The hadronic resonance masses and The Graphical law

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Abstract

We study the hadronic resonance masses. We rank those according to the masses. We draw the natural logarithm of the masses, normalised, starting with a rank vs the natural logarithm of the the rank, normalised. We conclude that the hadronic resonance masses, can be characterised by BP(4,$\beta H = 0.1$), the magnetisation curve for the Bethe-Peierls approximation of the Ising model with four nearest neighbours in the presence of external magnetic field, $\beta H = 0.1$. β is $\frac{1}{k_B T}$ where, T is temperature, H is external magnetic field and k_B is the tiny Boltzmann constant.

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resonance	$\rho(770)$	$\omega(782)$	η'	$f_0(980)$	$a_0(980)$	$\phi(1020)$	$h_1(1170)$	$b_1(1235)$	$a_1(1260)$	$f_2(1270)$	$f_1(1285)$	$\eta(1295)$	$\pi(1300)$
mass(Mev)	775	783	958	990	980	1019	1166	1230	1230	1275	1282	1294	1300
resonance	$a_2(1320)$	$\pi_1(1400)$	$\eta(1405)$	$h_1(1415)$	$f_1(1420)$	$\omega(1420)$	$a_0(1450)$	$\rho(1450)$	$\eta(1475)$	$f_0(1500)$	$f_2'(1525)$	$\pi_1(1600)$	$a_1(1640)$
mass(Mev)	1318	1354	1409	1416	1426	1410	1474	1465	1475	1506	1517	1661	1655
resonance	$\eta_2(1645)$	$\omega(1650)$	$\omega_3(1670)$	$\pi_2(1670)$	$\phi(1680)$	$\rho_3(1690)$	$\rho(1700)$	$a_2(1700)$	$f_0(1710)$	$\pi(1800)$	$\phi_3(1850)$	$\eta_2(1870)$	$\pi_2(1880)$
mass(Mev)	1617	1670	1667	1671	1680	1689	1720	1698	1704	1810	1854	1842	1874
resonance	$f_2(1950)$	$a_4(1970)$	$f_2(2010)$	$f_4(2050)$	$\phi(2170)$	$f_2(2300)$	$f_2(2340)$	$K_0^*(700)$	$K^*(892)$	$K_1(1270)$	$K_1(1400)$	$K^*(1410)$	$K_0^*(1430)$
mass(Mev)	1936	1967	2011	2018	2162	2297	2345	845	896	1253	1403	1414	1425
resonance	$K_2^*(1430)$	$K_1(1650)$	$K^*(1680)$	$K_2(1770)$	$K_3^*(1780)$	$K_2(1820)$	$K_2^*(1980)$	$K_4^*(2045)$	$D^*\ (2007^0)$	$D^*\ (2010^{\pm})$	$D_0^*(2300)$	$D_1\ (2420)^0$	$D_1\ (2430)^0$
mass(Mev)	1430	1650	1718	1773	1779	1819	1994	2048	2007	2010	2343	2422	2412
resonance	$D_2^*(2460)$	$D_3^*(2750)$	$D_S^{*\pm}$	$D_{S0}^*\ (2317)^{\pm}$	$D_{S1}\ (2460)^{\pm}$	$D_{S1}\ (2536)^{\pm}$	$D_{S2}^*(2573)$	$D_{S1}^*\ (2700)^{\pm}$	$D_{S3}^*\ (2860)^{\pm}$	B^*	$B_1\ (5721)^+$	$B_2^*(5747)$	$B_J\ (5970)^+$
mass(Mev)	2461	2763	2112	2318	2459	2535	2569	2714	2860	5325	5726	5737	5964
resonance	B_S^*	$B_{S1}\ (5830)^0$	$B_{S2}^*\ (5840)^0$	$\eta_c(1S)$	$\chi_{c0}(1P)$	$\chi_{c1}(1P)$	$h_c(1P)$	$\chi_{c2}(1P)$	$\eta_c(2S)$	$\psi(2S)$	$\psi(3770)$	$\psi_2(3823)$	$\psi_3(3842)$
mass(Mev)	5415	5829	5840	2984	3415	3511	3525	3556	3637	3686	3774	3824	3843
resonance	$\chi_{c1}(3872)$	$Z_c(3900)$	$\chi_{c0}(3915)$	$\chi_{c2}(3930)$	$X(4020)^{\pm}$	$\psi(4040)$	$\chi_{c1}(4140)$	$\psi(4160)$	$\psi(4230)$	$\chi_{c1}(4274)$	$\psi(4360)$	$\psi(4415)$	$Z_c(4430)$
mass(Mev)	3872	3887	3922	3922	4024	4039	4146	4191	4223	4286	4372	4421	4478
resonance	$\psi(4660)$	$\eta_b(1S)$	$\Upsilon(1S)$	$\chi_{b0}(1P)$	$\chi_{b1}(1P)$	$h_b(1P)$	$\chi_{b2}(1P)$	$\Upsilon(2S)$	$\Upsilon_2(1D)$	$\chi_{b0}(2P)$	$\chi_{b1}(2P)$	$h_b(2P)$	$\chi_{b2}(2P)$
mass(Mev)	4630	9399	9460	9859	9893	9899	9912	10023	10164	10232	10255	10259	10269
resonance	$\Upsilon(3S)$	$\chi_{b1}(3P)$	$\chi_{b2}(3P)$	$\Upsilon(4S)$	$Z_b(10610)$	$Z_b(10650)$	$\Upsilon(10860)$	$\Upsilon(11020)$					
mass(Mev)	10355	10513	10524	10579	10607	10652	10885	11000					

TABLE I. Mesonic masses(in Mev): the odd rows represent mesonic resonances, [1], [2], in the serial order, the even rows entries are the respective mesonic resonance masses.

I. INTRODUCTION

Half-integral spin mesons and integral spin baryons comprise hadrons. Hadronic resonances are short-lived particles created in a scattering or, decay process as intermediate objects. Those are characterised by the Dalitz plot analysis or, the partial wave analysis. The masses, [1], [2], of the more or, less established resonances are as tabulated in the tables, Table I. and Table II. as follow.

