The Riemann hypothesis has three types of non trivial zeros

Zhiyang Zhang

Nanning,Guangxi,China

438951211@qq.com

Abstract

This paper classifies non trivial zeros based on the Riemann Zeta function. Through this operation, we can clearly understand the distribution pattern of non trivial zeros and predict the position of the next zero point. You can know that the Riemann hypothesis has three types of non trivial zeros, and the first type of non trivial zeros is located on the critical line, while the second and third types of zeros are not. Meanwhile, through a series of equation derivations, we can also understand why it is so difficult to find counterexamples of the Riemann hypothesis.

Keywords: `Riemann hypothesis`, Riemann Zeta function, counterexample

1. Introduce

The current mainstream research on the Riemann hypothesis can be divided into two methods. One approach is to continue deriving the equation of the Riemann Zeta function in the hope of proving the Riemann hypothesis, while the other approach is to find counterexamples of the Riemann hypothesis through numerical calculations. Of course, this approach is correct, but lacks creativity. For the first method, since non trivial zeros are not classified, they are all mixed in the same equation, so no matter how mathematicians deduce, the Riemann hypothesis cannot be strictly proved. And it can be said with certainty that if anyone claims to prove the Riemann hypothesis, it is definitely not rigorous.

For the second method, due to the lack of guidance, mathematicians blindly search for answers in the vast numbers, wasting a lot of computer power.

2. The initial idea

Although my initial idea was very naive, mainly due to my lack of understanding of Riemann Zeta functions. But I still need to include it in such a rigorous paper. You can understand the ideas of an original creator, which plays an important role in controlling the overall situation. Firstly, I have not studied the historical research of the Riemann hypothesis, so I have avoided a lot of interference and can focus on using my imagination. If I am wrong, there is no pressure.

For the following formula

$$\zeta(s) = \frac{\eta(s)}{1-2^{1-s}}$$

$$\eta(s) = \eta(r+it) = \sum_{n=1}^{\infty} \frac{(-1)^n \cos(-t \ln n)}{n^r} + i \sum_{n=1}^{\infty} \frac{(-1)^n \sin(-t \ln n)}{n^r}$$

When $\eta(r+it) = 0$, means that also $\zeta(r+it) = 0$, define

$$f(r,t) = \sum_{n=1}^{\infty} \frac{(-1)^n \cos(-t \ln n)}{n^r}$$

$$g(r,t) = \sum_{n=1}^{\infty} \frac{(-1)^n \sin(-t \ln n)}{n^r}$$

Obtain

$$\eta(r+it) = f(r,t) + ig(r,t)$$

Define

$$l(r,t) = f(r,t)f(r,t) + g(r,t)g(r,t)$$

Obtain

$$l(r,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{(-1)^{n+m} \cos\left[-t(\ln n - \ln m)\right]}{(mn)^r}$$

When $l(r,t) = 0$, means that also $\eta(r+it) = 0$

I continuously input different r and t through the computer,and I am delighted to find that

$$l(1, \frac{2n\pi}{ln2}) = 0, (n! = 0)$$

Table 1: the relationship between l(r,t) and t

r	t	ł(r,t)
1	0	0.473580948377
1	9.06472012903	0.0000248022675172
1	18.1294402581	0.0000249491838336
1	27.1941603871	0.0000252094244129
1	36.2588805161	0.0000255752123801
1	45.3236006451	0.0000260401323658
1	54.3883207742	0.0000266592938922
1	63.4530409032	0.0000273918081272
1	72.5177610322	0.0000282623826698
1	81.5824811612	0.0000293412440093
1	90.6472012903	0.0000305578822113
1	99.7119214193	0.0000319784693282
1	108.776641548	0.0000337231089273
1	117.841361677	0.0000356533243235
1	126.906081806	0.0000379704668472
1	135.970801935	0.0000407336997967
1	145.035522064	0.0000438579157617
1	154.100242193	0.0000476496035856
1	163.164962322	0.0000522511729474
1	172.229682452	0.0000575956864926
1	181.294402581	0.0000642875732613
1	190.35912271	0.0000725123234525
1	199.423842839	0.0000824836609738
1	208.488562968	0.0000954054823879
1	217.553283097	0.000112024526437

After consulting some papers, it turns out that this is caused by analytical extension,$1 + i\frac{2n\pi}{ln2}$ is not the true zero point of Riemann hypothesis.

3. Three types of non trivial zeros

The most basic task of falsifying the Riemann hypothesis is computation. By making curve of $\mathrm{Re}(\zeta) = 0$ and $\mathrm{Im}(\zeta) = 0$,their intersection point can be found to obtain the zero point.

A large portion of the zeros of the Riemann hypothesis are located on the critical line, and mathematicians are also trying to find new zeros near the real number 1.

Through the study of Riemann Zeta functions, non trivial zeros can be classified into the following three categories.

