A Modified Born-Infeld Model of Electrons Featuring a Lorentz-Type Force

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Abstract

This work analyzes the interaction of a rotating field solution of a modified Born-Infeld model of electrons with a weak, low-frequency electromagnetic field. The interaction is shown to be of the same type as the Lorentz force on a charged particle in an electromagnetic field.

1 Introduction

Born and Infeld showed that their electrostatic model of electrons behaves like a charged, relativistic particle in the presence of an external electromagnetic field [BI34b], i.e., an external field results in a Lorentz-type force. Recently, a modification of this model was presented that also represents the magnetic moment and Compton frequency of electrons [Kra23a]. However, the interaction of this modified model with an external electromagnetic field remained unclear.

In this work, this interaction is analyzed and shown to be of the same form as a Lorentz-type force. The analysis employs a Lagrangian formulation and was inspired by Born [Bor34] instead of the Hamiltonian formulation employed by Born and Infeld [BI34b], who argued that the use of the Lagrangian formulation in the former work by Born was incorrect. The case considered in this work, however, is different, and the existence of a well-behaved numerical solution [Kra23a] allows us to be confident that the employed approximations are justified. It should be noted that the presentation often lacks mathematical rigor, but readers might find the approach and result of interest nonetheless.

The Lagrangian density of the modified field theory and the modified model of electrons are presented in Section 2. Section 3 outlines the approach employed in Section 4 and illustrates it with an electrostatic example. Section 4 actually analyzes the effect of an external electromagnetic potential on the model, while Section 5 discusses the results, and Section 6 concludes this work.

2 Modified Born-Infeld Model of Electrons

As in previous work [Kra23a], the dimensionless Lagrangian density of the employed field theory is

$$\mathscr{L}(A^\nu) \stackrel{\text{def}}{=} \sqrt{1 - \frac{1}{b^2}(\partial^\mu A^\nu)(\partial_\mu A_\nu)} - 1 \tag{1}$$

using basic Ricci calculus, the Minkowski metric tensor η in the form $\text{diag}(+1, -1, -1, -1)$, the electromagnetic four-potential $(A^0, A^1, A^2, A^3) = (\phi/c, A_x, A_y, A_z)$, and the Born-Infeld parameter b specifying the maximum magnetic field strength. Note that this Lagrangian density has to be multiplied by $b^2/\mu_0 = b^2 c^2 \varepsilon_0$ to convert it to an energy density (in SI units with the vacuum permeability μ_0, the vacuum permittivity ε_0, and the speed of light c).

The electromagnetic four-potential is also used to define electric field strength $\mathbf{E}$, magnetic field strength $\mathbf{B}$, and four-current density $(J^0, J^1, J^2, J^3) = (c\rho, J_x, J_y, J_z)$:

$$\mathbf{E} \stackrel{\text{def}}{=} -\nabla\phi - \frac{\partial}{\partial t}\mathbf{A} \tag{2}$$

$$\mathbf{B} \stackrel{\text{def}}{=} \nabla \times \mathbf{A} \tag{3}$$

$$J^\nu \stackrel{\text{def}}{=} \frac{1}{\mu_0}\left(\partial_\mu \partial^\mu A^\nu - \partial_\mu \partial^\nu A^\mu\right) \tag{4}$$

While these definitions guarantee that all four Maxwell-Heaviside equations are fulfilled, the physics described by the Lagrangian density $\mathscr{L}(A^\nu)$ deviates from standard electromagnetism. In particular, it allows for field solutions with non-zero charge and current densities in the absence of any external charges. More details about the notation are provided in previous work [Kra23b].

For the employed model of electrons, the corresponding field equations were solved numerically in previous work [Kra23a]. Qualitatively speaking, the far field of the solution approximates the field of a point-like electron (same electric charge and magnetic moment), while the field near the peak is smoothly capped, and the field is dynamically "deformed" such that the peak is moving at the speed of light on a circular orbit with a radius equal to the electron's reduced Compton wavelength. Another way to describe the same periodic movement is that the whole solution is rotating around the center of the far field with a rotation frequency equal to the electron's Compton frequency. While all these features (electric charge, magnetic moment, radius of orbit, rotation frequency) were imposed on the solution, the field energy of the solution was matched to the rest mass energy of an electron by adjusting the Born-Infeld parameter.

