COLLATZ CONJECTURE PROOF WITH "BRANCHES" OF TREE

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Collatz

Abstract. The Collatz's Conjecture is still unsolved as ancestors said that it is impossible. However, we did find the 'Infinite Beautiful Branches' in "Collatzs's Tree". And the Branches of tree shows us the way to prove "Collatz is right" like blue sky in autumn.

1. Introduction

1.1. **The definition of Collatz's Eq.** The Collatz's Conjecture is "Increasing the step, any selected natural number is arrive to 1 by below Eq.".

$$f(x) = \begin{cases} 3x+1, & (when\ x\ is\ odd\ number) \\ x/2, & (when\ x\ is\ even\ number). \end{cases} \tag{1.1}$$

For example, 9 -> 7 -> 11 -> 17 -> 13 -> 3- > 5 -> 1.
The 'Even' number is not so much useful, because
$f(3) = (3*3+1 = 10)$ -> $(10/2 = 5)$.
$f(3*2^3) = (3*2^2 = 12)$ -> $(3*2^1 = 6)$ -> $(3*3+1 = 10)$ -> $(10/2 = 5)$.
So, we can use equivalent Eq. in this proof. And if we prove that 'A_{n+1}' goes to '1', then Collatz's Conjecture is also true.

$$A_{n+1} = (3A_n+1)/2^z\ (z >= 1) \quad (A_n\ and\ A_{n+1}\ is\ Odd\ Number)$$

1.2. **'Parent' and 'Children' at a 'Node'.**

Definition 1.1. 'mX' = the multiple of 'X' $(X >= 1)$
'm3' means 'the multiple of 3'

Definition 1.2. $F_k = (4^k-1)/3$. (k>=1)

Then, $F_{n+1} = F_n$,
$F_1 = (4^1-1)/3 = 1$, $F_2 = (4^2-1)/3 = 5$,
$F_3 = 4*5+1 = 21$, $F_4 = (4*21)+1 = 85$.
And it is good visible in base-4 as

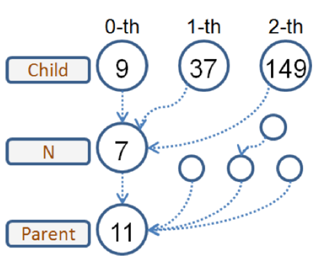

Figure 1. Parent and Children

$F_3 = 21 = 111_4, F_5 = 11111_4$.
In any Odd number 'N' have a 'Parent' by Collatz's Eq.

$$'Parent' = (3N+1)/2^1 \text{ (when N mod 4 = 3)}$$
$$'Parent' = (3N+1)/2^2/2^z \text{ (when N mod 4 <> 3, } z >= 0)$$
$$For\ example,\ (3(3)+1)/2^1 = 5,\ (3(9)+1)/2^2 = 7,\ (3(21)+1)/2^2/2^4 = 1.$$

For '$N = 1$', 'Parent' is itself in specially, and 'Parent' of the other is different with 'N'.

$$Parent = (3*(1)+1)/2^2 = 1$$

In Collatz's Reverse Eq., 'N' is 'Child' of the 'Parent'.

$$Child = not\ exist \qquad \text{(when 'Parent' mod 3 = 0)}$$
$$Child = (Parent * 2^2 - 1)/3 \qquad \text{(when 'Parent' mod 3 = 1)}$$
$$Child = (Parent * 2^1 - 1)/3 \qquad \text{(when 'Parent' mod 3 = 2)}$$
$$For\ example,\ ((5)*2^1-1)/3 = 3, ((7)*2^2-1)/3 = 9.$$

An Odd Number 'Parent' can have infinite 'Child'.

Theorem 1.3. $'K_{th}Child'$ $(k >= 0)$

$$'K_{th}Child' = not\ exist \qquad \textit{(when 'Parent' mod 3 = 0)}$$
$$'K_{th}Child' = ('Parent' * 2^2 * 4^k - 1)/3 \qquad \textit{(when 'Parent' mod 3 = 1)}$$
$$'K_{th}Child' = ('Parent' * 2^1 * 4^k - 1)/3 \qquad \textit{(when 'Parent' mod 3 = 2)}$$
$$For\ example,\ \textit{7 have 'Children' as } 9(0_{th}),\ 37(1_{th}),\ 149(2_{th}), \cdots.$$

For $'Parent' = 1$, '$0_{th}Child$' is itself in specially.

$$'Parent' = ((1)*2^2 - 1)/3 = 1$$

The Relation between K_{th} and $(K+1)_{th}$ Child as below.

$$'(K+1)_{th}Child' = 4 *' K_{th}Child' + 1$$

Theorem 1.4. *Any Odd Number N (N >= 3) have one 'Parent' and infinite 'Child' that all different with N.And the relation pair ('Parent', 'Child') is unique.*

2. Proof of Collatz' Conjecture

2.1. 'In a same time', Get the 'Child' or 'Parent' for all Odd Number.

