THE CLASSICAL DERIVATION OF THE REMNANT MASS OF A QUASI-BINARY BLACK HOLE.

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Abstract. In the present article, we classically derive an analytic formula of the Remnant Mass of a Quasi-Binary Black Hole. The Quasi Black Hole concept comes from a Theory Of Everything we have developed few years ago.

From the Quasi Black Hole concept from the Theory Of Everything we have developed previously, the radial distribution of a quasi black hole is the following :

$$r = \frac{2\ G\ M\left(r\right)}{c^2} \tag{1}$$

$$M\left(r\right) = \frac{c^2}{2\ G} r \tag{2}$$

The symmetric radial distribution is the following :

$$\frac{d\ M\left(r\right)}{dr} = 4\pi\ r^2\ \rho\left(r\right) \tag{3}$$

$$\rho\left(r\right) = \frac{1}{4\pi\ r^2} \frac{d\ M\left(r\right)}{dr} \tag{4}$$

$$\rho\left(r\right) = \frac{c^2}{8\pi\ G\ r^2} \tag{5}$$

The classical derivation of the gravitational binding of a quasi black hole is :

$$U = -\int_0^R \frac{G\ M\left(r\right)\ 4\pi\ r^2\ \rho\left(r\right)}{r} dr \tag{6}$$

$$U = -\int_0^R \frac{c^2}{2\ G} \times \frac{c^2}{2} dr \tag{7}$$

$$U = -\frac{c^4}{4\ G} r \tag{8}$$

$$U = -\frac{1}{2}\ M\ c^2 \tag{9}$$

(10)

An immediate corollary of that formula derivation is : the merging of a quasi-binary black hole into a radial symmetric quasi black hole does not emit gravitational waves and does not loose mass.

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From the concept of a quasi-black hole from the Theory Of Everything that we developed previously, to avoid any singularities of the spacetime metric, the probability of the paths of the quantum particles corresponding to these singularities is zero and the gravity can have a repulsive behavior in this case. Therefore, the merging of a quasi-binary black hole into a radial symmetric quasi black hole may not happen if that process create singularities into the the spacetime metric.
A very relevant criteria from the Theory Of Everything we have developed previously is : for any spacelike hypersurfaces and for any spatial geometric sphere with radius $\tilde{r}$ inside them, the mass $\tilde{m}(\tilde{r})$ contained in that spatial geometric sphere should be smaller or equal to $\frac{c^2}{2\ G}R$.

We can apply that relevant criteria by considering a spatial geometric sphere at the median point of a quasi-binary black hole and derive the minimal distance between both quasi black holes to avoid any singularities of the spacetime metric.

The first relevant case is to derive the mass function $\tilde{m}(\tilde{r})$ at the median point of both identical quasi black holes with radius $R = 1$, with $G = c = 1$ and a spatial separation $d_0/2 = cot\,(2/3)$.

The ordinate $\tilde{h}$ of the intersection between the spatial geometric sphere with radius $\tilde{r}$ and one of the unit quasi black hole is:

$$h^2 + x_1^2 = \tilde{r}^2 \tag{11}$$

$$h^2 + x_2^2 = R^2 \tag{12}$$

$$x_1 + x_2 = d_0/2 \tag{13}$$

$$R = 1 \tag{14}$$

The analytic solution h of that set of equation is :

$$h = \frac{\sqrt{4\ \tilde{r}^2\ R^2 - \left(\tilde{r}^2 + R^2 - d_0^2/4\right)^2}}{d_0} \tag{15}$$

Therefore, the mass $\tilde{m}(\tilde{r})$ contained in that spatial geometric sphere is :

$$\tilde{m} = 4\pi\ H\left(d_0^2/4 + R^2 - \tilde{r}^2\right)\int_{\sqrt{R^2-h^2}}^{R} dh' \int_0^{\sqrt{R^2-h'^2}} \frac{r\ dr}{8\pi\left(h'^2 + r^2\right)} \tag{16}$$

$$+ 4\pi\ H\left(\tilde{r}^2 - d_0^2/4 - R^2\right)\int_{-R}^{\sqrt{R^2-h^2}} dh' \int_0^{\sqrt{R^2-h'^2}} \frac{r\ dr}{8\pi\left(h'^2 + r^2\right)} \tag{17}$$

$$+ 4\pi \int_{\sqrt{\tilde{r}^2-h^2}}^{\tilde{r}} dh' \int_0^{\sqrt{\tilde{r}^2-h'^2}} \frac{r\ dr}{8\pi\left(\left(d_0/2 - h'\right)^2 + r^2\right)} \tag{18}$$

