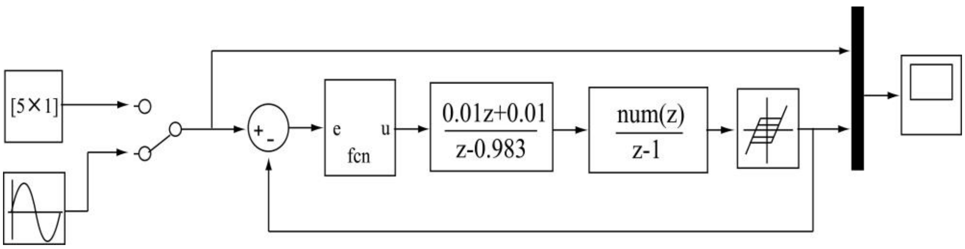
(a)



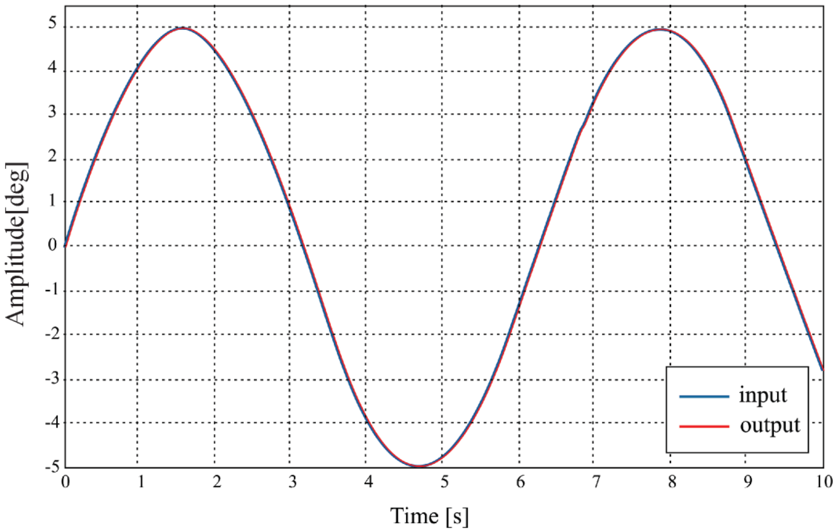
(b)

1) output without backlash property, 2) output with backlash property

Fig 6.Block diagram of the non-linear control system with backlash prope rty (a- block diagram , b-simulation result)



(a)



(b)

Fig 7. Advanced block diagram of the non-linear control system with bac klash property and its simulation result (a- block diagram, b-simu lation result)

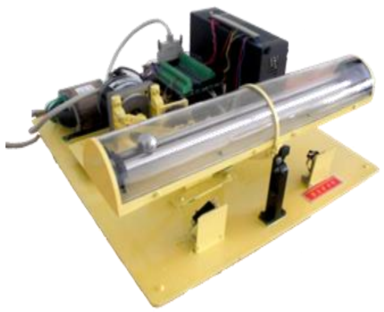
In the simulation results, we can see that the performance of the system with th e introduced controller is improved and the error between input and output is abo ut 0.0001.

It is difficult to obtain this result by any other controller.

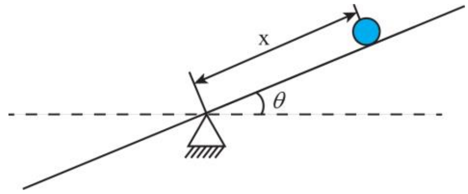
Controller is as follows.

$$A_{cont} = 1.0e+04 * \begin{bmatrix} 1.393 & 0 & 0 & 0 & 0 \\ -1.454 & 0.4889 & 0 & 0 & 0 \\ 0.4807 & -0.9697 & 0.4889 & 0 & 0 \\ 0 & 0.4807 & -0.9697 & 0.4889 & 0 \\ 0 & 0 & 0.9614 & -2.9008 & 1.9393 \end{bmatrix}$$

Example 3: Let's consider the following ball stabilization device.