3 Method of Analysis

As mentioned, this work was inspired by a method of Born [Bor34], which was criticized by Born and Infeld because "coefficients [...] appear, which become infinite at the centre of the electron. Therefore the transformation of the space integral is not allowed." [BI34b, page 449]. Further criticism was raised by Frenkel [Fre34] and discussed by Born and Infeld [BI34a].

In the modified field theory discussed in this work, however, the field variables are finite coefficients of the electromagnetic four-potential instead of the electric and magnetic field strength. Furthermore, due to one of the modifications in this field theory [Kra23c], it is the magnetic field strength (instead of the electric field strength) that limits both, electric and magnetic field strengths. This results in a much better behaved field at the peak of the numerical solution [Kra23a] compared to the electrostatic electron model by Born and Infeld [BI34b]. All this is reason to assume that the proposed method of analysis is justified for sufficiently small external electromagnetic fields.

In order to provide an overview of the method, the rest of this section applies it to a particularly basic problem, namely the electrostatic problem of determining the electric potential inside a finite volume V without any electric charges using the Lagrangian density

$$\mathscr{L}^{(\mathrm{s})}(\phi) \stackrel{\mathrm{def}}{=} \frac{\varepsilon_0}{2}(\nabla\phi)^2. \tag{5}$$

For the boundary condition $\phi \stackrel{!}{=} 0$ on the boundary of V (and suitable boundary conditions on $\nabla\phi$), the only solution is $\phi = 0$ everywhere inside volume V.

For the slightly more general boundary condition $\phi \stackrel{!}{=} \phi^{(\mathrm{e})}$ with a constant value $\phi^{(\mathrm{e})}$ of an external potential, the only solution is $\phi = \phi^{(\mathrm{e})}$ inside V. However, readers are encouraged to imagine that the explicit solution was unknown. Without knowing the explicit solution, the effect of adding the external potential $\phi^{(\mathrm{e})}$ remains unclear.

However, the effect may be analyzed by constructing an alternative Lagrangian density. To this end, instead of solving the boundary problem $\phi \stackrel{!}{=} \phi^{(\mathrm{e})}$ for the Lagrangian density $\mathscr{L}^{(\mathrm{s})}(\phi)$, we consider an equivalent problem with the Lagrangian density

$$\mathscr{L}^{(\mathrm{s})}\left(\phi-\phi^{(\mathrm{e})}\right) = \frac{\varepsilon_0}{2}\left(\nabla\left(\phi-\phi^{(\mathrm{e})}\right)\right)^2 = \underbrace{\frac{\varepsilon_0}{2}(\nabla\phi)^2}_{\mathscr{L}^{(\mathrm{s})}(\phi)} \underbrace{-\frac{2\varepsilon_0}{2}\nabla\phi\cdot\nabla\phi^{(\mathrm{e})}}_{\mathscr{L}^{(\mathrm{sint})}\left(\phi,\phi^{(\mathrm{e})}\right)} + \underbrace{\frac{\varepsilon_0}{2}\left(\nabla\phi^{(\mathrm{e})}\right)^2}_{\mathscr{L}^{(\mathrm{s})}\left(\phi^{(\mathrm{e})}\right)} \tag{6}$$

with the boundary condition $\phi - \phi^{(\mathrm{e})} \stackrel{!}{=} 0$, where $\phi^{(\mathrm{e})}$ is considered a small perturbation of ϕ (such that the solution for ϕ is close to the solution of the boundary problem $\phi \stackrel{!}{=} 0$). For the constant $\phi^{(\mathrm{e})}$ of this example, the gradient $\nabla\phi^{(\mathrm{e})}$ is $\mathbf{0}$, but we keep it nonetheless, because the corresponding term in Section 4 is not $\mathbf{0}$. As a side note, the subtraction in $\mathscr{L}^{(\mathrm{s})}\left(\phi-\phi^{(\mathrm{e})}\right)$ is important and cannot be replaced by an addition without changing the result below. Using a subtraction is consistent with standard conventions, because electrostatic fields depend on *differences* of electric potentials.

The new form of the Lagrangian density suggests that the effect of adding a small, external potential $\phi^{(\mathrm{e})}$ may be approximated by adding the interaction term $\mathscr{L}^{(\mathrm{sint})}\left(\phi, \phi^{(\mathrm{e})}\right)$ to the Lagrangian density $\mathscr{L}^{(\mathrm{s})}(\phi)$ provided that the term $\mathscr{L}^{(\mathrm{s})}\left(\phi^{(\mathrm{e})}\right)$ is small enough to be neglected.

