

Four-Variable Jacobian Conjecture in a Topological Quantum Model of Intersecting Fields

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Abstract

This paper introduces a topological model based on two intersecting fields varying either same or opposite phase that may graphically illustrate the Jacobian conjecture for four variables. It also suggests connections with other mathematical topics as Gorenstein Liaison, Tomita-Takesaki modular theory, the Mass gap problem, Reflection positivity, or T-duality in string theory, all considered within the same framework.

Being applicable to the Jacobian conjecture, the model would represent a confirmation case in both the symmetric and antisymmetric systems. However, in the symmetric system the mirror reflection property would not be considered by the conjecture when it comes to the vertical subfields.

The model has been developed in the context of the physics beyond the Standard theory as an unconventional atomic system whose nucleus is formed by mirror matter and antimatter. It is generally described in the last section.

1 Antisymmetric system

We start by considering an antisymmetric system of two intersecting fields that vary with opposite phase, when the right field contracts the left expands and vice versa. In their intersection they form two transverse and two vertical subfields. The transverse sub-

fields are mirror antisymmetric, when the right subfield expands the left contract and vice versa.

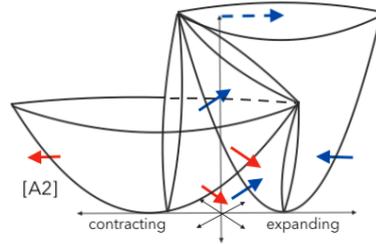


Figure 1: *Antisymmetric system: at moment A2, the right transverse subfield contracts and the left transverse subfield expands; the vertical subfield moves rightward.A4*

The curvature of the transverse subfields is half positive and half negative, and they are determined by the forces of pressure caused by the inward displacement of the negative curvature of the contracting field determined, and by the outward displacement of the positive curvature of the expanding field.

Those forces of pressure are represented by four eigenvectors with eigenvalue 1 or -1 .

An actual inversion equivalent to a 180 degrees rotation is operated when the right-hand contracting field expands and the left expanding field contracts. Then, the right contracting subfield at A2 is mapped to the left contracting subfield at A4, and the left expanding subfield at A2 is mapped to the right expanding subfield of A4.

In that way, the left and right transversal subfields

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exhibit chiral mirror symmetry at different times. This is because half of the system follows a purely imaginary time dimension, delayed with respect the real time dimension that follows the other half of the system.

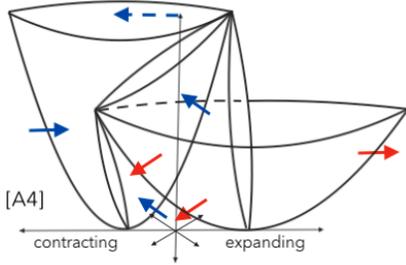


Figure 2: *Antisymmetric system: at moment A4, the right transverse subfield expands and the left transverse subfield contracts; the vertical subfield moves leftward.*A4

However, both left and right transverse subfields are described by the same spatial dimensions. Those dimensions cannot be the same that are used to describe the intersecting fields, because the Y coordinate one of the transversal subfields will be considered a diagonal axis from the point of view of the coordinates system of the intersecting fields. Misleading the coordinates would introduce a relativistic space-time elongated metric.

Each eigenvector has two possible directions, given by their positive or negative sign. In that sense, the antisymmetric system can be described by a complex conjugate function of two variables, and a pair of 2×2 complex matrices whose elements the mentioned eigenvectors:

$$A_2 \left\{ \begin{array}{l} \rightarrow \\ \leftarrow \end{array} \right\} \quad A_4 \left\{ \begin{array}{l} \leftarrow \\ \rightarrow \end{array} \right\}$$

Figure 3: *Pair of 2×2 complex conjugate matrices whose elements are eigenvectors with eigenvalue 1 or -1 , related to the antisymmetric system.*

2 Symmetric system

In the symmetric system the two intersecting field vary with the same phase, they simultaneously contract and, later, they will simultaneously expand.

The transversal subfields, being isomorphic, exhibit chiral mirror symmetry at the same time. That implies that performing the inversion of the system, which would be equivalent to a 180-degrees rotation, both subfields would be interchangeable, and so the right and left transverse subfields would be mapped to the left and right transverse subfields.

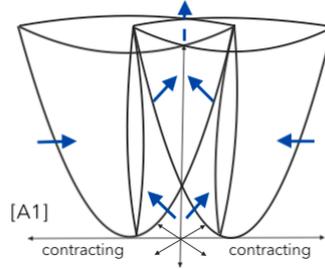


Figure 4: *Symmetric system: at moment A1, both left and right transversal subfields expand. The vertical subfield moves upward while contracting.*A4

However, considering the dynamics given by the periodic contraction and expansion of the intersecting fields, it can occur that when the contracting system with expanding transverse subfield is inverted, the curvatures of the intersecting fields have already change and so the transverse subfields are now contracting.