The second relevant case is to derive the mass function $\tilde{m}(\tilde{r})$ at the median point of two different quasi black holes with radius $R_1 = 1$ and $R_2 = log\,(e-1)$, with $G = c = 1$ and a unit spatial separation $d/2 = 1$. The radius ratio of the quasi-binary black hole is $\xi_0 = R_2/R_1 = log\,(e-1)$.

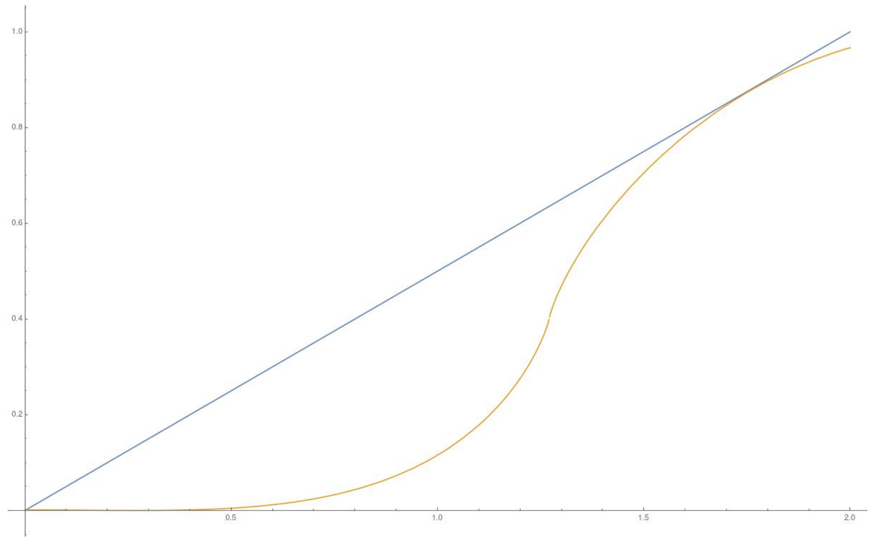

FIGURE 1. The abscissa is the radius of the spatial geometric sphere centered at the origin. The ordinate is the mass $\tilde{m}(\tilde{r})$ contained inside the spatial geometric sphere of radius $\tilde{r}$ from both unit quasi black hole of radius 1 and centered at $(x, y) = (\pm d/2, 0)$.

The ordinate $\tilde{h}$ of the intersection between the spatial geometric sphere with radius $\tilde{r}$ and the unit quasi black hole is:

$$h_1^2 + x_1^2 = \tilde{r}^2 \tag{19}$$

$$h_1^2 + x_2^2 = R^2 \tag{20}$$

$$x_1 + x_2 = R_1 \tag{21}$$

$$R_1 = 1 \tag{22}$$

The ordinate $\tilde{h}$ of the intersection between the spatial geometric sphere wit radius $\tilde{r}$ and the smaller quasi black hole is:

$$h_2^2 + x_1^2 = \tilde{r}^2 \tag{23}$$

$$h_2^2 + x_2^2 = R_2^2 \tag{24}$$

$$x_1 + x_2 = R_2 \tag{25}$$

(26)

The analytic solution $\tilde{h}$ of that set of equation is :

$$h_1 = \frac{\sqrt{4\ \tilde{r}^2\ R_1^2 - \tilde{r}^4}}{2\ R_1} \tag{27}$$

$$h_2 = \frac{\sqrt{4\ \tilde{r}^2\ R_2^2 - \tilde{r}^4}}{d\ R_2} \tag{28}$$

Therefore, the mass $\tilde{m}(\tilde{r})$ contained in that spatial geometric sphere is :