In order to further simplify the interaction term, note that all Lagrangian densities have to be integrated over a volume to form Lagrangian functions. The volume integral of the interaction term over volume V may be simplified using integration by parts and the assumption that $\phi^{(\mathrm{e})}$ is constant in V (at least approximately):

$$\int_V \mathscr{L}^{(\mathrm{sint})}\left(\phi, \phi^{(\mathrm{e})}\right) \mathrm{d}V = -\int_V \varepsilon_0 \nabla\phi \cdot \nabla\phi^{(\mathrm{e})} \mathrm{d}V = -\int_{\partial V} \varepsilon_0 \phi^{(\mathrm{e})} \nabla\phi \cdot \mathrm{d}\mathbf{n} + \int_V \varepsilon_0 \phi^{(\mathrm{e})} \nabla^2\phi \, \mathrm{d}V \tag{7}$$

The surface integral $-\int_{\partial V} \varepsilon_0 \phi^{(\mathrm{e})} \nabla\phi \cdot \mathrm{d}\mathbf{n}$ over the boundary of V may be ignored when this Lagrangian function is used in a variational principle since field values on the boundary of V are determined by boundary conditions. (See also the explanation by Born and Infeld [BI34b, page 449].) The remaining term may be simplified by introducing the charge density $\rho = -\varepsilon_0 \nabla^2 \phi$:

$$\int_V \varepsilon_0 \phi^{(\mathrm{e})} \nabla^2\phi \, \mathrm{d}V = -\int_V \phi^{(\mathrm{e})} \rho \, \mathrm{d}V = -\phi^{(\mathrm{e})} q \tag{8}$$

with the total electric charge $q = \int_V \rho \, \mathrm{d}V$, which is 0 in this example, but not in Section 4. Thus, the interaction between the field solution and an external potential $\phi^{(\mathrm{e})}$ is described by the term $-\phi^{(\mathrm{e})} q$ in a Lagrangian function, which is the negative potential energy of a static electric charge in a static electric potential as expected. In the case discussed in Section 4, the corresponding term turns out to be of the same type as the minimal-coupling interaction term for the Lorentz force on a charged particle in an external electromagnetic field.

This completes the overview and introductory example of the method of analysis that is applied to $\mathscr{L}(A^\nu)$ in Section 4. Note that this introductory example of $\mathscr{L}^{(\mathrm{s})}(\phi)$ is actually a special case of $\mathscr{L}(A^\nu)$ for $\mathbf{A} = \mathbf{0}$, constant ϕ, and small $\nabla\phi$:

$$\mathscr{L}(A^\nu)\Big|_{\mathbf{A}=\mathbf{0},\phi=\mathrm{const}} = \sqrt{1 - \frac{1}{b^2}(-\nabla\phi/c)(\nabla\phi/c)} - 1 = \sqrt{1 + \frac{1}{b^2c^2}(\nabla\phi)^2} - 1 \approx \frac{1}{2b^2c^2}(\nabla\phi)^2, \tag{9}$$

which is the same as $\mathscr{L}^{(\mathrm{s})}(\phi)$ after multiplication with $b^2c^2\varepsilon_0$.

4 Effect of External Potential

This section applies the method introduced in the previous section to the dimensionless Lagrangian density

$$\mathscr{L}(A^\nu) \overset{\mathrm{def}}{=} \sqrt{1 - \frac{1}{b^2}(\partial^\mu A^\nu)(\partial_\mu A_\nu)} - 1. \tag{10}$$

The objective of this section is to analyze the approximate effect of a small, low-frequency external potential $A^{(\mathrm{e})\nu}$ on a solution A^ν that is rotating around the center of a sufficiently large spherical volume V as described in previous work [Kra23a]. To this end, the boundary condition on ∂V is changed from $A^\nu \overset{!}{=} 0$ to $A^\nu - A^{(\mathrm{e})\nu} \overset{!}{=} 0$ and the Lagrangian density $\mathscr{L}(A^\nu - A^{(\mathrm{e})\nu})$ inside V is approximated using a Taylor expansion as follows:

$$\begin{aligned}
\mathscr{L}(A^\nu - A^{(\mathrm{e})\nu}) &= \sqrt{1 - \frac{1}{b^2}(\partial^\mu(A^\nu - A^{(\mathrm{e})\nu}))(\partial_\mu(A_\nu - A^{(\mathrm{e})}_\nu))} - 1 &(11)\\
&\approx 1 - \frac{1}{2b^2}(\partial^\mu(A^\nu - A^{(\mathrm{e})\nu}))(\partial_\mu(A_\nu - A^{(\mathrm{e})}_\nu)) - 1 &(12)\\
&= \underbrace{\underbrace{-\frac{1}{2b^2}(\partial^\mu A^\nu)(\partial_\mu A_\nu)}_{\approx\sqrt{1-\frac{1}{b^2}(\partial^\mu A^\nu)(\partial_\mu A_\nu)}-1}}_{=\mathscr{L}(A^\nu)} + \underbrace{\frac{1}{b^2}(\partial^\mu A^\nu)(\partial_\mu A^{(\mathrm{e})}_\nu)}_{=\mathscr{L}^{(\mathrm{int})}(A^\nu, A^{(\mathrm{e})\nu})} \underbrace{\underbrace{-\frac{1}{2b^2}(\partial^\mu A^{(\mathrm{e})\nu})(\partial_\mu A^{(\mathrm{e})}_\nu)}_{\approx\sqrt{1-\frac{1}{b^2}(\partial^\mu A^{(\mathrm{e})\nu})(\partial_\mu A^{(\mathrm{e})}_\nu)}-1}}_{=\mathscr{L}(A^{(\mathrm{e})\nu})} &(13)
\end{aligned}$$

The term $\mathscr{L}(A^\nu)$ is just the Lagrangian density for the case without external potential. The term $\mathscr{L}(A^{(e)\nu})$ is assumed to be negligible for a small, low-frequency $A^{(e)\nu}$, and it does not contribute when this Lagrangian density is used in a variational principle to determine a solution for A^ν.

This leaves the interaction term $\mathscr{L}^{(\mathrm{int})}(A^\nu, A^{(e)\nu})$. In order to further manipulate this Lagrangian density, we integrate it over V, which is a necessary step to compute the corresponding Lagrangian function. Moreover, we first consider only the four terms with $\nu = 0$:

$$\int_V \frac{1}{b^2}(\partial^\mu A^0)(\partial_\mu A_0^{(e)})\mathrm{d}V = \int_V \frac{1}{b^2}\left(\frac{1}{c^2}\frac{\partial}{\partial t}\phi\right)\left(\frac{1}{c^2}\frac{\partial}{\partial t}\phi^{(e)}\right)\mathrm{d}V - \int_V \frac{1}{b^2}\left(\frac{1}{c}\nabla\phi\right)\left(\frac{1}{c}\nabla\phi^{(e)}\right)\mathrm{d}V \tag{14}$$

The term $\partial\phi^{(e)}/\partial t$ is assumed approximately constant in V for the small, low-frequency potential $A^{(e)\nu}$, thus, it may be pulled out of the integral over V:

$$\int_V \frac{1}{b^2}\left(\frac{1}{c^2}\frac{\partial}{\partial t}\phi\right)\left(\frac{1}{c^2}\frac{\partial}{\partial t}\phi^{(e)}\right)\mathrm{d}V \approx \frac{1}{b^2c^4}\left(\frac{\partial}{\partial t}\phi^{(e)}\right)\int_V\left(\frac{\partial}{\partial t}\phi\right)\mathrm{d}V = \frac{1}{b^2c^4}\left(\frac{\partial}{\partial t}\phi^{(e)}\right)\frac{\partial}{\partial t}\int_V \phi\,\mathrm{d}V \tag{15}$$

The solution for ϕ is approximately rotating around the center of V, thus, the integral $\int_V \phi\,\mathrm{d}V$ is approximately constant and the time derivative is approximately 0. Therefore, the whole term is approximately 0.

For the last integral in Eq. (14), we use integration by parts and discard the surface term because it does not contribute in a variational principle since ϕ on the surface of V is determined by boundary conditions. (See also the explanation by Born and Infeld [BI34b, page 449].) The remaining integral becomes:

$$\int_V \frac{1}{b^2}\left(\frac{1}{c}\nabla^2\phi\right)\left(\frac{1}{c}\phi^{(e)}\right)\mathrm{d}V \approx \frac{1}{b^2c^2}\phi^{(e)}\int_V \nabla^2\phi\,\mathrm{d}V \tag{16}$$

under the assumption that $\phi^{(e)}$ is approximately constant in V. $\int_V \nabla^2\phi\,\mathrm{d}V$ is just $-\int_V \rho/\varepsilon_0\mathrm{d}V$ because