That would imply that the unrotated right expanding field would be mapped to the left rotated contracting subfield, and the left unrotated expanding subfield would be mapped to the rotated right expanding subfield.

The evolution of the symmetric system can be described by a complex function of two variables, as the four eigenvectors only can have two possible directions, and a pair of 2×2 complex matrices of eigenvectors with eigenvalue 1 or -1 :

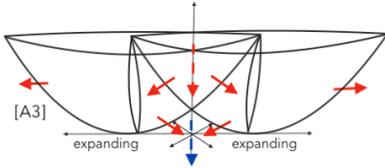


Figure 5: *Symmetric system: at moment A3, both left and right transversal subfields contract. The vertical subfield moves downward while expanding. A4*

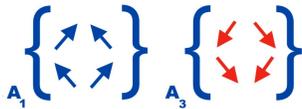


Figure 6: *Pair of 2×2 complex conjugate matrices of eigenvectors related to the antisymmetric system.*

3 Rotational system

So far, we have described the inversion that performs the mapping between the transverse subfields in terms of a symbolic 180-degree rotation, where the symmetric and antisymmetric systems are considered as separate and unrelated systems described by a complex and a complex conjugated function of two variables, respectively.

However, in the context of a system that physically rotates, the symmetric and the antisymmetric systems may turn to be a same system that is topologically transformed after each 90-degree rotation, becoming periodically symmetric or antisymmetric.

That implies that the evolution of the system must be described by two interpolated functions, the complex function, and its harmonic conjugate solution.

The evolution of the rotational system can be represented by a set of 2×2 complex rotational 2 matrices of eigenvectors with eigenvalue 1 or -1 : Taking A_2 as the identity matrix, the inversion operated at A_4 mapping A_2 implies a partial conjugation given by the 90-degree rotation of A_2 performed at A_3 . In that way, the two positive eigenvectors of A_2 change their sign at A_3 . This operation can be interpreted as a $(\frac{1}{2})$ order derivative.

It also implies the transposition of A_3 , given by

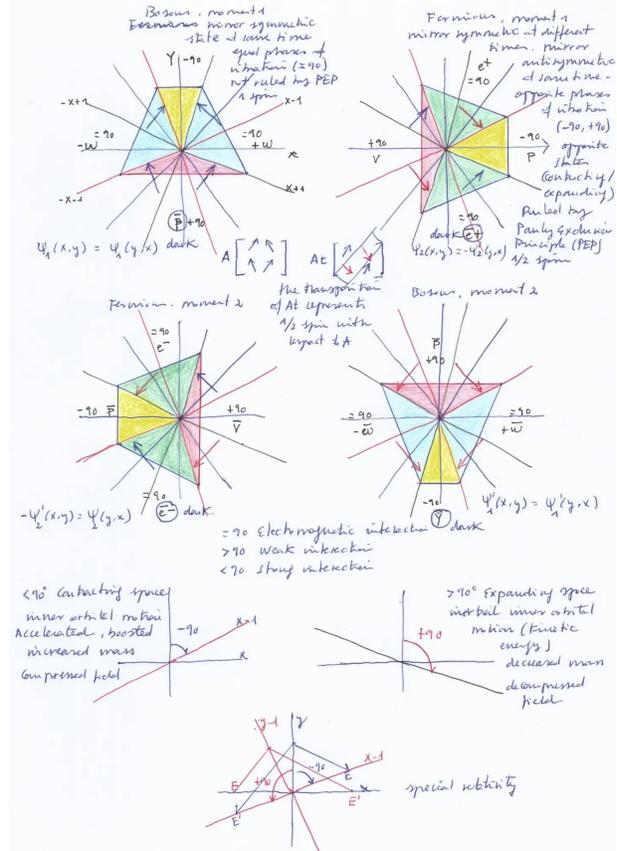


Figure 7: *Rotational interpolation of vector symmetric and antisymmetric system spaces represented in a 2D scheme*

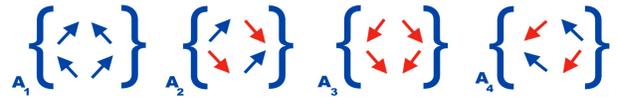


Figure 8: *Set of 2×2 rotational matrices related by pairs to the complex symmetric (A_1 and A_3) and the conjugate antisymmetric (A_2 and A_4) systems.*

the partial conjugation operated at $A4$. Two of the negative eigenvectors of $A3$ become positive at $A4$. Their sign commutation can be interpreted as a $(\frac{1}{2})$ order antiderivative. In that sense, the first order differentiation that operates the inversion of the antisymmetric system will involve a $(\frac{1}{2})$ derivative and a $(\frac{1}{2})$ antiderivative.

On the other hand, in the rotational system, the vectors do not simply change their sign by a 180-degrees permutation, they are physically rotated with the whole complex plane. Their change of spatial position can be observed if we identify them with letters.