2.1.1. *'Core Eq.' for getting A_n.*
We can set any Odd Number 'A' ($A >= 3$) as 'A_0' and can get 'A_1', 'A_2', $\cdots$ by Collatz's Eq.And we can hope to calculate for All 'TOGETHER' as below.

A_0	$3 =$ $2*(1)+1$	$5 =$ $2*(2)+1$	$9 =$ $2*(4)+1$	$21 =$ $2*(10)+1$	...	$123456789 =$ $2*(61728394)+1$	...
A_1	5	1	7	1		5787037	
A_2	1	1	11	1		????	
A_3	1	1	17	1		???	
A_4	1	1	13	1		??	
A_{10000}	1	1	1	1		?	

TABLE 1. Get A_n of all Odd Number in a same time

For this 'Mass Calculation', we must find the powerful logic.
Any 'A_0' can have 4 cases as below.

$$A_0 = (as\ 'x011'_2) = 2x * 4^b + (4^b - 1) \quad (b >= 1)$$
$$A_0 = (as\ 'x0111'_2) = 2 * (2x * 4^b + 4^b) - 1 \quad (b >= 1)$$
$$A_0 = (as\ 'x001'_2) = (2x)4^b + 1 \quad (b >= 1)$$
$$A_0 = (as\ 'x101'_2) = 2x * 4^b + F_b \quad (b >= 1)$$

So, the Core Eq. can be set as '$8a + b$', it can includes upper 4 cases and can express All Odd Number.

Definition 2.1. $\|a, b\| = 8a + b$ ($a >= 0$, b=1 or 3 or 5 or 7)
For example, $\|3, 1\| = 8(3) + 1 = 25$
$\|4(3(a + 1)) + 1, 3\| = 8(13) + 3,\ 8(25) + 3,\ \ldots, \infty\ (for\ a >= 0)$

2.1.2. *Logic to calculate A_n.* For all Odd Number Group $A_0 = 2k + 1$ ($k >= 1$) and $A_1 = (3(2k + 1) + 1)/2/2^z = (3k + 2)/2^z = \|a, b\| = 8a + b$ ($z >= 0$),
'$k = 4s + t$' or '$k = 2s + t$' seems the best for matching the relation as below. With '4s' or '2s', the A_0 can be divided 4 or 2 groups and the count of number in each group is almost same.
In first, divide A_0 as $\|a, b\|$ style. Set $k = 4x + y$ ($x >= 0,\ 0 <= y < 4$).

$$A_0 = 2k + 1 = 2(4x + y) + 1\ (k >= 1)$$

$$= 8(x) + (2y + 1) = \|x, 2y + 1\|$$

A_0 can be divided as 4 parts $\|x, 2y + 1\| = \|x, 1\|, \|x, 3\|, \|x, 5\|, \|x, 7\|.(x >= 0)$

A_1 Group can be got with A_0, and we can divide A_1 as 2 or 4 cases as below.

$$
\begin{aligned}
A_1 &= [3(8(x) + (2y+1)) + 1]/2/2^z \quad (z >= 0)\\
(when\ 2y+1=1)\ A_1 &= [3(8(4s+t)+1)+1]/2/2^1 \quad (x = 4s+t, t < 4)\\
&= 3*2*(4s+t)+1 = 8(3s)+6t+1\\
A_1 &= \|3s+0,1\| \quad (when\ t=0)\\
A_1 &= \|3s+1,5\| \quad (when\ t=2)\\
(when\ 2y+1=3)\ A_1 &= [3(8(2s+t)+3)+1]/2/2^0 \quad (x = 2s+t, t < 2)\\
&= 3*4*(2s+t)+5 = 8(3s)+12t+5\\
A_1 &= \|3s+2,1\| \quad (when\ t=1)\\
(when\ 2y+1=5)\ A_1 &= [3(8x+5)+1]/2/2^z \quad (z > 0)\\
&= 8[3x+2]/2/2^z\\
&\text{('}3x+2\text{' can be many cases. So we can't solve this easily.)}\\
(when\ 2y+1=7)\ A_1 &= [3(8(2s+t)+7)+1]/2/2^0 \quad (x = 2s+t, t < 2)\\
&= 3*4*(2s+t)+11 = 8(3s)+12t+11\\
A_1 &= \|3s+2,7\| \quad (when\ t=1)
\end{aligned}
$$