- $\mathrm{Re}(\zeta) = 0$ and $\mathrm{Im}(\zeta) = 0$, two curves have an intersection point located on the critical line
- $\mathrm{Re}(\zeta) = 0$ and $\mathrm{Im}(\zeta) = 0$,two curves have at least two intersections, one of which is on $\mathrm{Re}(s) = 1$ and the others are on the critical band
- $\mathrm{Re}(\zeta) = 0$ and $\mathrm{Im}(\zeta) = 0$,the intersection of two curves is located on the critical band, and no intersection is located on $\mathrm{Re}(s) = 1$ or $\mathrm{Re}(s) = 0.5$

According to the order, we will name the three types of zeros mentioned above as the first type of non trivial zero, the second type of non trivial zero, and the third type of non trivial zero, respectively.

4. Proof of the existence of the second type of non trivial zeros

To find the second type of zero point, we try to draw two curves of $\mathrm{Re}(\zeta) = 0$ and $\mathrm{Im}(\zeta) = 0$,and see if there is a second or even more intersection points.

Table 2: To find the second type of zero point

r	t	f(r,t)	g(r,t)
1	9.064720129	-0.00289917062059	0.00404933
0.9	9.08	0.095347403	-0.000370664
0.8	9.09	0.203139393	0.001855571
0.7	9.1	0.321959588	0.006985978
0.6	9.122	0.455973173	0.000533384
0.5	9.143	0.605286999	-0.000376742
0.4	9.168	0.775160638	-4.69097E-05
0.3	9.199	0.974298396	0.000594383
0.2	9.237	1.218292336	0.000786957
0.1	9.281	1.532413773	-0.000188059
0	9.327	1.951419697	-0.000497317

Table 3: To find the second type of zero point

r	t	f(r,t)	g(r,t)
1	9.064720129	-0.00289917062059	0.00404933
0.9	9.61	-0.000615012	-0.560817135
0.8	9.78	-0.006251523	-0.801335854
0.7	9.89	-0.002289649	-1.006983001
0.6	9.98	-0.004050748	-1.215990271
0.5	10.05	-0.004322117	-1.42717373595
0.4	10.1	0.004570374	-1.64078151758
0.3	10.14	0.009076636	-1.8666005112
0.2	10.17	0.006305885	-2.10473419254
0.1	10.188	-0.006009272	-2.35658587218
0	10.18	0.000572646	-2.63938022765

It can be seen that there is a significant difference in the rate of change between f(r,t) and g(r,t), making it impossible to form a second intersection point. Meanwhile, after testing nearly 50 numbers, it was still found that the pattern was like this.

Below, we can only use formulas to calculate, so that they have a second intersection point.

For

$$f(r,t) = \sum_{n=1}^{\infty} \frac{(-1)^n \cos(-t \ln n)}{n^r}$$

$$g(r,t) = \sum_{n=1}^{\infty} \frac{(-1)^n \sin(-t \ln n)}{n^r}$$

We get

$$\frac{df(r,t)}{dt} = \sum_{n=1}^{\infty} \frac{(-1)^n \ln n \sin(-t \ln n)}{n^r}$$

$$\frac{dg(r,t)}{dt} = -\sum_{n=1}^{\infty} \frac{(-1)^n \ln n \cos(-t \ln n)}{n^r}$$

When

$$r = 1, t = \frac{2k\pi}{ln2}$$

We get

$$sin(-t \ln n) = 0, cos(-t \ln n) = 1$$

$$n = 2^c$$

When c is an integer, the above equation holds

In most cases, the rate of change of g(r,t) is much higher than that of f(r,t).If we can find a counterexample that makes f(r,t)'s rate of change greater than g(r,t),then the second type of non trivial zero exists. There are two ways to prove it

- The first method is probability theory, even if g(r,t) is much larger than f(r,t) in a given value. However, assuming a random distribution, the remaining values of f(r,t) can still obtain relatively large numbers with an extremely low probability, as long as the expected value is not 0, there is still a possibility

- The second method is to find a common multiple, and when the value of k approaches infinity, the discretization becomes a continuous state. We can use the method of finding common multiples, where there exists a k that makes f(r,t) a very large value

Regardless of which method is used, the value of the second type of non trivial zeros is bound to be very large. That is to say, the counterexample of the Riemann hypothesis is a very large number, but it actually exists.

5. Does the third type of non trivial zero exist?

In fact, I cannot prove the existence of the third type of non trivial zeros, so from the beginning, I divided the non trivial zeros of the Riemann hypothesis into three cases.

Method

Open website https://www.desmos.com/
Inputting formulas and parameters, the numerical values of this paper can be calculated.

Acknowledgements

I wrote this paper is to commemorate Professor Gong-Sheng.In the process of proving the Riemann hypothesis, British mathematician Hardy made significant contributions. His student Hua-Luogeng brought modern mathematics, especially number theory,to China in 1950s. Professor Gong-Sheng was one of the best students of Hua-Luogeng, and he cultivated my mathematical literacy at University of Science and Technology of China. From Hardy to me, it has been four generations of research,a full hundred years.

References

- 1.viXra:2005.0284 The Riemann Hypothesis Proof. Authors:Isaac Mor
- 2.https://www.desmos.com/