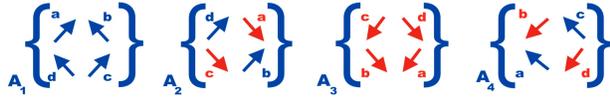


Figure 9: *Actual rotation of the vectors in the context of the rotational matrices, changing their position after each 90-degrees rotation.*

Each of the eigenvectors only can have two possible directions. But considering them as rotational vectors, they will have four possible directions. In that sense, the interpolating rotational system must be thought as a four variables polynomial.

Considering $A2$ as the starting point, and performing its inversion, from the point of view of the rotational vectors the number of variables would be three.

However, it does not seem possible to arbitrarily separate matrix $A2$ from matrix $A1$, as it is the transposition of matrix $A1$ by means of its partial conjugation what gives birth to matrix $A2$.

The inversion of $A2$ involves the transposition of $A3$, which is the negative reflection of $A1$. $A2$ is the first order $(\frac{1}{2} + \frac{1}{2})$ derivative of $A1$ and $A1$ the first order $(\frac{1}{2} + \frac{1}{2})$ antiderivative of $A3$. $A1$ and $A3$ are intertwined to $A2$ and $A4$, and vice versa, being both pairs interdependent.

The set of transformation matrices result from the operations of transposition, complex conjugation (as the sum of two partial complex conjugations), and inversion:

- $A1$ (0-degrees rotation) represents the eigenvectors in the symmetric system, when the transver-

sal subspaces have mirror symmetry at the same moment; performing its partial conjugation (rotating the plane 90-degrees) only two eigenvectors (acting as two variables) change their sign at $A2$.

- $A2$ (90-degrees rotation) represents the eigenvectors when half of the system has a delayed its phase, introducing a purely imaginary time dimension) having mirror antisymmetry with respect to the other half side. $A2$ represents the partial complex conjugation of $A1$ and its $\frac{1}{2}$ order derivative.
- $A3$ (180-degrees rotation) represents the partial conjugation of $A2$ (only the yet two uncommuted eigenvectors commute now their sign with respect to $A2$); $A3$ also represents the negative reflection of $A1$; its four eigenvectors (acting as four variables) have already commuted their sign with respect to $A1$; $A3$ represents the $\frac{1}{2}$ order derivative of $A2$, and the first order $(\frac{1}{2} + \frac{1}{2})$ derivative of $A1$.
- $A4$ (270-degrees rotation with respect to $A1$, 180-degrees with respect to $A2$, and 90-degrees with respect to $A3$) represents the transpose of $A3$, the $\frac{1}{2}$ order antiderivative of $A3$, the second transposition of $A1$, and the first order $(\frac{1}{2} + \frac{1}{2})$ derivative of $A2$; $A4$ is also the negative mirror reflection of $A2$, having commuted their sign the four eigenvectors.
- An additional 90-degrees rotation produces $A1$ which represents the positive reflection of $A3$, a $\frac{1}{2}$ order antiderivative of $A4$, and the first order $(\frac{1}{2} + \frac{1}{2})$ antiderivative of $A3$.

4 Jacobian conjecture

The Jacobian conjecture [1] formulated by Keller in 1939 states that if a polynomial map from an n -dimensional space to itself has Jacobian determinant which is a non-zero constant, then the function has a polynomial inverse.

Expressed in terms of vectorial functions, it would state that if a vector-valued function (or map) from

an n-dimensional space to itself has Jacobian determinant which is a non-zero constant, then the function (or map) has a vector-valued inverse.

The Jacobian determinant is a measure of how much a transformation stretches or shrinks the space it maps to, and it is defined for continuous transformations. The Jacobian conjecture applies to maps between homeomorphic spaces, which are those that can be continuously deformed into one another.

In the context of the rotational system, the transformations are continuous but not in a linear way. The smooth continuity passes through the interpolation of the antisymmetric and the symmetric systems after each 90-degrees rotation.

In that way, the topological structure of the transverse subfields is preserved, being automorphic, even when their size is not identical as it happens in the symmetric system when the contracting subfields map the expanding subfields. Their curvatures are always half positive and half negative, as they are formed by the inner curvature of an intersecting field and by the outer curvature of the other intersecting field.

In the antisymmetric system, the topological structure of the automorphic vertical subfield that maps to itself when moving leftward or rightward is also preserved because it's always formed by a negative curvature formed by the inner curvatures of both left and right intersecting fields.

In the case of the symmetric system, the top vertical subfield that moves upward while contracting when the two intersecting fields contract, or downward while expanding when the intersecting fields expand, has a negative curvature. However, the inverted subfield that exists at the convex side of the system, that maps the top vertical subfield, has a double positive curvature.

In that sense, the topological structure of the concave vertical subfield is not preserved in the inverse convex subfield. But, still, the ascending contracting concave subfield can be considered mapped to itself when a moment later descends while expanding. However, in that case the mirror reflection property of the vertical subfield would not be considered by the conjecture.