Because $\|x,5\|$'s result is very variable, we can't make the rule. So, the best way is 'STOP' of calculation. We take $\|x,5\|$ as 'Terminal' of 'A_n' that is end of getting 'A_n'.

In this way, we can make the full 'Logic Table (from A_n Group to A_{n+1} Group)' as below.'$A_n = \|4s+P,Q\|$' is converted to '$A_{n+1} = \|3s+M,N\|$'. And $\|X,5\|$ is no need to get because that is terminal.

Divider	$\|X,1\|$	Divider	$\|X,3\|$	$\|X,7\|$
X=4s+0	$\|3s+0,1\|$	X=2s+0	$\|3s+0,5\|$	$\|3s+1,3\|$
X=4s+1	$\|3s+0,7\|$	X=2s+1	$\|3s+2,1\|$	$\|3s+2,7\|$
X=4s+2	$\|3s+1,5\|$			
X=4s+3	$\|3s+2,3\|$			

Table 2. Logic Table of 'Parent' Direction

Because 's' can be translated freely such as
'$A_0 = \|4s+3,1\| = \|4(123a^2+45b+6c)+3,1\|$', this 'Logic' can be used for any 'A_n Group'.
For A_0, $A_1 = \|3(123a^2+45b+6c)+2,3\|$ (got in $\|X,1\|$ column and '$X = 4s+3$' row)
We can see $A_1 = \|3s_0+M_1,M_1\|$ is got from
$A_0 = \|4s_0+P_0,Q_0\|$ by 'Logic Table',
then for next step, A_1 must be as $\|4s_1+P_1,Q_1\|$.

We check how '3s' is changed to '4s' by example. (K_x is one of 4 or 2 cases for example in here)

$$
\begin{aligned}
X_0 &= \|k_0, 1\| \ (k_0 >= 1) \\
(At\ k_0 = 4k_1 + 1)\ X_1 &= \|3k_1, 7\| \ (k_1 >= 0) \\
(At\ k_1 = 2k_2 + 0) &= \|3(2k_2 + 0), 7\| \ (k_2 >= 0) \\
&= \|2(3k_2 + 0) + 0, 7\| \\
X_2 &= \|3(3k_2 + 0) + 1, 3\| \\
(At\ k_2 = 2k_3 + 1) &= \|3(3[2k_3 + 0] + 0) + 1, 3\| \ (k_3 >= 0) \\
&= \|3(2[3k_3 + 0] + 0) + 1, 3\| \\
&= \|2(3[3k_3 + 0] + 0) + 1, 3\| \\
&= \|2(3[3k_3 + 0] + 0) + 1, 3\| \\
X_3 &= \|3(3[3k_3 + 0] + 0) + 2, 1\| \\
(At\ k_3 = 4k_4 + 0) &= \|3(3[3(4k_4 + 0) + 0] + 0) + 2, 1\| \ (k_4 >= 0) \\
&= \|3(3[4(3k_4 + 0) + 0] + 0) + 2, 1\| \\
&= \|3(4[3(3k_4 + 0) + 0] + 0) + 2, 1\| \\
&= \|4(3[3(3k_4 + 0) + 0] + 0) + 2, 1\| \\
X_4 &= \|3(3[3(3k_4 + 0) + 0] + 0) + 1, 5\|
\end{aligned}
$$

So, $X_0 = (9, 17, 25, 33, 41, 49, \ldots, \infty)$ can be $X_4 = \|X, 5\|$ altogether in a same time.
As 'Logic Table', $\|x, b\|$ is classified and rotate as step increase. The figure shows the 'Flows' of A_n.
$\|X, Y\|$ works as below.

$\|X, 7\|$ works as 'Buffer' of $\|X, 3\|$.

A_n stay for k-step $(k >= 1)$.

$\|X, 3\|$ works as 'Classifier' of $\|X, 7\|$.