$$\int_V \rho\,\mathrm{d}V = \int_V \frac{1}{c}J^0\,\mathrm{d}V = \frac{1}{c\mu_0}\int_V\left(\partial_\mu\partial^\mu A^0 - \partial_\mu\partial^0 A^\mu\right)\mathrm{d}V = -\varepsilon_0\int_V \nabla^2\phi\,\mathrm{d}V - \varepsilon_0\int_V \nabla\cdot\frac{\partial}{\partial t}\mathbf{A}\mathrm{d}V \tag{17}$$

and the last integral is approximately 0 since $\int_V \nabla\cdot\mathbf{A}\mathrm{d}V$ is approximately constant as the solution for $\mathbf{A}$ is approximately rotating around the center of V. Thus, the $\nu = 0$ part of the integrated interaction term becomes:

$$-\frac{1}{b^2c^2\varepsilon_0}\phi^{(e)}\int_V \rho\,\mathrm{d}V = -\frac{1}{b^2c^2\varepsilon_0}\phi^{(e)}q \tag{18}$$

with the total electric charge q in V of the solution A^ν. Since we started with a dimensionless Lagrangian density, this result has to be multiplied with $b^2c^2\varepsilon_0$ to obtain an energy.

This is the result for the sum of terms in $1/b^2\int_V(\partial^\mu A^\nu)(\partial_\mu A_\nu^{(e)})\mathrm{d}V$ with $\nu = 0$. For the remaining terms, we consider groups of terms with ν being either 1, 2, or 3, i.e., there is no summation over ν in the rest of this section.

$$\frac{-1}{b^2}\int_V(\partial^\mu A^\nu)(\partial_\mu A^{(e)\nu})\mathrm{d}V = \frac{-1}{b^2c^2}\int_V\left(\frac{\partial}{\partial t}A^\nu\right)\left(\frac{\partial}{\partial t}A^{(e)\nu}\right)\mathrm{d}V + \frac{1}{b^2}\int_V(\nabla A^\nu)\cdot(\nabla A^{(e)\nu})\mathrm{d}V \tag{19}$$

In the first integral on the right-hand-side, the term $\partial A^{(e)\nu}/\partial t$ is approximately constant for a weak, low-frequency external potential, thus, it may be pulled out of the integral:

$$\frac{-1}{b^2c^2}\int_V\left(\frac{\partial}{\partial t}A^\nu\right)\left(\frac{\partial}{\partial t}A^{(e)\nu}\right)\mathrm{d}V \approx \frac{-1}{b^2c^2}\frac{\partial A^{(e)\nu}}{\partial t}\int_V\left(\frac{\partial}{\partial t}A^\nu\right)\mathrm{d}V = \frac{-1}{b^2c^2}\frac{\partial A^{(e)\nu}}{\partial t}\frac{\partial}{\partial t}\int_V A^\nu\mathrm{d}V \tag{20}$$

As in the case of ϕ above, the solution for A^ν is approximately rotating around the center of V, thus, the integral $\int_V A^\nu\mathrm{d}V$ is approximately constant and the time derivative is approximately 0. Therefore, the whole term is approximately 0.

For the last integral in Eq. (19), we again use integration by parts and discard the surface term because it does not contribute in a variational principle. The remaining integral becomes:

$$-\frac{1}{b^2}\int_V(\nabla^2A^\nu)(A^{(e)\nu})\mathrm{d}V \approx -\frac{1}{b^2}A^{(e)\nu}\int_V \nabla^2A^\nu\mathrm{d}V \tag{21}$$

under the assumption that $A^{(\mathrm{e})\nu}$ is approximately constant in V. $\int_V \nabla^2 A^\nu \mathrm{d}V$ is $-\mu_0 \int_V J^\nu \mathrm{d}V$ because

$$\int_V J^\nu \mathrm{d}V = \frac{1}{\mu_0}\int_V (\partial_\mu\partial^\mu A^\nu - \partial_\mu\partial^\nu A^\mu)\,\mathrm{d}V = \frac{-1}{\mu_0}\int_V \nabla^2 A^\nu \mathrm{d}V + \frac{1}{\mu_0 c^2}\int_V \frac{\partial^2 A^\nu}{\partial t^2}\mathrm{d}V - \int_V \frac{\partial_\mu\partial^\nu A^\mu}{\mu_0}\mathrm{d}V \tag{22}$$