An additional complexity could be introduced in

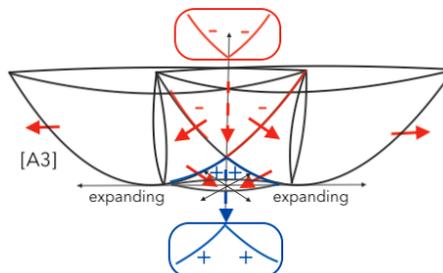


Figure 10: *Antisymmetric system: at moment A3, when the top vertical subfield descends while expanding it has negative curvatures; but its mirror counterpart at the convex side has positive curvatures.*

the case that the intersecting fields periodically synchronize and desynchronize their phases of variation while the whole system rotates.

5 Gorenstein Liaison

In algebraic geometry, the Gorenstein theory [2] establishes that two modules in projective space are linked if they are isomorphic. This implies that the curves contained within these modules share the same algebraic deficiency, which reflects the lack of regularity in the curve. For instance, in the case of a curve exhibiting a change in curvature sign, the deficiency of the curve is equivalent to the degree of that change.

The deficiency module of a curve is isomorphic to the deficiency module of any other curve with the same deficiency. Consequently, if two curves share the same deficiency, then the modules that encompass them in the projective space will also exhibit the same deficiency module. This establishes that the two modules are "linked" by their deficiency in a Gorenstein sense.

To be linked in projective space, it suffices for two modules to preserve the algebraic structure imparted by the deficiency in the curves. It is not mandatory for the curves to exhibit the same curvature.

Under the model of intersecting fields, the algebraic modules of the Gorenstein theory can be interpreted as the transversal subfields in both the symmetric and

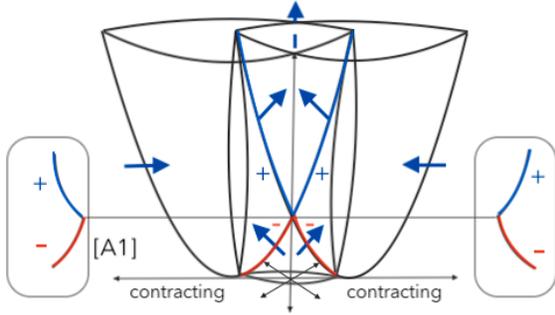


Figure 11: *The left and right modulus, represented by the mirror transversal subfields, encompassing curvatures that exhibit a same irregularity, being half + and half - from the point of intersection between the left and right fields.*

antisymmetric systems. The projective space can be interpreted in terms of mirror reflection between the left and right sides of the system.

The curvature of each modular transversal subspace exhibits a deficiency due to its sign change, being half positive and half negative. In this context, both modular transversal subspaces are automorphic (or isomorphic) because the topological (or algebraic) structure of their deficiency is preserved under their mirror reflection (or projection), creating a Gorenstein-style entanglement (or linkage).

The intersecting fields model conceptually would in this way illustrate the relationship between the Gorenstein and Jacobian conjectures for the case of vectorial functions with four variables, as previously observed.

6 Tomita-Takesaki theory

Considering the rotational fields system as a specific case of the Jacobian conjecture, it is possible to conceptually infer its possible relations to Tomita-Takesaki (TT) modular theory [3].

In TT theory two intersecting algebras form two shared “modular inclusions” (with + and - half sided subalgebras) and a “modular intersection” (with an integer sided subalgebra).

The left and right half handed subalgebras will be images of each other, when they are commutative, or they will not be their mirror image when they are noncommutative.

Mapping the modular inclusion to its reflection image, the left and right subalgebras will be the opposite image of each other (reverting their initial signs) if they are commutative; if they are noncommutative, the initial left sided subalgebra will be the image of the right sided mapped subalgebra, and the initial right-handed subalgebra will be the image of the left sided mapped subalgebra.

TT theory decomposes a linear transformation into its modular building blocks, revealing its automorphisms.

Decomposing the bounded operator, it obtains the modular operator and the modular conjugation (or modular involution) which is a transformation that reverses the orientation, preserving distances and angles.

Translating the abstract algebraic terms to the fields model, two intersecting algebras would represent the two intersecting fields fluctuating with the same or opposite phase.

The half handed subalgebras (or “modular inclusions”) will be the transversal subfields of the nucleus shared by the intersecting fields, while the integer handed subalgebra (or “intersection inclusion”) will be our vertical subfields. In this context, we identify commutativity and noncommutativity with mirror symmetry and mirror antisymmetry, respectively.

The bounded operator that is decomposed will be the 90-degree rotational matrix; The modular building blocks are the set of matrices that are obtained when applying the operator.

The modular operator will be the $\frac{1}{2}$ partial conjugate $A2$ matrix; And the modular conjugation will be the conjugate matrix $A4$, which forms the whole conjugation by adding the fractional conjugations ($\frac{1}{2} + \frac{1}{2}$).

Therefore, by separating the conjugate matrix from the complex one the automorphism of the antisymmetric conjugate system is found.