Send $1/2 * \|X, 7\|$ to $\|X, 1\|$

Send $1/2 * \|X, 7\|$ to $\|X, 5\|$

$\|X, 1\|$ works as 'Classifier' of itself and $\|X, 3\|$

As whole, Send $1/4 * \|X, 5\|$ to $\|X, 5\|$ simply.

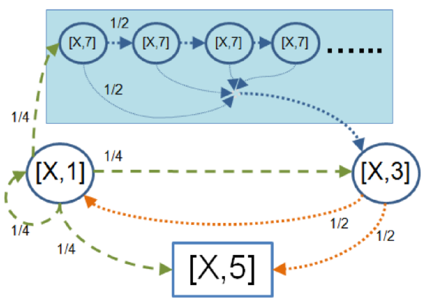

FIGURE 2. [a,b] flow chart

2.1.3. *'Reverse Core Eq. and Logic' for getting A_{-n}.* With onlt the Logic of 'Parent Direction(A_n to A_{n+1})', it is not enough for proving "Possibility of isolated loop". We can get the 'Reverse' Eq. and Logic that gets '$0_{th}Child$' for A_0 to A_{-n} with same way of 'Core Eq.'.

Definition 2.2. $\|a, b\|_6 = 6a + b$ ($a >= 0$, b=1 or 3 or 5)
For example, $\|3, 1\|_6 = 6(3) + 1 = 19$

$\|4(3(a+1)+1,3\|_6 = 6(13)+3,\ 6(25)+3,\ \ldots,\infty\ (for\ a >= 0)$

For all Odd Number Group $A_0 = 2k+1$ $(k >= 1)$, '$k = 3s+t$' seems the best for matching the relation as below. For '3s', the A_0 can be divided 3 groups and the count of number in each group is almost same.
In first, divide A_0 as $\|a,b\|_6$ style. Set $k = 3x+y$ $(x >= 0, 0 <= y <= 2)$.

$$A_0 = 2k+1 = 2(3x+y)+1$$

$$= 6x+(2y+1)$$

A_0 can be divided as 3 parts $\|x,2y+1\|_6 = \|x,1\|_6, \|x,3\|_6, \|x,5\|_6.(x >= 0)$
A_{-1} Group can be got from A_0, and we can divide A_{-1} as 3 cases as below.

$$A_1 = [(6x+(2y+1)) * 2 * 2^z - 1]/3 \quad (z = 0\ or\ 1)$$

$$(when\ 2y+1=1)\ A_1 = [(6(3s+t)+(2y+1)) * 2 * 2^1 - 1]/3 \quad (x = 3s+t, t <= 2)$$

$$= 6(4s)+8t+1$$

$$A_1 = \|4s,1\|_6\ (when\ t=0)$$

$$A_1 = \|4s+2,5\|_6\ (when\ t=2)$$

$$(when\ 2y+1=3)\ A_1 = \text{ can not get 'Child' because '(2y+1)-1' is not 'm3'}$$

$$(when\ 2y+1=5)\ A_1 = [(6(3s+t)+(2y+1)) * 2 * 2^0 - 1]/3 \quad (x = 3s+t, t <= 2)$$

$$= 6(2s)+4t+3$$

$$A_1 = \|2s+1,1\|_6\ (when\ t=1)$$

$$A_1 = \|2s+1,5\|_6\ (when\ t=2)$$

As same with '$\|x,5\|$', we take $\|x,3\|_6$ as 'Terminal' of 'A_{-n}' that end of getting 'Child'.

In this way, we can make the full 'Logic Table (from A_{-n} Group to A_{-n-1} Group)' as below.

Divider	$\|X,1\|_6$	$\|X,5\|_6$
X=3s+0	$\|4s+0,1\|_6$	$\|2s+0,3\|_6$
X=3s+1	$\|4s+1,3\|_6$	$\|2s+1,1\|_6$
X=3s+2	$\|4s+2,5\|_6$	$\|2s+1,5\|_6$

Table 3. Logic Table of 'Child' Direction

Because 's' can be translated freely such as
'$A_0 = \|3s+1,5\| = \|3(123a^2+45b+6c)+1,5\|$', this 'Logic' can be used for any 'A_{-n} Group'.
For A_0, $A_{-1} = \|2(123a^2+45b+6c)+1,1\|_6$ (got in $\|X,5\|_6$ column and '$X = 3s+1$' row)

2.2. **The Branch of Collatz.**

2.2.1. *Establish Collatz's Branch.*
With the 'Reverse Core Logic', we can get all A_{-1}('$0_{th}Child$' of All odd number Group 'A_0') in altogether as below.