Looking for the Graphical Law in the hadronic masses, we proceed narrating the development. We have started considering magnetic field pattern in [3], in the languages we converse with. We have studied there, a set of natural languages, [3] and have found the existence

resonance	$N(1440)\frac{1}{2}^{+}$	$N(1520)\frac{3}{2}^{-}$	$N(1535)\frac{1}{2}^{-}$	$N(1650)\frac{1}{2}^{-}$	$N(1675)\frac{5}{2}^{-}$	$N(1680)\frac{5}{2}^{+}$	$N(1700)\frac{3}{2}^{-}$	$N(1710)\frac{1}{2}^{+}$	$N(1720)\frac{3}{2}^{+}$	$N(1900)\frac{3}{2}^{+}$	$N(2190)\frac{7}{2}^{-}$	$N(2220)\frac{9}{2}^{+}$	$N(2250)\frac{9}{2}^{-}$	$N(2600)\frac{11}{2}^{-}$
mass(Mev)	1440	1515	1530	1650	1675	1680	1700	1710	1720	1900	2190	2220	2250	2600
resonance	$\Delta(1232)\frac{3}{2}^{+}$	$\Delta(1600)\frac{3}{2}^{+}$	$\Delta(1620)\frac{1}{2}^{-}$	$\Delta(1700)\frac{3}{2}^{-}$	$\Delta(1900)\frac{1}{2}^{-}$	$\Delta(1905)\frac{5}{2}^{+}$	$\Delta(1910)\frac{1}{2}^{+}$	$\Delta(1920)\frac{3}{2}^{+}$	$\Delta(1930)\frac{5}{2}^{-}$	$\Delta(1950)\frac{7}{2}^{+}$	$\Delta(2420)\frac{11}{2}^{-}$	$\Lambda(1405)\frac{1}{2}^{-}$	$\Lambda(1520)\frac{3}{2}^{-}$	$\Lambda(1600)\frac{1}{2}^{+}$
mass(Mev)	1232	1570	1610	1700	1900	1905	1910	1920	1930	1950	2420	1405	1519	1600
resonance	$\Lambda(1670)\frac{1}{2}^{-}$	$\Lambda(1690)\frac{3}{2}^{-}$	$\Lambda(1800)\frac{1}{2}^{-}$	$\Lambda(1810)\frac{1}{2}^{+}$	$\Lambda(1820)\frac{5}{2}^{+}$	$\Lambda(1830)\frac{5}{2}^{-}$	$\Lambda(1890)\frac{3}{2}^{+}$	$\Lambda(2100)\frac{7}{2}^{-}$	$\Lambda(2110)\frac{5}{2}^{+}$	$\Lambda(2350)\frac{9}{2}^{+}$	$\Sigma(1385)\frac{3}{2}^{+}$	$\Sigma(1660)\frac{1}{2}^{+}$	$\Sigma(1670)\frac{3}{2}^{-}$	$\Sigma(1750)\frac{1}{2}^{-}$
mass(Mev)	1670	1690	1880	1810	1820	1830	1890	2100	2110	2350	1383	1660	1670	1750
resonance	$\Sigma(1775)\frac{5}{2}^{-}$	$\Sigma(1915)\frac{5}{2}^{+}$	$\Sigma(1940)\frac{3}{2}^{-}$	$\Sigma(2030)\frac{7}{2}^{+}$	$\Sigma(2250)$	$\Xi(1530)\frac{3}{2}^{+}$	$\Xi(1690)$	$\Xi(1820)\frac{3}{2}^{-}$	$\Xi(1950)$	$\Xi(2030)$	$\Omega\ (2012)^{-}$	$\Omega\ (2250)$	$\Lambda_c\ (2595)^{+}$	$\Lambda_c\ (2625)^{+}$
mass(Mev)	1775	1915	1940	2030	2250	1530	1690	1823	1950	2025	2012	2250	2592	2628
resonance	$\Lambda_c\ (2860)^{+}$	$\Lambda_c\ (2880)^{+}$	$\Lambda_c\ (2940)^{+}$	$\Sigma_c(2455)$	$\Sigma_c(2520)$	$\Sigma_c(2800)$	Ξ_c^{+}	Ξ_c^{0}	$\Xi_c(2645)$	$\Xi_c(2790)$	$\Xi_c(2815)$	$\Xi_c(2970)$	$\Xi_c(3055)$	$\Xi_c(3080)$
mass(Mev)	2856	2882	2940	2453	2518	2800	2465	2470	2645	2792	2817	2965	3056	3078
resonance	$\Omega_c\ (2770)^{0}$	$\Omega_c\ (3000)^{0}$	$\Omega_c\ (3050)^{0}$	$\Omega_c\ (3065)^{0}$	$\Omega_c\ (3090)^{0}$	$\Omega_c\ (3120)^{0}$	$\Lambda_b\ (5912)^{0}$	$\Lambda_b\ (5920)^{0}$	$\Lambda_b\ (6070)^{0}$	$\Lambda_b\ (6146)^{0}$	$\Lambda_b\ (6152)^{0}$	$\Sigma_b(5811)$	Σ_b^{*}	$\Sigma_b\ (6097)^{+}$
mass(Mev)	2766	3000	3050	3066	3090	3119	5912	5920	6072	6146	6153	5811	5832	6096
resonance	$\Sigma_b\ (6097)^{-}$	$\Xi_b^{'}\ (5935)^{-}$	$\Xi_b\ (5945)^{0}$	$\Xi_b\ (5955)^{-}$	$\Xi_b\ (6100)^{-}$	$\Xi_b\ (6227)^{-}$	$\Xi_b\ (6227)^{0}$							
mass(Mev)	6098	5935	5952	5955	6100	6228	6227							

TABLE II. Baryonic masses(in Mev): the odd rows represent baryonic resonances, [1], [2], in the serial order, the even rows entries are the respective baryonic resonance masses.

of a magnetisation curve under each language. We have termed this phenomenon as the Graphical Law. Then, we moved on to investigate into, [4], dictionaries of five disciplines of knowledge and found the existence of a curve of magnetisation under each discipline. This was followed by finding of the graphical law in references from [5] to [75].