The half sided algebras that form a modular inclusion are noncommutative, it means we are in the antisymmetric system where the left intersecting field

contracts while the right one contracts and vice versa; in that system, the left transversal subfield will be the mirror symmetric image (it will be the mapped image) of the right transversal subfield when, later, the left intersecting field expands and the right one contracts.

In that sense, a past half handed subalgebra is being mapped with its future image. A time delay will exist between both subalgebras.

Considering Δ as the modular operator $A2$, J the modular conjugation $A4$, and M the intersection of two Von Neumann algebras, $\Delta^{-Yt}M\Delta^{it}$ will represent the positive and negative $\frac{1}{2}$ sided modular inclusions of the modular operator, being t a real time dimension and it an imaginary time dimension given by the partial conjugation of $A1$ or $A3$.

It is this different time dimension what makes non-commutative, as non-interchangeable, the modular $+$ and $-$ inclusions related to Δ in the antisymmetric system.

Applying the modular involution, yields $J^{yt}M'J^{-it}$.

Δ^{-yt} is transformed into J^{yt} and Δ^{it} is transformed into J^{-it} , being $J^{yt}M'J^{-it}$ the involutive automorphism of $\Delta^{-yt}M\Delta^{it}$.

The noncommutative, as non-interchangeable, Δ^{-yt} and Δ^{it} become commutative or interchangeable through time at $J^{yt}M'J^{-it}$, fixing their antisymmetry (restoring the lost mirror symmetry) in that way.

7 Reflection positivity

Related to the delay in time in the antisymmetric system, it can also be mentioned a property that all unitary quantum field theories are expected to hold: "reflection positivity" (RP). [4]

The positive increasing energy that appears in one side of the mirror system should also be reflected in the other side. However, in the context of the antisymmetric system, the positive or increasing energy of the contracting right transverse subfield does not mirror simultaneously in the expanding left transverse subfield, which exhibits negative or decreasing energy.

Therefore, to obtain a positive energy reflected at the left side, making the sides of the system virtually symmetric, a time reversal operation is needed.

To observe the positive energy reflected at the left side, it will be needed to go back in time to the moment where the left transversal subfield was contracting and had a positive energy. This operation is performed by a type of "Wick rotation". [5]

The main time phase of the symmetric system can be represented with the Y coordinate.

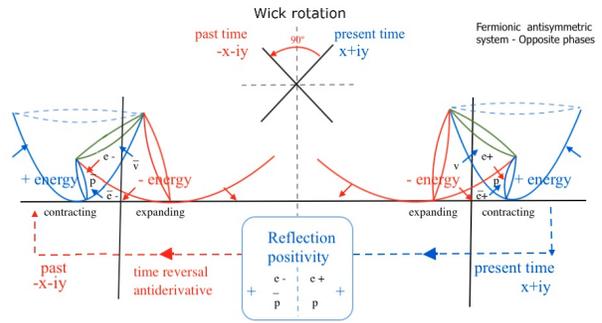


Figure 12: Reflection positivity in the antisymmetric system.

By performing a partial conjugation that involves a fractional derivative, the time coordinate Y undergoes a rotation into the purely imaginary dimension within the complex plane. At that moment, the mirror system becomes antisymmetric as one side of the system keeps following the imaginary time of Y while the other side follows a harmonic phase. A positive or negative time lag has been introduced.

Reversion time on one side of the system serves as a symbolic tool to virtually restore symmetry to the time phases. To revert to the previous time, one could perform a reverse rotation of the complex time axis ($X+iY$) to achieve a full complex conjugation at ($-X-iY$). In the A matrices context, that time backwards rotation represents an antiderivative of $-A$.

When the time reverse has been symbolically completed, in the left side of the mirror system the left subfield will be contracting, having an increased positive energy; this is a past reflection of the future positive energy that there will be a moment later in right side.

In the reverse past time, at the right side of the system the right subfield will be expanding having a decreased negative energy. In regard to the symmet-

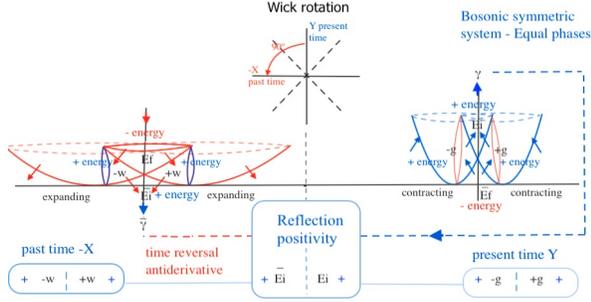


Figure 13: *Reflection positivity the symmetric system.*

ric system, positivity is reflected between the right and left transverse subfields at the same time. In that sense, it's not necessary to use the Wick operation to reverse time.

Both left and right transversal subfields will be the mirror reflection of each other at the same time. However, in the case of the strong interaction in the symmetric system, when the contracting vertical subfield has an increased positive energy while ascending to emit a pushing force, it will be necessary to virtually visit a past moment to look for a previous state where positivity could be reflected.