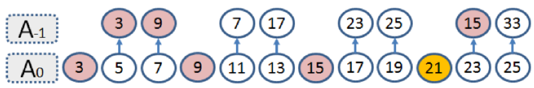

FIGURE 3. Origin of Branch

$$A_0 = \|x, 2y+1\|_6 \qquad (A_0 >= 3,\ x >= 0,\ 0 <= y <= 2)$$

With $A_0 = \|X, 1\|_6$,

$$(in\ X = 3s+0)\ A_{-1} = \|4s+0, 1\|_6,\ (s >= 1)$$
$$(in\ X = 3s+1)\ A_{-1} = \|4s+1, 3\|_6,\ (s >= 0)$$
$$= (6(1)+3,\ 6(5)+3,\ 6(9)+3,\ \dots)$$
$$(in\ X = 3s+2)\ A_{-1} = \|4s+2, 5\|_6,\ (s >= 0)$$

With $A_0 = \|X, 5\|_6$,

$$(in\ X = 3s+0)\ A_{-1} = \|2s+0, 3\|_6,\ (s >= 0)$$
$$= (6(0)+3,\ 6(2)+3,\ 6(4)+3,\ 6(6)+3, \dots)$$
$$(in\ X = 3s+1)\ A_{-1} = \|2s+1, 1\|_6,\ (s >= 0)$$
$$(in\ X = 3s+2)\ A_{-1} = \|2s+1, 5\|_6,\ (s >= 0)$$

So, in A_{-1}, we can check the result of '$6k+3$' as below.

$$6(0)+3,\ 6(1)+3,\ 6(2)+3,\ ?,\ 6(4)+3,\ 6(5)+3,\ 6(6)+3,\ ...$$
$$(not\ exist\ 6(3)+3 = 21,\ 6(13)+3 = 85, ...)$$

Because '6(3)+1, ...' have no 'Parent (>=3)', it can't be in A_1 and be only in A_0.

$$6(3)+3 = (1*4*4^2-1)/3,\ 6(13)+3 = (1*4*4^3-1)/3$$

A_{-1} includes all '$0_{th}Child$'
of all Odd A_0 ($A_0 >= 3$) in if A_0 can have 'Child'.
So,
all connection of '$6k+3$' and its 'Parent' is established,
that means all odd number is 'Parent' of '$6k+3$'.

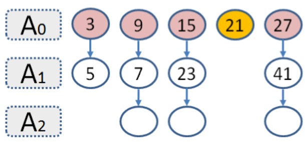

FIGURE 4. New Series

Because '$6k+3$' can be the 'Terminal', we can make new Series with group '$A_0 = 6k+3$' by the rule that is "getting 'A_n' for all $6k+3$' together until all series meets '$8k+5$' in once".

As now,
we can prove that there is no 'Isolated Loop' in Collatz's Tree.
1. All '$6k+3$'
is connected to all Odd Number by '$0_{th}Child$' connection.
2. If any Odd Number have a
isolated loop, then the 'Merged Node(C in Figure)' is 'Parent's of '$6k+3$', so '$6k+3$' must be '$0_{th}Child$' of 'Merged Node'.

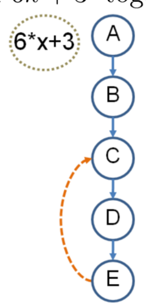

FIGURE 5. Isolated Loop

3. Our rule of 'Making Series' is 'STOP of step' at '$8k+5$', so "Node(E in Figure) of making wrong path" can't be '$8k+5$' that means it is '$0_{th}Child$' of 'Merged Node'.
4. So, there is conflict that 'Merged Node' have 2 '$0_{th}Child$' (B and E in Figure).
5. 'Isolated Loop' is never occur because of our 'Safe Rules', and we can do going on step in safety.

Theorem 2.3. *In Collatz's Tree, there is no 'Isolated Loop'.*

For A_n must have new odd A_{n+1} that is not used in other series, without 'Isolated Loop' even in any rule of connection. So, all series includes all Odd numbers N ($N >= 3$).In a result, we can get infinite count of series that have terminal(s) of '$6k+3$' and '$8k+5$' that sometimes 2 is same one.