The planning of the paper is as follows. In the next section, we describe the Graphical Law analysis of the hadronic masses, [1], [2]. The section III, we give an introduction to the standard curves of magnetisation of Ising model. The section IV is Acknowledgment. The last section is Bibliography.

II. THE GRAPHICAL LAW ANALYSIS

For the purpose of exploring graphical law, we assort the hadronic resonances according to the masses, in the descending order, denoted by f and the respective rank, [76], denoted by k. k is a positive integer starting from one. Moreover, the minimum non-zero mass is seven hundred seventy five(in Mev). Hence, we attach a limiting mass one(in Mev). The limiting rank is maximum rank plus one, here it is two hundred one. As a result both $\frac{lnf}{lnf_{max}}$ and $\frac{lnk}{lnk_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table, Table III., and plot $\frac{lnf}{lnf_{max}}$ against $\frac{lnk}{lnk_{lim}}$ in the figure fig.1. We then ignore the nineteen resonances from the top, tabulate in the adjoining table, Table III.,and redo the plot, normalising the lnfs with $lnf_{19n-max}$ in the figure fig.2.

k	lnk/lnk_{lim}	f	lnf/lnf_{max}	lnf/$lnf_{19n-max}$
1	0	11000	1	Blank
2	0.131	10885	0.999	Blank
3	0.207	10652	0.997	Blank
4	0.261	10607	0.996	Blank
5	0.303	10579	0.996	Blank
6	0.338	10524	0.995	Blank
7	0.367	10513	0.995	Blank
8	0.392	10355	0.994	Blank
9	0.414	10269	0.993	Blank
10	0.434	10259	0.993	Blank
11	0.452	10255	0.992	Blank
12	0.469	10232	0.992	Blank
13	0.484	10164	0.992	Blank
14	0.498	10023	0.990	Blank
15	0.511	9912	0.989	Blank
16	0.523	9899	0.989	Blank
17	0.534	9893	0.989	Blank
18	0.545	9859	0.988	Blank
19	0.555	9460	0.984	Blank
20	0.565	9399	0.983	1
21	0.574	6228	0.939	0.955
22	0.583	6227	0.939	0.955
23	0.591	6153	0.938	0.954
24	0.599	6146	0.937	0.954
25	0.607	6100	0.937	0.953
26	0.614	6098	0.937	0.953
27	0.621	6096	0.937	0.953
28	0.628	6072	0.936	0.952
29	0.635	5964	0.934	0.950
30	0.641	5955	0.934	0.950
31	0.648	5952	0.934	0.950
32	0.654	5935	0.934	0.950
33	0.659	5920	0.933	0.949
34	0.665	5912	0.933	0.949
35	0.670	5840	0.932	0.948
36	0.676	5832	0.932	0.948
37	0.681	5829	0.932	0.948
38	0.686	5811	0.931	0.947
39	0.691	5737	0.930	0.946
40	0.696	5726	0.930	0.946
41	0.700	5415	0.924	0.940
42	0.705	5325	0.922	0.938
43	0.709	4630	0.907	0.923
44	0.714	4478	0.903	0.919
45	0.718	4421	0.902	0.918
46	0.722	4372	0.901	0.916
47	0.726	4286	0.899	0.914
48	0.730	4223	0.897	0.913
49	0.734	4191	0.896	0.912
50	0.738	4146	0.895	0.911

k	$\ln k/\ln k_{lim}$	f	$\ln f/\ln f_{max}$	$\ln f/\ln f_{19n-max}$
51	0.741	4039	0.892	0.908
52	0.745	4024	0.892	0.907
53	0.749	3922	0.889	0.904
54	0.752	3887	0.888	0.903
55	0.756	3872	0.888	0.903
56	0.759	3843	0.887	0.902
57	0.762	3824	0.886	0.902
58	0.766	3774	0.885	0.900
59	0.769	3686	0.883	0.898
60	0.772	3637	0.881	0.896
61	0.775	3556	0.879	0.894
62	0.778	3525	0.878	0.893
63	0.781	3511	0.877	0.892
64	0.784	3415	0.874	0.889
65	0.787	3119	0.865	0.879
66	0.790	3090	0.864	0.878
67	0.793	3078	0.863	0.878
68	0.796	3066	0.863	0.878
69	0.798	3056	0.862	0.877
70	0.801	3050	0.862	0.877
71	0.804	3000	0.860	0.875
72	0.806	2984	0.860	0.875
73	0.809	2965	0.859	0.874
74	0.812	2940	0.858	0.873
75	0.814	2882	0.856	0.871
76	0.817	2860	0.855	0.870
77	0.819	2856	0.855	0.870
78	0.822	2817	0.854	0.868
79	0.824	2800	0.853	0.868
80	0.826	2792	0.853	0.867
81	0.829	2766	0.852	0.866
82	0.831	2763	0.852	0.866
83	0.833	2714	0.850	0.864
84	0.835	2645	0.847	0.861
85	0.838	2628	0.846	0.861
86	0.840	2600	0.845	0.860
87	0.842	2592	0.845	0.859
88	0.844	2569	0.844	0.858
89	0.846	2535	0.842	0.857
90	0.848	2518	0.842	0.856
91	0.851	2470	0.839	0.854
92	0.853	2465	0.839	0.854
93	0.855	2461	0.839	0.854
94	0.857	2459	0.839	0.853
95	0.859	2453	0.839	0.853
96	0.861	2422	0.837	0.852
97	0.863	2420	0.837	0.852
98	0.865	2412	0.837	0.851
99	0.866	2350	0.834	0.848
100	0.868	2345	0.834	0.848