Going back in time, the vertical subfield will be losing its energy while expanding, moving downwards. Therefore, at that past moment, the vertical subfield will not display a positive energy.

Reflection positivity, however, can be found at that past moment in the convex side of the system of the two intersecting fields, where an inverted subfield with convex curvatures will be experiencing an increased energy.

That inverted subfield can mirror the vertical subfield which in a future state will be ascending in the concave side of the system through the Y axis.

The missing reflection positivity in the concave side of the system in the strong interaction can be related to a mass gap problem when it comes to the weak interaction.

8 Mass gap problem

There will be a mass gap [6] in the system when the two intersecting fields simultaneously expand, and the vertical subfield experiences a decay of energy.

This case represents the ground state with the lowest possible energy of the vertical subfield, which is always greater than 0 because the highest rate of expansion of the intersecting fields prevents them from having zero curvature.

The zero point of the vacuum, where there should be no energy nor mass, is placed at the point of intersection of the XY coordinates, and that point is never reached by the vertical subfield that descends through the Y axis while expanding during its decay.

An “upper” mass gap would be referred to the highest possible mass of a particle in the strong interaction. Its limit would be given by the greatest rate of contraction of the intersecting spaces.

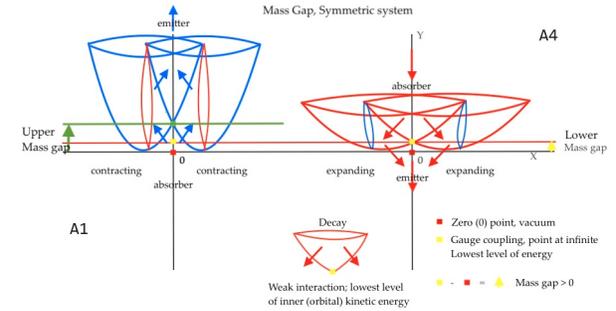


Figure 14: *Graphical representation of the mass gap in the symmetric system; the upper gap occurs in the compressed photonic subfield when both intersecting fields contract, while the lower gap occurs in the decompressed subfield when both intersecting fields expand.*

The zero point of the vertical subfield is marked in yellow on the above diagram, at the point of intersection of the left and right intersecting fields.

The gap is given by the distance from that point to the zero point where the X and Y coordinates intersect, represented by a red mark. An arrow shows the gap distance between those critical points.

However, in this model, the zero point does not

represent a vacuum where neither energy nor mass exists.

When the mass and energy of the vertical subfield reach their weakest level in the concave side of the symmetric system, an equivalent amount of energy and mass arises in the convex side, where the zero point is located, as the result of the double pushing force caused by the displacement of the positive curvature of the expanding intersecting fields.

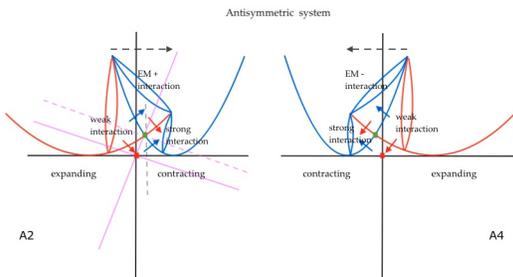


Figure 15: *Visual representation of the map gap in the antisymmetric system, with the left and right displacements of the point of intersection.*

That mass and energy at this zero point will be considered dark from the point of view of the concave side of the system.

In the antisymmetric system, the lowest energy level occurs when a transverse subfield experiences a double decompression due to the displacement of the concave curvature of the contracting intersecting field and the displacement of the positive curvature of the expanding intersecting field.

The corresponding double compression is then experienced by its mirror antisymmetric transverse subfield.

9 N = 1 Supersymmetric dual atomic model

The fields model emerges in the context of the development of a supersymmetric quantum field model of an atom formed by the two intersecting fields that share a nucleus of two transvers and two vertical subfields that represent the matter and antimatter of the

dual structure. [7]

The composition of the atomic antisymmetric nucleus will depend on the specific moment of the system's evolution. It may consist of a proton, a positron and a neutrino, or an antiproton, an electron, and an antineutrino.

9.0.1 Antisymmetric system: the left intersecting field expands while the right one contracts (A2)

- The right contracting transversal subspace will represent a proton.
- The left expanding transversal subspace will represent a neutrino.
- The vertical subspace moving toward the right will represent a positron.

9.0.2 Antisymmetric system, the left intersecting field contracts while the right one expands (A4)

- The right contracting proton will expand, becoming a right expanding antineutrino.
- The left expanding neutrino will contract, becoming a left-handed contracting antiproton.
- The vertical positron will move toward the left, becoming an electron.

However, it does not reflect the moment when the top vertical subfield passes through the central axis, which is the reference center of symmetry of the system, carrying a neutral charge.

It is considered neutral because it is placed in the location used to distinguish between positive or negative: from that central point to the right the charge will be positive, and from that point to the left it will be negative.

This neutrality will occur during the intermediate expansion or contraction of the intersecting fields.

