Knot in weak velocity of fluid flow

Miftachul Hadi[1,2]
[1] *Badan Riset dan Inovasi Nasional (BRIN), KST Habibie (Puspiptek) Gd 442, Serpong, Tangerang Selatan 15314, Banten, Indonesia.*
[2] *Institute of Mathematical Sciences, Kb Kopi, Jalan Nuri I, No.68, Pengasinan, Gn Sindur 16340, Bogor, Indonesia. E-mail: instmathsci.id@gmail.com*

We propose there exists a knot in the weak velocity of fluid flow.

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Let us observe[1]

$$C(\vec{v}) \equiv \int \varepsilon^{ijk}\ v^i\ \partial_j v^k\ dr = \int \vec{v} \cdot \vec{\omega}\ dr \tag{1}$$

where $C(\vec{v})$ is the fluid helicity, $\vec{v}$ is velocity field, $\vec{\omega}$ is vorticity.

In 3-dimensional space, the Chern-Simons integral could be written as[2–4]

$$h = \int_M \varepsilon^{\alpha\mu\nu}\ \vec{A}_\alpha\ \vec{F}_{\mu\nu}\ d^3r \tag{2}$$

where h is the electromagnetic helicity, a non-zero integer number (if h is zero it implies zero energy), M denotes 3-dimensional manifold, $\varepsilon^{\alpha\mu\nu}$ is the Levi-Civita symbol, $\alpha, \mu, \nu = 1, 2, 3$ denote the 3-dimensional space, $\vec{A}_\alpha$ is the gauge potential, $\vec{F}_{\mu\nu}$ is the gauge field tensor[3] (the field strength tensor). In Maxwell's theory, this integer h determines the $\pi_3(S^2)$ topology of the electromagnetic knots[2].

We consider that equations (1), and (2), are identical where velocity field and vorticity are identical to the gauge potential and the field strength tensor, respectively. So, roughly speaking, we could write the velocity field and the vorticity in the case of Abelian (weak velocity) respectively as[5,6]

$$\vec{v}_\nu = f\ \partial_\nu q \tag{3}$$

and

$$\vec{\omega}_{\mu\nu} = \partial_\mu \vec{v}_\nu - \partial_\nu \vec{v}_\mu \tag{4}$$

where

$$f = -1/[2\pi(1 + \rho_c^2)] \tag{5}$$

ρ_c is a constant amplitude so f is also a constant. We consider ρ_c as an analogy with a constant amplitude of electromagnetic wave in a vacuum space[7], q is the phase.

The fluid helicity or knot in weak velocity of fluid flow could be written as

$$h = \int_M \varepsilon^{\alpha\mu\nu}\ \vec{v}_\alpha\ \vec{\omega}_{\mu\nu}\ d^3r \tag{6}$$

or

$$h = \int_M \varepsilon^{\alpha\mu\nu} f\ \partial_\alpha q \left\{\partial_\mu(f\ \partial_\nu q) - \partial_\nu(f\ \partial_\mu q)\right\} d^3r \tag{7}$$

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