k	lnk/lnk_{lim}	f	lnf/lnf_{max}	lnf/$lnf_{19n-max}$
101	0.870	2343	0.834	0.848
102	0.872	2318	0.833	0.847
103	0.874	2297	0.832	0.846
104	0.876	2250	0.829	0.844
105	0.878	2220	0.828	0.842
106	0.879	2190	0.827	0.841
107	0.881	2162	0.825	0.839
108	0.883	2112	0.823	0.837
109	0.885	2110	0.823	0.837
110	0.886	2100	0.822	0.836
111	0.888	2048	0.819	0.833
112	0.890	2030	0.818	0.832
113	0.891	2025	0.818	0.832
114	0.893	2012	0.817	0.832
115	0.895	2011	0.817	0.831
116	0.896	2010	0.817	0.831
117	0.898	2007	0.817	0.831
118	0.900	1994	0.816	0.831
119	0.901	1967	0.815	0.829
120	0.903	1950	0.814	0.828
121	0.904	1940	0.814	0.828
122	0.906	1936	0.813	0.827
123	0.907	1930	0.813	0.827
124	0.909	1920	0.812	0.826
125	0.910	1915	0.812	0.826
126	0.912	1910	0.812	0.826
127	0.913	1905	0.812	0.826
128	0.915	1900	0.811	0.825
129	0.916	1890	0.811	0.825
130	0.918	1874	0.810	0.824
131	0.919	1854	0.809	0.823
132	0.921	1842	0.808	0.822
133	0.922	1830	0.807	0.821
134	0.924	1823	0.807	0.821
135	0.925	1820	0.807	0.821
136	0.926	1819	0.807	0.820
137	0.928	1810	0.806	0.820
138	0.929	1800	0.805	0.819
139	0.930	1779	0.804	0.818
140	0.932	1775	0.804	0.818
141	0.933	1773	0.804	0.818
142	0.934	1750	0.802	0.816
143	0.936	1720	0.801	0.814
144	0.937	1718	0.800	0.814
145	0.938	1710	0.800	0.814
146	0.940	1704	0.800	0.813
147	0.941	1700	0.799	0.813
148	0.942	1698	0.799	0.813
149	0.944	1690	0.799	0.812
150	0.945	1689	0.799	0.812

k	lnk/lnk_{lim}	f	lnf/lnf_{max}	lnf/$lnf_{19n-max}$
151	0.946	1680	0.798	0.812
152	0.947	1675	0.798	0.811
153	0.949	1671	0.797	0.811
154	0.950	1670	0.797	0.811
155	0.951	1667	0.797	0.811
156	0.952	1661	0.797	0.811
157	0.953	1660	0.797	0.811
158	0.955	1655	0.796	0.810
159	0.956	1650	0.796	0.810
160	0.957	1617	0.794	0.808
161	0.958	1610	0.793	0.807
162	0.959	1600	0.793	0.806
163	0.960	1570	0.791	0.804
164	0.962	1530	0.788	0.802
165	0.963	1519	0.787	0.801
166	0.964	1517	0.787	0.801
167	0.965	1515	0.787	0.800
168	0.966	1506	0.786	0.800
169	0.967	1475	0.784	0.798
170	0.968	1474	0.784	0.797
171	0.970	1465	0.783	0.797
172	0.971	1440	0.782	0.795
173	0.972	1430	0.781	0.794
174	0.973	1426	0.780	0.794
175	0.974	1425	0.780	0.794
176	0.975	1416	0.780	0.793
177	0.976	1414	0.780	0.793
178	0.977	1410	0.779	0.793
179	0.978	1409	0.779	0.793
180	0.979	1405	0.779	0.792
181	0.980	1403	0.779	0.792
182	0.981	1383	0.777	0.791
183	0.982	1354	0.775	0.788
184	0.983	1318	0.772	0.785
185	0.984	1300	0.771	0.784
186	0.985	1294	0.770	0.783
187	0.986	1282	0.769	0.782
188	0.987	1275	0.768	0.782
189	0.988	1253	0.767	0.780
190	0.989	1232	0.765	0.778
191	0.990	1230	0.765	0.778
192	0.991	1166	0.759	0.772
193	0.992	1019	0.744	0.757
194	0.993	990	0.741	0.754
195	0.994	980	0.740	0.753
196	0.995	958	0.738	0.750
197	0.996	896	0.731	0.743
198	0.997	845	0.724	0.737
199	0.998	783	0.716	0.728
200	0.999	775	0.715	0.727
201	1	1	0	0

TABLE III. Hadronic resonance masses: ranking, natural logarithm, normalisations

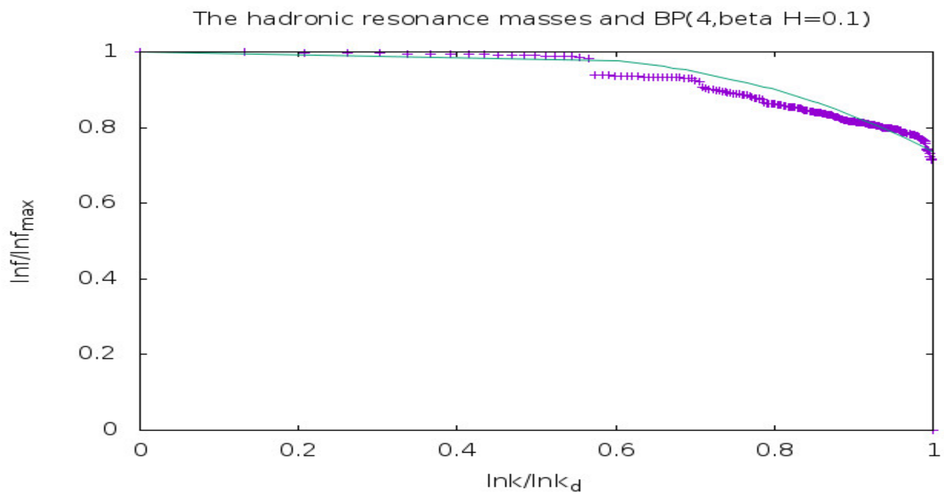

FIG. 1. The vertical axis is $\frac{lnf}{lnf_{max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the hadronic resonance masses, [1], [2], with the fit curve, BP(4,βH = 0.1), being the Bethe-Peierls curve in the presence of four nearest neighbours and external magnetic field, $m = 0.05$ or, $\beta H = 0.1$.