$$\tilde{m} = 2\pi\ H\left(2\ R_1^2 - \tilde{r}^2\right) \int_{\sqrt{R_1^2-h_1^2}}^{R_1} dh' \int_0^{\sqrt{R_1^2-h'^2}} \frac{r\ dr}{8\pi\left(h'^2+r^2\right)} \tag{29}$$

$$+ 2\pi\ H\left(\tilde{r}^2 - 2\ R_1^2\right) \int_{-R_1}^{\sqrt{R_1^2-h_1^2}} dh' \int_0^{\sqrt{R_1^2-h'^2}} \frac{r\ dr}{8\pi\left(h'^2+r^2\right)} \tag{30}$$

$$+ 2\pi \int_{\sqrt{\tilde{r}^2-h_1^2}}^{\tilde{r}} dh' \int_0^{\sqrt{\tilde{r}^2-h'^2}} \frac{r\ dr}{8\pi\left(\left(R_1-h'\right)^2+r^2\right)} \tag{31}$$

$$+ 2\pi\ H\left(2\ R_2 - \tilde{r}\right)\ H\left(2\ R_2^2 - \tilde{r}^2\right) \int_{\sqrt{R^2-h_2^2}}^{R_2} dh' \int_0^{\sqrt{R_2^2-h'^2}} \frac{r\ dr}{8\pi\left(h'^2+r^2\right)} \tag{32}$$

$$+ 2\pi\ H\left(2\ R_2 - \tilde{r}\right)\ H\left(\tilde{r}^2 - 2\ R_2^2\right) \int_{-R_2}^{\sqrt{R_2^2-h_2^2}} dh' \int_0^{\sqrt{R^2-h'^2}} \frac{r\ dr}{8\pi\left(h'^2+r^2\right)} \tag{33}$$

$$+ 2\pi\ H\left(2\ R_2 - \tilde{r}\right) \int_{\sqrt{\tilde{r}^2-h_2^2}}^{\tilde{r}} dh' \int_0^{\sqrt{\tilde{r}^2-h'^2}} \frac{r\ dr}{8\pi\left(\left(R_2-h'\right)^2+r^2\right)} \tag{34}$$

$$+ H\left(\tilde{r} - 2\ R_2\right)\ R_2/2 \tag{35}$$

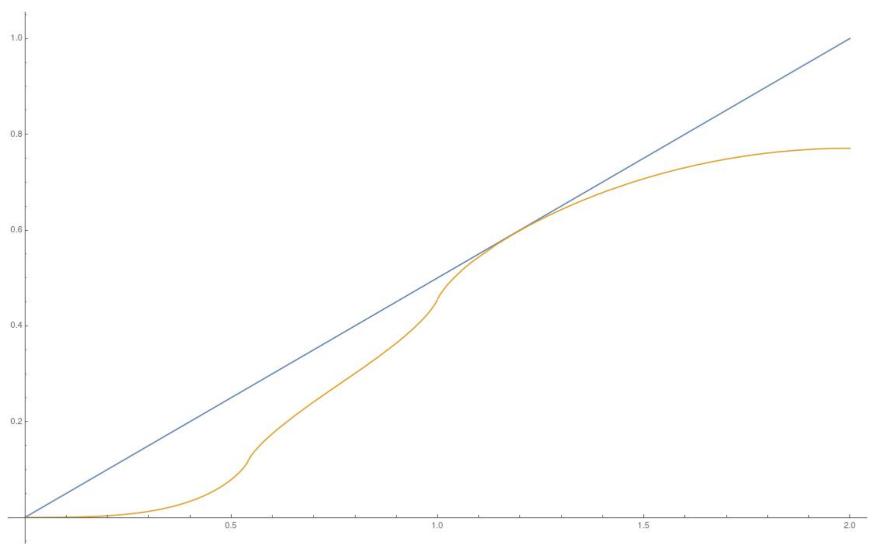

FIGURE 2. The abscissa is the radius of the spatial geometric sphere centered at the origin. The ordinate is the mass $\tilde{m}(\tilde{r})$ contained inside the spatial geometric sphere of radius $\tilde{r}$ from a quasi unit black hole of radius $R_1 = 1$ and centered at $(x, y) = (-1, 0)$ and from a quasi black hole of radius R_2 and centered at $(x, y) = (R_2, 0)$. The mass $\tilde{m}(\tilde{r})$ contained in that spatial geometric sphere should be smaller than $\tilde{r}/2$ to avoid any singularities of the spacetime metric.