In that case, the proton (or antiproton) transversal subfield, and the neutrino (or antineutrino) transversal subfield will show an isomorphic shape and their

positive and negative charges will be in compensation. It may be at that moment when the notion of neutron and antineutron arises.

Simultaneously, in the left side of the antisymmetric system an antiproton and an electron arise.

Later, the left-handed antiproton of $A4$ will decay into a left-handed neutrino at $A2$, while in the right side of the mirror system a proton and a positron will arise.

Proton and antiproton, and neutrino and antineutrino, will be Dirac antiparticles at different times.

Positron and electron are the same subfield, acting at different times as their own mirror reflection Majorana antimatter.

The existence of an electron and a positron in the same atom, also known as “positronium”, was predicted by Dirac in 1928. However, positronium was formulated as an exotic atom with no proton in its nucleus.

The coexistence of proton and antiproton in the same atom is currently accepted as an exotic structure called “protonium” with no electrons nor positrons.

In the dual atomic model, matter and antimatter coexist and are related to each other by means of their chiral mirror reflection symmetry at the same or different times.

All the subfields in the antisymmetric system are fermions with noninteger $\frac{1}{2}$ spin, represented by the commuted eigenvector, being ruled by the Pauli exclusion principle. In that regard, they should adhere to Fermi-Dirac statistics, although the dual atomic nucleus is a causal model that can be described without using probability.

Additionally, in that same context, considering an antisymmetric Schrödinger’s cat as a figurative example, it could be said that the right alive contracting cat will be the delayed reflection of the left dead expanding cat, and vice versa.

It can be discussed whether they are the future or the passed reflection of each other, but that will only be a way to speak.

There will not be a single alive and dead cat, but two identical cats with opposite states and positions.

Their simultaneous states of being “alive” and “dead” can be considered “superposed” but in the

context of their mirror antisymmetry.

9.0.3 Symmetric system, when the left and right intersecting fields contract ($A1$)

- The right and left expanding transversal subspaces represent a right-handed positive and a left-handed negative gluon.
- The top vertical ascending subspace that contracts receiving a double force of compression will be the electromagnetic subfield that emits a photon while pushing upward.
- The inverted bottom vertical subspace at the convex side of the system represents the dark decay of a previous dark antiphoton.

9.0.4 Symmetric system, when the left and right intersecting fields expand ($A3$)

- The right and left expanding transverse subspaces may represent $-W$ and $+W$ bosons.
- The top vertical descending subspace will be the electromagnetic subfield losing its previous energy, after having emitted a photon.
- The bottom vertical subspace at the convex side of the system is the dark anti electromagnetic subfield that emits a dark antiphoton.

The left and right transversal subspaces will be mirror symmetric antimatters at the same time, being bosons not ruled by the Pauli exclusion principle. They should then obey the Fermi-Dirac statistics.

However, the photon and the dark antiphoton – or the vertical subfield from which they emerge – are mutually exclusive. Therefore, they are governed by the Pauli exclusion principle, even though they have an integer spin represented by the two converging eigenvectors.

The identity of the symmetric transversal subfields, labeled before as “ W bosons” and “gluons” requires further clarification.

Each of those subfields receives a bottom inward pushing force and a top outward decompression – in the strong interaction – or a top inward pushing force

and a bottom outward decompression – in the weak interaction.

In the strong interaction, the magnitudes of the pushing forces caused by contracting or expanding intersecting fields will be different, because the contracting field exhibits a higher density, intensifying the propulsive force caused by the displacement of its negative curvature.

The vertical photonic subfield receives an inward double pushing force from right to left and from left to right caused by the displacement of the negative curvatures of the intersecting fields.

These pushing forces are the same as those that decompress the transversal subfields – labeled as gluons – at that moment. The emitted photon would have a double helix spin. The pushing forces received at different moments by the moving right positron and the moving left electron in the antisymmetric system, now converge simultaneously in the photonic subfield.

From the perspective of this model, the transversal subspaces are the same topological subfields that contract when the intersecting fields expand in the weak interaction or expand when the intersecting fields contract in the strong interaction.

The strong and weak interactions, then, are related by the same mechanism. And the mirror transversal subfields that mediate the strong and weak interactions are the same topological subspaces that are transformed through time.

The model is labeled as $N = 1$ because it relates in a supersymmetric way, through time, each fermionic subfield of the antisymmetric system with a bosonic subfield of the symmetric system and vice versa. In the case of the ascending photonic field related to the symmetric system, it will be related to the electronic field in the antisymmetric system in a double way related to different times: when it moves toward left acting as electron, and when it moves towards right when acting as a positron.

In that way:

- The fermionic electron-positron subfield will be the superpartner of the bosonic vertical subfield that emits the photon when ascending.
- The fermionic proton-antineutrino subfield, and the fermionic antiproton-neutrino subfields will

be the superpartners of the symmetric transversal right and left subfields respectively, when they contract or expand.

The symmetry of the system is preserved through time. The modular Hamiltonian of the system also remains invariant through time.