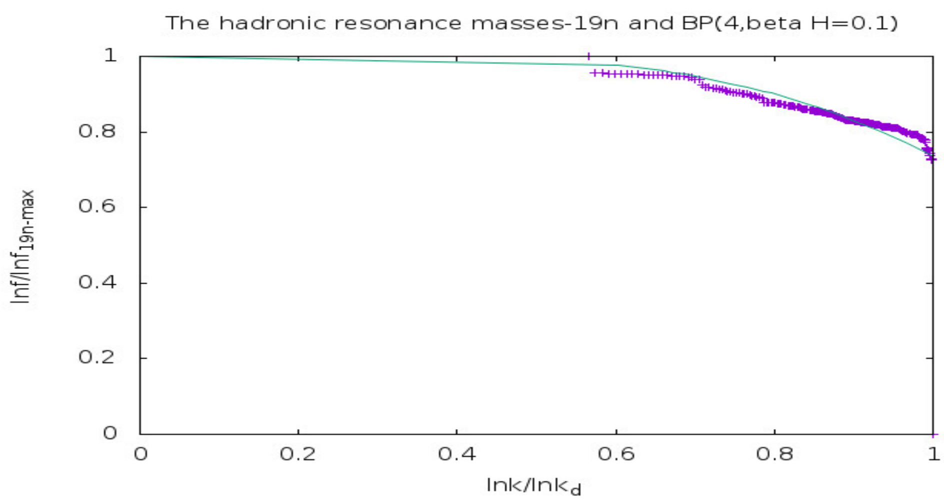

FIG. 2. The vertical axis is $\frac{lnf}{lnf_{19n-max}}$ and the horizontal axis is $\frac{lnk}{lnk_{lim}}$. The + points represent the hadronic resonance masses, [1], [2], with the fit curve, BP(4,βH = 0.1), being the Bethe-Peierls curve in the presence of four nearest neighbours and external magnetic field, $m = 0.05$ or, $\beta H = 0.1$.

A. conclusion

From the figures (fig.1-fig.2), we observe that there is a curve of magnetisation, behind the hadronic resonance masses, [1], [2]. This is BP(4,$\beta H = 0.1$), the magnetisation curve for the Bethe-Peierls approximation of the Ising model with four nearest neighbours in the presence of external magnetic field, $\beta H = 0.1$. β is $\frac{1}{k_B T}$ where, T is temperature, H is external magnetic field and k_B is the Boltzmann constant.
Moreover, the associated correspondence is,

$$\frac{lnf}{lnf_{19n-max}} \longleftrightarrow \frac{M}{M_{max}},$$

$$lnk \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [83].

III. APENDIX: MAGNETISATION

A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\Sigma_i \sigma_i$, where σ_i is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where N_+ is the number of up spins, N_- is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1+L)$ and $N_- = \frac{N}{2}(1-L)$. Magnetisation or, net magnetic moment , M is $\mu\Sigma_i\sigma_i$ or, $\mu(N_+ - N_-)$ or, $\mu N L$, $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$. $\frac{M}{M_{max}}$ is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[77], for the lattice of spins, setting μ to one, is $-\epsilon\Sigma_{n.n}\sigma_i\sigma_j - H\Sigma_i\sigma_i$, where n.n refers to nearest neighbour pairs. The difference $\triangle E$ of energy if we flip an up spin to down spin is, [78], $2\epsilon\gamma\bar{\sigma} + 2H$, where γ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $exp(-\frac{\triangle E}{k_B T})$, [79]. In the Bragg-Williams approximation,[80], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$ln\frac{1+L}{1-L} = 2\frac{\gamma\epsilon L + H}{k_B T} = 2\frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2\frac{L+c}{\frac{T}{T_c}} \tag{1}$$

where, $c = \frac{H}{\gamma\epsilon}$, $T_c = \gamma\epsilon/k_B$, [81]. $\frac{T}{T_c}$ is referred to as reduced temperature.
Plot of L vs $\frac{T}{T_c}$ or, reduced magentisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [78]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [77],[78],[79],[80],[81], due to Bethe-Peierls, [82], reduced magnetisation varies with reduced temperature, for γ neighbours, in absence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \tag{2}$$

$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe

BW	BW(c=0.01)	BP(4,βH = 0)	reduced magnetisation
0	0	0	1
0.435	0.439	0.563	0.978
0.439	0.443	0.568	0.977
0.491	0.495	0.624	0.961
0.501	0.507	0.630	0.957
0.514	0.519	0.648	0.952
0.559	0.566	0.654	0.931
0.566	0.573	0.7	0.927
0.584	0.590	0.7	0.917
0.601	0.607	0.722	0.907
0.607	0.613	0.729	0.903
0.653	0.661	0.770	0.869
0.659	0.668	0.773	0.865
0.669	0.676	0.784	0.856
0.679	0.688	0.792	0.847
0.701	0.710	0.807	0.828
0.723	0.731	0.828	0.805
0.732	0.743	0.832	0.796
0.756	0.766	0.845	0.772
0.779	0.788	0.864	0.740
0.838	0.853	0.911	0.651
0.850	0.861	0.911	0.628
0.870	0.885	0.923	0.592
0.883	0.895	0.928	0.564
0.899	0.918		0.527
0.904	0.926	0.941	0.513
0.946	0.968	0.965	0.400
0.967	0.998	0.965	0.300
0.987		1	0.200
0.997		1	0.100
1	1	1	0

TABLE IV. Reduced magnetisation vs reduced temperature data s for Bragg-Williams approximation, in absence of and in presence of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours.

data s generated from the equation(1) and the equation(2) in the table, IV, and curves of magnetisation plotted on the basis of those data s. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.3. Empty spaces in the table, IV, mean corresponding point pairs were not used for plotting a line.