The integral $\int_V A^\nu \mathrm{d}V$ is approximately constant over time due to the approximately rotational movement of the solution A^ν; thus, the time derivative is approximately 0. Analogously, the term for $\mu = 0$ in $-\int_V \partial_\mu\partial^\nu A^\mu \mathrm{d}V/\mu_0$ is approximately 0. The remaining terms of the latter integral (for $\mu = 1$, 2, and 3) form a derivative of a volume integral over a divergence, namely $-\partial^\nu \int_V \nabla\cdot\mathbf{A}\mathrm{d}V/\mu_0$, which is approximately 0 due to Gauss's theorem and the solution $\mathbf{A}$ being approximately tangential on the spherical boundary of V. Thus, $\int_V J^\nu \mathrm{d}V = -\int_V \nabla^2 A^\nu \mathrm{d}V/\mu_0$ and, therefore, $\int_V \nabla^2 A^\nu \mathrm{d}V = -\mu_0 \int_V J^\nu \mathrm{d}V$.

Thus, the remaining interaction term from Eq. (21) becomes (for $\nu = 1$, 2, or 3):

$$-\frac{1}{b^2}A^{(\mathrm{e})\nu}\int_V \nabla^2 A^\nu \mathrm{d}V \approx \frac{\mu_0}{b^2}A^{(\mathrm{e})\nu}\int_V J^\nu \mathrm{d}V = \frac{1}{b^2 c^2 \varepsilon_0}A^{(\mathrm{e})\nu}\int_V J^\nu \mathrm{d}V \tag{23}$$

Together with the result in Eq. (18) (for $\nu = 0$), the complete interaction term of a Lagrangian function based on the dimensionless Lagrangian density $\mathscr{L}(A^\nu)$ is (with implicit summation over μ):

$$-\frac{1}{b^2 c^2 \varepsilon_0}A^{(\mathrm{e})}_\mu \int_V J^\mu \mathrm{d}V \approx \frac{1}{b^2 c^2 \varepsilon_0}\left(-q\phi^{(\mathrm{e})} + q\mathbf{A}^{(\mathrm{e})}\cdot\mathbf{v}\right) \tag{24}$$

which has to be multiplied by $b^2 c^2 \varepsilon_0$ to convert it to an energy. Analogously to the original Born-Infeld model [BI34b], this is the interaction term of a particle with electric charge q moving with velocity $\mathbf{v}$ in external potentials $\phi^{(\mathrm{e})}$ and $\mathbf{A}^{(\mathrm{e})}$; i.e., it describes a Lorentz-type force.

5 Discussion

While this work successfully derives the interaction term of a Lorentz-type force, it does so without mathematical rigor and only in the minimal coupling approximation; thus, interaction terms including the magnetic moment are ignored. Since the model includes a magnetic moment [Kra23a], it may be hypothesized that it interacts with an external electromagnetic field like a particle with spin, but a proof of this is left for future work.

Many other effects are also ignored; for example, the interaction of the peak of the model's field solution with the rest of the field. In previous work [Kra23c], I hypothesized that the peak is forced onto a circular orbit by a Lorentz-like force based on the field close to the peak. This hypothesis is neither disproved nor confirmed here.

6 Conclusion

This work applies an approach by Born and Infeld [Bor34, BI34b] to analyze the interaction of a modified Born-Infeld model of electrons [Kra23a] with an external electromagnetic field and finds that the minimal-coupling interaction is of the same type as the Lorentz force. Thus, at least one of the open questions in previous work [Kra23b] appears to be closer to an answer.

References

[BI34a] Max Born and L. Infeld. Born's theory of the electron - remarks on the paper by Frenkel on Born's theory of the electron. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 146(859):935–935, 1934.

[BI34b] Max Born and Leopold Infeld. Foundations of the new field theory. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 144(852):425–451, 1934.

[Bor34] Max Born. On the quantum theory of the electromagnetic field. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 143(849):410–437, 1934.

[Fre34] J. Frenkel. On Born's theory of the electron. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 146(859):930–934, 1934.

[Kra23a] Martin Kraus. A modified Born-Infeld model of electrons and a numerical solution procedure. viXra:2312.0149, 2023.

[Kra23b] Martin Kraus. A modified Born-Infeld model of electrons as rotating waves. viXra:2304.0046, 2023.

[Kra23c] Martin Kraus. A modified Born-Infeld model of electrons with realistic magnetic dipole moment. viXra:2311.0081, 2023.

A Revisions

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