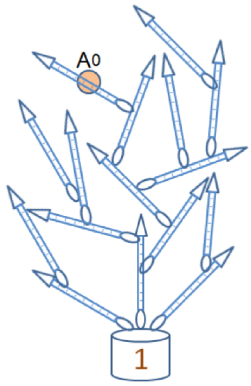

FIGURE 6. Concept of Collatz's Tree

2.2.2. *Definition of Collatz's Branch.*
We define these groups as "Branch'es of Collatz's Tree". And define the '$8k+5$' as the 'Head(Parent Direction)' of 'Branch' and the '$6k+3$' as the 'Tail(Child Direction)'.

Definition 2.4.
The 'Branch' of Collatz's Tree can be expressed as 2 types.
BR_{8X+5} is 'Branch' including 'Head=$8k+5$ (k>=0)'.
or BR_{6X+3} is 'Branch' including 'Tail=$6k+3$ (k>=0)'.

Because all 'Node' except 'Tail' is $0_{th}Child$, and '$8k+5$' can be only $X_{th}Child$ ($X >= 1$) of 'Node' of another 'Branch'. Moreover, if '$N = 8k+5 = 4(2k+1)+1$', then have a elder brother that is 'k'. So, $8k+5$ can not be 'Node' of a Branch except at 'Tail'.

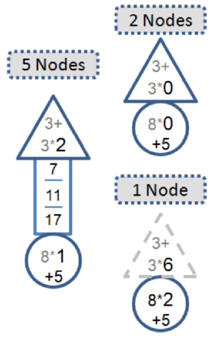

FIGURE 7. Branch Type

Theorem 2.5. *In a branch. there is no other '8k+5' except 'Tail' and there is no other '6k+3' except 'Head'.*

Theorem 2.6. *All Odd Number 'N' (N>=3) is only in one 'Branch', and can not duplicated in a 'Branch'.And every branch is unique.*

And, There is only 1 'Head=$8k+5$' and 1 'Tail=$6k+3$' in a 'Branch', and that 2 is same number in some cases.

Theorem 2.7. *Branch have 3 types as below.*

In Case Branch have 1 node. Head = Tail.

For example, Branch=[21],

$$'Head = 8(2) + 5', \ 'Tail = 6(3) + 3'$$

In Case Branch have 2 node. Only have Head and Tail.

For example, Branch=[61, 81],

$$'Head = 61 = 8(7) + 5', \ 'Tail = 81 = 6(13) + 3'$$

In Case Branch have more than 3 nodes.

For example, Branch=[53, 35, 23, 15],

$$'Head = 53 = 8(6) + 5', \ 'Tail = 15 = 6(2) + 3'$$

2.3. The Connection of branches.

Any '$Node_A$' in the 'BR_A' have a 'Parent' in its own 'Branch'. But only '$Tail_A$' have a 'Parent' in another 'BR_B'. And that 'Parent' have infinite 'Child' including 'BR_A' that is '$K_{th}Child$ $(k >= 1)$' of 'Parent'.

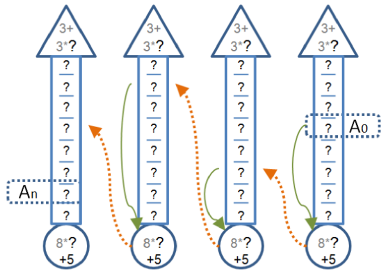

FIGURE 8. A_n's Movement in Branch

In '$Tail_A$',
we can divide all Branch as 2 groups as below.

$$'Sub - Branch'Group = \ BR_A \text{ and all Sub-Branch of } 'BR_A'$$

$$'Unknown - Branch'Group = \ \text{'All Branch' - 'Sub-Branch Group' .}$$

When we do one step of Collatz's Eq., Sub-Branch Group is more bigger with including new Sub-Branch of 'Parent'. And Unknown-Branch Group is more smaller with lost of infinite Sub-Branch.

The added amount is infinte that is the sum of Sub-Branches which 'Tail' is the Brother of '$Tail_A$'.

In global sight, the '$Node_A$' moves to '$Tail_A$', and moves to 'Node' of 'BR_B'.And '$Node_B$' moves to '$Tail_B$' of 'BR_B'.

Recursively,
'Sub-Branch' Group of the 'Tail' of the current 'Branch' is more bigger, and Unknown-Branch Group is very smaller than 'Sub-Branch' as to remain only last 'Node' that is 'Parent'.