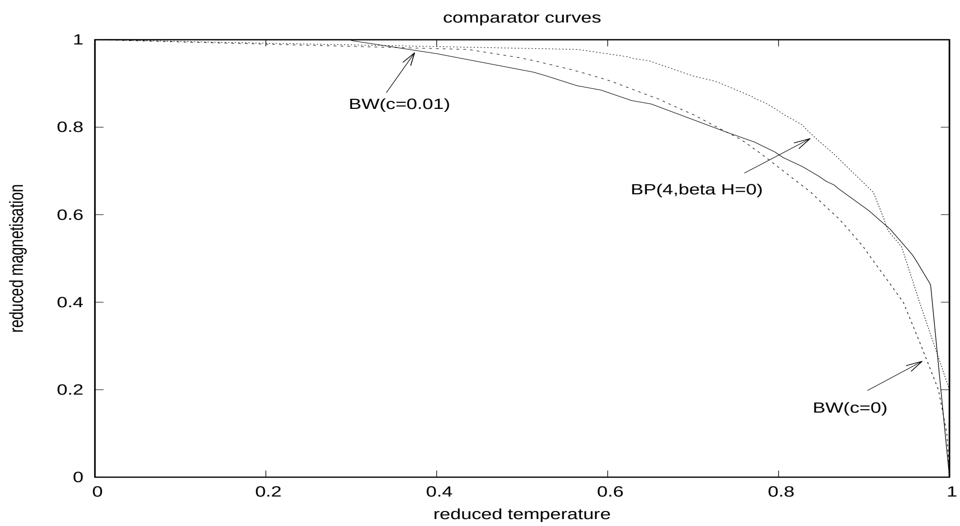

FIG. 3. Reduced magnetisation vs reduced temperature curves for Bragg-Williams approximation, in absence(dark) of and presence(inner in the top) of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours (outer in the top).

C. Bethe-peierls approximation in presence of four nearest neighbours, in the presence of external magnetic field

In the Bethe-Peierls approximation scheme , [82], reduced magnetisation varies with reduced temperature, for γ neighbours, in presence of external magnetic field, as

$$\frac{ln\frac{\gamma}{\gamma-2}}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{3}$$

Derivation of this formula ala [82] is given in the appendix of [8].
$ln\frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For four neighbours,

$$\frac{0.693}{ln\frac{factor-1}{e^{\frac{2\beta H}{\gamma}}factor^{\frac{\gamma-1}{\gamma}}-e^{-\frac{2\beta H}{\gamma}}factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}}+1}{1-\frac{M}{M_{max}}}. \tag{4}$$

In the following, we describe datas in the table, V, generated from the equation(4) and curves of magnetisation plotted on the basis of those datas. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.06$. calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that

$\beta H = 0.05.$ calculated from the equation(4). BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.04.$ calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.02.$ calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.01.$ calculated from the equation(4). The data set is used to plot fig.4. Empty spaces in the table, V, mean corresponding point pairs were not used for plotting a line.

BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
				0.964	0.513
			1.00		0.500
				1.00	0.400
					0.300
					0.200
					0.100
					0

TABLE V. Bethe-Peierls approx. in presence of little external magnetic fields

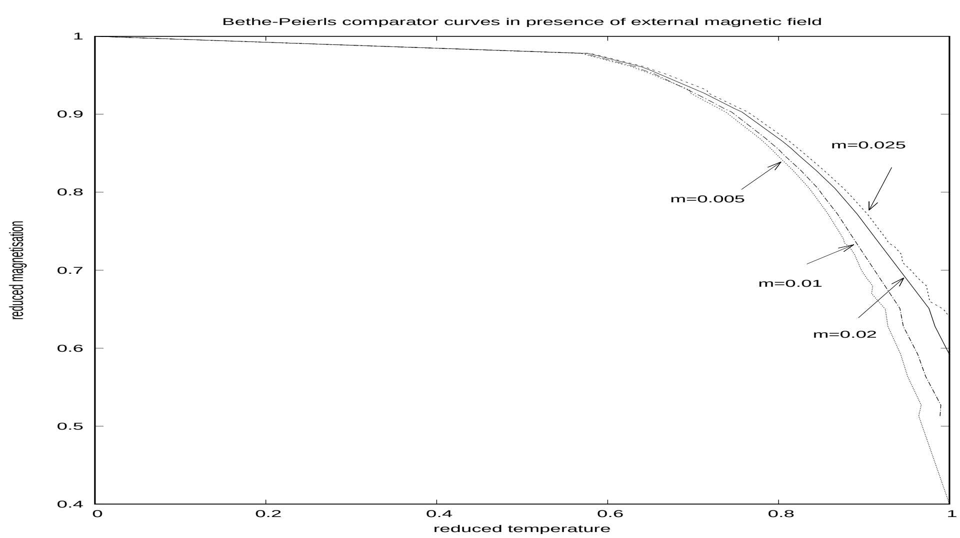

FIG. 4. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

IV. ACKNOWLEDGMENT

We have used gnuplot for plotting the figures in this paper. We have benefited from going through the site, [84].

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[1] PDG particle data group, 2022, Particle Physics Booklet; Extracted from the Review of Particle Physics, R.L.Workman et al.(Particle Data Group), Prog. Theor. Exp. Phys.2022, 083C01(2022).

[2] Review of Particle Properties, Phys.Rev.D.50,1173.

[3] Anindya Kumar Biswas, "Graphical Law beneath each written natural language", arXiv:1307.6235v3[physics.gen-ph]. A preliminary study of words of dictionaries of twenty six languages, more accurate study of words of dictionary of Chinese usage and all parts of speech of dictionary of Lakher(Mara) language and of verbs, adverbs and adjectives of dictionaries of six languages are included.

[4] Anindya Kumar Biswas, "A discipline of knowledge and the graphical law", IJARPS Volume 1(4), p 21, 2014; viXra: 1908:0090[Linguistics].

[5] Anindya Kumar Biswas, "Bengali language and Graphical law", viXra: 1908:0090[Linguistics].

[6] Anindya Kumar Biswas, "Basque language and the Graphical Law", viXra: 1908:0414[Linguistics].

[7] Anindya Kumar Biswas, "Romanian language, the Graphical Law and More", viXra: 1909:0071[Linguistics].

[8] Anindya Kumar Biswas, "Discipline of knowledge and the graphical law, part II", viXra:1912.0243 [Condensed Matter],International Journal of Arts Humanities and Social Sciences Studies Volume 5 Issue 2 February 2020.

[9] Anindya Kumar Biswas, "Onsager Core of Abor-Miri and Mising Languages", viXra: 2003.0343[Condensed Matter].

[10] Anindya Kumar Biswas, "Bengali language, Romanisation and Onsager Core", viXra: 2003.0563[Linguistics].