(a)



(b)

$q_1 = \theta, \quad q_2 = x$

J - rotary inertia moment of the beam

J_b - rotary inertia moment of the ball

Fig 8. ball stabilization device and generalized coordinates

(a)- Ball control device (b) generalized coordinates

The motion equation of this device is as follows.

$$(J + J_b + mx^2)\ddot{\theta} + mgx\cos\theta + 2mx\dot{x}\theta = u$$

Representing above formula to transferfunction

$$\frac{x(s)}{\theta(s)} = -\frac{k}{s^2} \quad , \quad \text{where} \quad k = \frac{mg}{m + \frac{J_b}{r^2}}$$

The step response for simulation is as follows.

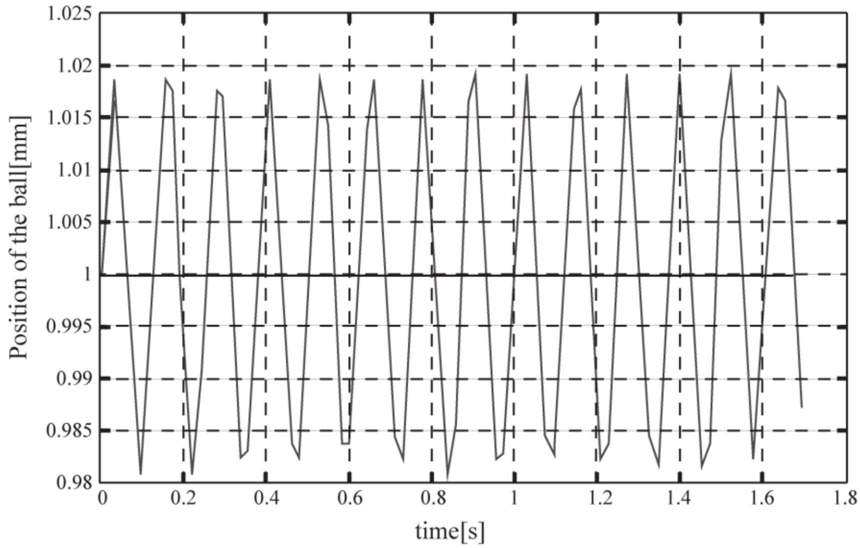


Fig 9. The step response for simulation

The control signal for simulation is as follows.

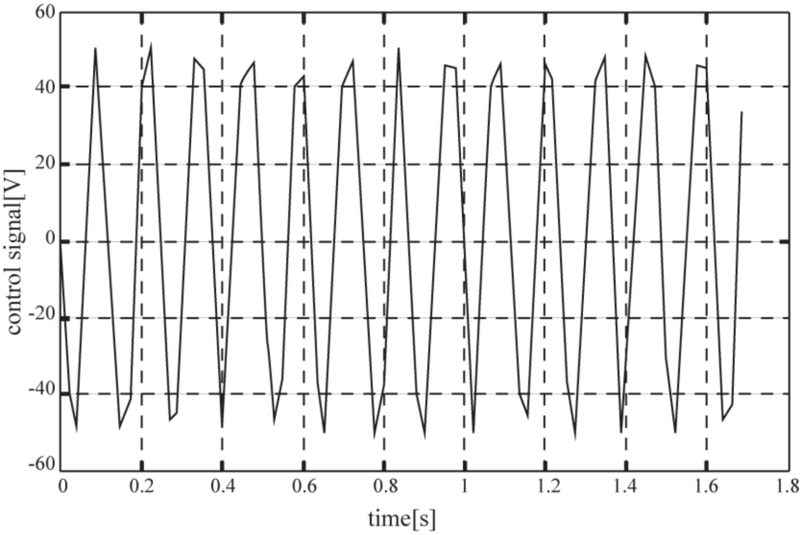


Fig 10. The control signal

7. Conclusion

In this paper, we propose a modeling method for TVNLG plants by integral equations and a design method for controllers by lattice matrix operators.

Here, the controller may consist of a non-regression type such as Example 1 or a regression type as in Example 2, and the performance of the regression controller improves better than the non-regression type, but it can cause very small oscillations or Alias effects.

The controller using lattice matrix operators is general as a solution of matrix algebraic equations, and this design method can be applied to plants with time-dependent or non-linear properties.

This method can meet any requirement of system design as the initial condition is approached near the input value.

Thus, this method can be used for terminal control that requires precise control in conjunction with other controls.

In addition, this method can be applied not only to control but also to measurement and identification.

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