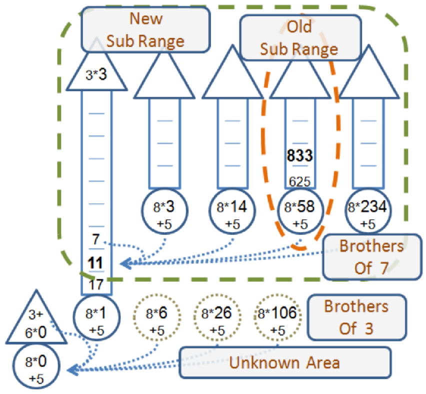

FIGURE 9. Connection of Branch

For Example, '$Node_A = 833$' in $BR_{8(58)+5}$.
And $Node_A$ moves to $Node_B = 11$ in $BR_{8(1)+5}$,
because $Tail_A = 8(58) + 5$ is '$3_{th}Child$' of 11.
And $Node_A$ moves to '$Tail_B = 8(1) + 5$' and ready for meet its own 'Parent=5' with its 'Brothers (6(0)+3, 8(1)+5, 8(6)+5, ...)'.

2.4. **At the final Connection of branches.** As Collatz's step increased, The 'Unknown-Branch Group' is as smaller as before quickly.
Finally, at the 'Tail' of a 'Current-Branch', that 'Tail' is '$0_{th}Child$' of the 'Parent' that is unknown.
Then, this '$0_{th}Child$' and its 'Brother' can be in a trouble that is "Who is our's 'Parent'?", because there is no remained Unknown Branches.

This last Brothers have a common feature that is X, 4X

Because 'Parent' must be Odd natural number and all odd number 'N' ($N >= 3$) is in all Branch, and there is no remained Unknown-Branch that means there is no Unknown Odd Number.
Because
any 'Tail' must have 'Parent' that is odd number.
So, last 'Parent' of that 'Brother'
is only '1', and last remained 'Brother' is

$$\frac{4^k - 1}{3} = 5,\ 21,\ 85,\ ...\ (k >= 2)$$

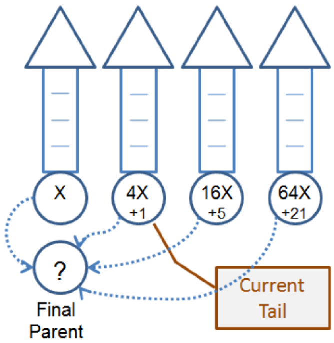

FIGURE 10. Final Parent

And we can check the 'Child' of '1' include all upper 'Brother'.

$$'Child'\ of\ 1\ = \frac{1 * 2^2 * 4^k - 1}{3} = 5, 21, 85, \ldots (k >= 1)$$

Proof. Because there is no 'Isolated Loop' and all Odd natural number N ($N >= 1$) become '1' by Collatz's Equation, so the 'Collatz's Conjecture' is true perfectly. □

3. Reverse of Collatz' Conjecture

3.1. **Reverse Eq. of Collatz's.**

3.1.1. *Specification.*
In simple, we can say that "The Reverse Eq. is huge indexed tree structure.".
This 'Tree' seems the biggest 'Tree Structure' that is bigger more than the sum of all data in the world. and specially that is indexed already.

Theorem 3.1. *The specification of Collatz's Tree.*
1. Tree is consist of 'Odd' number (>= 1).
2. The main 'Parent' R_0 *number is '1', but we can use any odd number in occasion.*
3. From any 'Node' can make infinite '$K_{th}Child(K >= 0)$*', but* R_n *is also big as K is bigger. (And most of cases,* R_n *seems to be smaller again)*
4. We can define the position of 'R_n*' as '*$[K_0, K_1, \ldots]$ *.*
5. In any 'Child', the 'Path' to '1' is twisted so much, as to it seems only calculation can find it.
(in K_x*, 'K' is of '*$K_{th}Child$*', x is the generation') from '1'*

3.1.2. *Imagination with Reverse.*
How we can use this for us?
Question 1) Can we use it for 'Data Structure of classification in real world'?.
Question 2) With this huge indexed structure, can it be the 'Key' of solving other problems?
Question 3) We have found some special equivalent Eq. Is it useful for another region of mathe?

$$A_{n+1} = \frac{1 - cos(\prod A_n)}{2}(\frac{5}{2}A_n + 1) + \frac{A_n}{2}$$

References

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