[11] Anindya Kumar Biswas, "Little Oxford English Dictionary and the Graphical Law", viXra: 2008.0041[Linguistics].

[12] Anindya Kumar Biswas, ”Oxford Dictionary Of Social Work and Social Care and the Graphical law”, viXra: 2008.0077[Condensed Matter].

[13] Anindya Kumar Biswas, ”Visayan-English Dictionary and the Graphical law”, viXra: 2009.0014[Linguistics].

[14] Anindya Kumar Biswas, ”Garo to English School Dictionary and the Graphical law”, viXra: 2009.0056[Condensed Matter].

[15] Anindya Kumar Biswas, ”Mursi-English-Amharic Dictionary and the Graphical law”, viXra: 2009.0100[Linguistics].

[16] Anindya Kumar Biswas, ”Names of Minor Planets and the Graphical law”, viXra: 2009.0158[History and Philosophy of Physics].

[17] Anindya Kumar Biswas, ”A Dictionary of Tibetan and English and the Graphical law”, viXra: 2010.0237[Condensed Matter].

[18] Anindya Kumar Biswas, ”Khasi English Dictionary and the Graphical law”, viXra: 2011.0011[Linguistics].

[19] Anindya Kumar Biswas, ”Turkmen-English Dictionary and the Graphical law”, viXra: 2011.0069[Linguistics].

[20] Anindya Kumar Biswas, ” Webster’s Universal Spanish-English Dictionary, the Graphical law and A Dictionary of Geography of Oxford University Press”, viXra: 2103.0175[Condensed Matter].

[21] Anindya Kumar Biswas, ”A Dictionary of Modern Italian, the Graphical law and Dictionary of Law and Administration, 2000, National Law Development Foundation”, viXra: 2107.0171[Condensed Matter].

[22] Anindya Kumar Biswas, ”Langenscheidt’s German-English English-German Dictionary and the Graphical law”, viXra: 2107.0179[Linguistics].

[23] Anindya Kumar Biswas, ”Essential Dutch dictionary by G. Quist and D. Strik, the Graphical law Classification”, viXra: 2108.0040[Linguistics].

[24] Anindya Kumar Biswas, ”Swahili, a lingua franca, Swahili-English Dictionary by C. W. Rechenbach and the Graphical law”, viXra: 2108.0101[Linguistics].

[25] Anindya Kumar Biswas, ”The French, Larousse Dictionnaire De Poche and the Graphical law”, viXra: 2109.0080[Linguistics].

[26] Anindya Kumar Biswas, "An Arabic dictionary: "al-Mujam al-wáfi" or, "adhunik arabi-bangla abhidhan" and the Onsager's solution", viXra: 2109.0119[Condensed Matter].

[27] Anindya Kumar Biswas, "Langenscheidt Taschenwörterbuch Deutsch-Englisch / Englisch-Deutsch, Völlige Neubearbeitung and the Graphical law", viXra: 2109.0141[Linguistics].

[28] Anindya Kumar Biswas, Bawansuk Lyngkhoi, "The Graphical law behind the NTC's Hebrew and English Dictionary by Arie Comey and Naomi Tsur", viXra: 2109.0164[Linguistics].

[29] Anindya Kumar Biswas, "Oxford Dictionary Of Media and Communication and the Graphical law", viXra: 2109.0202[Social Science].

[30] Anindya Kumar Biswas, "Oxford Concise Dictionary Of Mathematics, Penguin Dictionary Of Mathematics and the Graphical law", viXra: 2112.0054[Social Science].

[31] Anindya Kumar Biswas, "An Arabic dictionary: "al-Mujam al-wáfi" or, "adhunik arabi-bangla abhidhan" and the Onsager's solution Second part", viXra: 2201.0021[Condensed Matter].

[32] Anindya Kumar Biswas, "The Penguin Dictionary Of Sociology and the Graphical law", viXra: 2201.0046[Social Science].

[33] Anindya Kumar Biswas, "The Concise Oxford Dictionary Of Politics and the Graphical law", viXra: 2201.0069[Social Science].

[34] Anindya Kumar Biswas, "A Dictionary Of Critical Theory by Ian Buchanan and the Graphical law", viXra: 2201.0136[Social Science].

[35] Anindya Kumar Biswas, "The Penguin Dictionary Of Economics and the Graphical law", viXra: 2201.0169[Economics and Finance].

[36] Anindya Kumar Biswas, "The Concise Gojri-English Dictionary by Dr. Rafeeq Anjum and the Graphical law", viXra: 2201.0205[Linguistics].

[37] Anindya Kumar Biswas, "A Dictionary of the Kachin Language by Rev.O.Hanson and the Graphical law" ("A Dictionary of the Kachin Language by Rev.o.Hanson and the Graphical law", viXra: 2202.0030[Linguistics]).

[38] Anindya Kumar Biswas, "A Dictionary Of World History by Edmund Wright and the Graphical law", viXra: 2202.0130[History and Philosophy of Physics].

[39] Anindya Kumar Biswas, "Ekagi-Dutch-English-Indonesian Dictionary by J. Steltenpool and the Onsager's solution", viXra: 2202.0157[Condensed Matter].

[40] Anindya Kumar Biswas, "A Dictionary of Plant Sciences by Michael Allaby and the Graphical law", viXra: 2203.0011[Mind Science].

[41] Anindya Kumar Biswas, "Along the side of the Onsager's solution, the Ekagi language", viXra: 2205.0065[Condensed Matter].

[42] Anindya Kumar Biswas, "Along the side of the Onsager's solution, the Ekagi language-Part Three", viXra: 2205.0137[Condensed Matter].

[43] Anindya Kumar Biswas, "Oxford Dictionary of Biology by Robert S. Hine and the Graphical law", viXra: 2207.0089[Phyiscs of Biology].

[44] Anindya Kumar Biswas, "A Dictionary of the Mikir Language by G. D. Walker and the Graphical law", viXra: 2207.0165[Linguistics].

[45] Anindya Kumar Biswas, "A Dictionary of Zoology by Michael Allaby and the Graphical law", viXra: 2208.0075[Phyiscs of Biology].