The intersecting spaces model can also be thought in terms of the topology of a two genus torus or two related tori.

The outer positive and the inner negative curvatures of the torus can be seen as the simultaneous representation of the expanding or contracting moments of the vibrating fields when looking at them from above, in an orthographic projection.

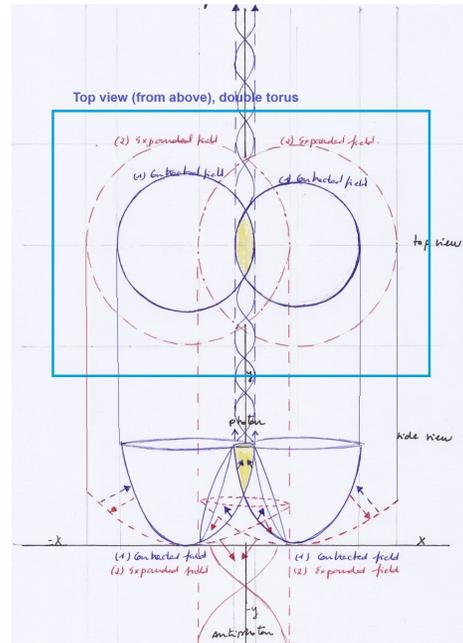


Figure 16: *Symmetric system, Orthographic projection.*

The symmetric and antisymmetric subfields can be described as cobordant [8] subspaces. The vertical subspaces share borders with the left and right transversal subspaces, and they all share borders with the two intersecting spaces.

These borders can be thought of as de unidimen-

sional lines described by the curvatures of the intersecting fields.

From that point of view, the fields model could be related to string theories.

In the antisymmetric fields system, the transverse subspaces periodically change their role becoming the negative or positive reflection of each other at different times.

This type of automorphic mirror reflection symmetry at different times may be related to the SYZ conjecture [9] in String theory, which states that there exists a special type of Calabi-Yau manifold [10] that is related to another Calabi-Yau manifold by a T-duality [11] transformation.

In string theory, mirror symmetry emerges from the notion of T-duality, which relates the spaces described by Type IIA and type IIB strings theories. In Type IIA string theory, the strings can move freely in the Calabi-Yau transverse space with a larger radius, while in type IIB string theory, the strings are confined to the boundaries of the transverse space of shorter radius.

T-duality relates these two different types of larger and smaller transversal spaces by means of a type of inversion that exchanges the roles of the large and small radii transverse spaces.

In the context of the dual fields model, the Calabi-Yau spaces of smaller or larger radius may be considered equivalent to the transverse contracting or expanding subspaces that are mapped to each other in a mirror symmetric way by means of their topological transformation through time, as described before in the antisymmetric rotational system.

The elliptic orbits inside of the transversal subfields, caused by their periodical expansion and contraction, can be visually related to the notion of elliptic fibrations.

10 Additional diagrams

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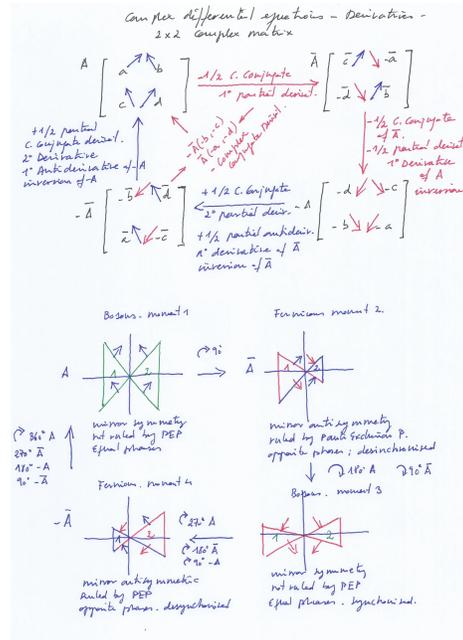


Figure 17: 2D scheme of smooth rotational interpolation of vectorial systems; Related matrices.

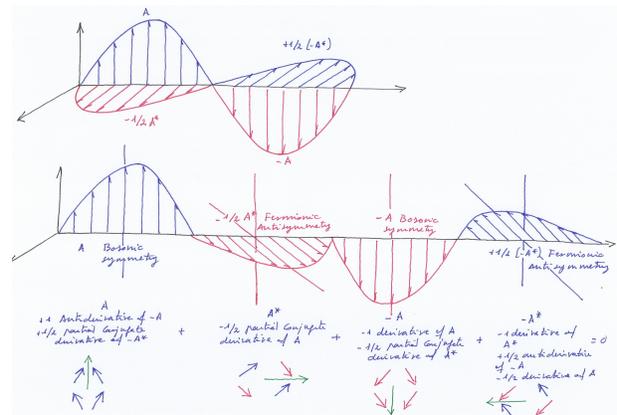


Figure 18: Symbolic representation of the interpolated symmetric and antisymmetric systems as the electric and magnetic moments, respectively, of the rotational supersymmetric system.

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