[46] Anindya Kumar Biswas, "Dictionary of all Scriptures and Myths by G. A. Gaskell and the Graphical law", viXra: 2208.0093[Religion and Spiritualism].

[47] Anindya Kumar Biswas, "Dictionary of Culinary Terms by Philippe Pilibossian and the Graphical law", viXra: 2211.0061[Social Sciences].

[48] Anindya Kumar Biswas, "A Greek and English Lexicon by H.G.Liddle et al simplified by Didier Fontaine and the Graphical law", viXra: 2211.0087[Linguistics].

[49] Anindya Kumar Biswas, "Learner's Mongol-English Dictionary and the Graphical law", viXra: 2211.0101[Linguistics].

[50] Anindya Kumar Biswas, "Complete Bulgarian-English Dictionary and the Graphical law", viXra: 2212.0009[Linguistics].

[51] Anindya Kumar Biswas, "A Dictionary of Sindhi Literature by Dr. Motilal Jotwani and the Graphical Law", viXra: 2212.0015[Social Sciences].

[52] Anindya Kumar Biswas, "Penguin Dictionary of Physics, the Fourth Edition, by John Cullerne, and the Graphical law", viXra: 2212.0072[History and Philosophy of Physics].

[53] Anindya Kumar Biswas, "Oxford Dictionary of Chemistry, the seventh edition and the Graphical Law", viXra: 2212.0113[Chemistry].

[54] Anindya Kumar Biswas, "A Burmese-English Dictionary, Part I-Part V, by J. A. Stewart and C. W. Dunn et al, head words and the Graphical Law", viXra: 2212.0127[Linguistics].

[55] Anindya Kumar Biswas, "The Graphical Law behind the head words of Dictionary Kannada and English written by W. Reeve, revised, corrected and enlarged by Daniel Sanderson", viXra: 2212.0185[Linguistics].

[56] Anindya Kumar Biswas, ”Sanchayita and the Graphical Law”, viXra: 2301.0075[Social Science].

[57] Anindya Kumar Biswas, ”Samsad Bangla Abhidan and The Graphical Law”, viXra: 2302.0026[Linguistics].

[58] Anindya Kumar Biswas, ”Bangiya Sabdakosh and The Graphical Law”, viXra: 2302.0060[Linguistics].

[59] Anindya Kumar Biswas, ”Samsad Bengali-English Dictionary and The Graphical Law ”, viXra: 2304.0047[Linguistics].

[60] Anindya Kumar Biswas, ”Rudyard Kipling’s Verse and the Graphical Law”, viXra: 2304.0207[Social Science].

[61] Anindya Kumar Biswas, ”W. B. Yeats, The Poems and the Graphical Law”, viXra: 2305.0008[Social Science].

[62] Anindya Kumar Biswas, ”The Penguin Encyclopedia of Places by W. G. Moore and the Graphical Law”, viXra: 2305.0147[Archaeology].

[63] Anindya Kumar Biswas, ”The Poems of Tennyson and the Graphical Law”, viXra: 2305.0157[Social Science].

[64] Anindya Kumar Biswas, ”Khasi-Jaintia Jaids(Surnames) and the Graphical law”, viXra:2307.0135[Social Science].

[65] Anindya Kumar Biswas, ”Age, Amplitude of accommodation and the Graphical law”, viXra:2311.0110[Physics of Biology].

[66] Anindya Kumar Biswas, ”Dictionary of Ayurveda by Dr. Ravindra Sharma and the Graphical law”, viXra:2401.0030[General Science and Philosophy].

[67] Anindya Kumar Biswas, ”The Practical Sanskrit-English Dictionary by Vaman Shivram Apte and The Graphical Law”, viXra:2402.0041[Linguistics].

[68] Anindya Kumar Biswas, ”The Langenscheidt’s Pocket Russian Dictionary and The Graphical Law”, viXra:2402.0049[Linguistics]

[69] Anindya Kumar Biswas, ”The Scholar Dictionary Portuguese and The Graphical Law”, viXra:2402.0044[Linguistics]

[70] Anindya Kumar Biswas, ”The Langenscheidt’s Pocket Japanese Dictionary and the Onsager’s solution”, viXra:2402.0052[Condensed Matter]

[71] Anindya Kumar Biswas, "Langenscheidt's Pocket Chinese Dictionary and The Graphical Law", viXra:2403.0066[Linguistics]

[72] Anindya Kumar Biswas, "Oxford Hindi-English Dictionary and The Graphical Law", viXra:2403.0129[Linguistics]

[73] Anindya Kumar Biswas, "Concise Urdu to English Dictionary and The Graphical Law", viXra:2404.0006[Linguistics]

[74] Anindya Kumar Biswas, "The Standard Urdu-English Dictionary by Abdul Haq and The Graphical Law", viXra:2404.0034[Linguistics]

[75] Anindya Kumar Biswas, "The Urdu-Hindi Shabdakosh by Muhammad Sajjad Osmani, Sudhindra Kumar and The Graphical Law", viXra:2404.0114[Linguistics]

[76] A. M. Gun, M. K. Gupta and B. Dasgupta, Fundamentals of Statistics Vol 1, Chapter 12, eighth edition, 2012, The World Press Private Limited, Kolkata.

[77] E. Ising, Z.Physik 31,253(1925).

[78] R. K. Pathria, Statistical Mechanics, p400-403, 1993 reprint, Pergamon Press,© 1972 R. K. Pathria.

[79] C. Kittel, Introduction to Solid State Physics, p. 438, Fifth edition, thirteenth Wiley Eastern Reprint, May 1994, Wiley Eastern Limited, New Delhi, India.

[80] W. L. Bragg and E. J. Williams, Proc. Roy. Soc. A, vol.145, p. 699(1934);

[81] P. M. Chaikin and T. C. Lubensky, Principles of Condensed Matter Physics, p. 148, first edition, Cambridge University Press India Pvt. Ltd, New Delhi.

[82] Kerson Huang, Statistical Mechanics, second edition, John Wiley and Sons(Asia) Pte Ltd.

[83] Sonntag, Borgnakke and Van Wylen, Fundamentals of Thermodynamics, p206-207, fifth edition, John Wiley and Sons Inc.

[84] https://pdg.ibl.gov