In both cases, the mass $\tilde{m}(\tilde{r})$ contained in that spatial geometric sphere should be smaller than $\tilde{r}/2$ to avoid any singularities of the spacetime metric.

From the final distance between both quasi lack holes derived by the linear interpolation of the two previous cases, we can classically derive the remnant mass of the quasi-binary black hole from the lost mass between the final remnant mass and the initial quasi-binary black hole mass :

(36)

$$\Delta M\ c^2 = (M_1 + M_2 - M_3)\ c^2 =$$

(37)

$$\frac{G\ M_1\ M_2}{C_{GR}\ \left(\frac{2\ G\ M_1}{c^2} + \frac{2\ G\ M_2}{c^2}\right)\left(H\left(\xi_0 - \xi\right) + H\left(\xi - \xi_0\right)\left(d_0/2 + \left(1 - d_0/2\right)\frac{1-\xi}{1-\xi_0}\right)\right)}$$

(38)

$$C_{GR} = 4\pi/5$$

(39)

$$d_0/2 = cot\left(2/3\right)$$

(40)

$$\xi_0 = log\left(e - 1\right)$$

(41)

$$\xi = min\left(M_1, M_2\right)/max\left(M_1, M_2\right)$$

(42)

To perfectly match the list of gravitational wave observations, we need to introduce the constant $C_{GR} = 4\pi/5$ to the previous classical derivation in order to take in account the effects of general relativity. $C_{GR} = 4\pi/5$ is chosen in order to have a unit median GW ratio (the median GW ratio is the 42th-43th GW ratio over the 85 GW ratio in total) for the ratios between the theoretical lost masses and the observed lost masses. Only 7 Gravitational Waves Events (GW Events) are far from the theoretical ratios of which 5 of them involve very light masses (a light mass is more close to a neutron star than a quasi black hole) :

Enumeration	GW Event Name	Primary Mass	Secondary Mass	Remnant Mass	Lost Mass Observed	Ratio Lost Mass Theoretical/Observed
1	GW200322	34.	14.	53.	-5.	-0.364814
2	GW200115	5.7	1.5	7.8	-0.6	-0.364047
3	GW170817	1.27	0.72	2.8	-0.81	-0.102791
4	GW190620	57.	36.	87.	6.	0.642191
5	GW200308	36.4	13.8	47.4	2.8	0.657346
6	GW190521	85.	66.	142.	9.	0.666715
7	GW190828	32.1	26.2	54.9	3.4	0.671427
8	GW200114	78.	70.	140.	8.	0.700833
9	GW200105	8.9	1.9	10.4	0.4	0.720005
10	GW200224	40.	32.5	68.6	3.9	0.728951
11	GW190706	67.	38.	99.	6.	0.732176
12	GW191222	45.1	34.7	75.5	4.3	0.73931
13	GW190519	66.	41.	101.	6.	0.740365
14	GW190727	38.	29.4	63.8	3.6	0.744721
15	GW190521	42.2	32.8	71.	4.	0.74488
16	GW190517	37.4	25.3	59.3	3.4	0.756084
17	GW170729	50.6	34.3	80.3	4.6	0.756436
18	GW200128	42.2	32.6	71.	3.8	0.783311
19	GW191230	49.4	37.	82.	4.4	0.787759
20	GW200208	51.	12.3	61.	2.3	0.792536
21	GW200129	34.5	28.9	60.3	3.1	0.794129
22	GW190602	69.	48.	111.	6.	0.795325
23	GW200112	35.6	28.3	60.8	3.1	0.813636
24	GW170823	39.6	29.4	65.6	3.4	0.815921
25	GW200219	37.5	27.9	62.2	3.2	0.821259
26	GW190424	39.5	31.	67.1	3.4	0.821511
27	GW150914	35.6	30.6	63.1	3.1	0.821919
28	GW200311	34.2	27.7	59.	2.9	0.8378
29	GW170814	30.7	25.3	53.4	2.6	0.840808
30	GW190701	53.9	40.8	90.2	4.5	0.841978

Enumeration	GW Event Name	Primary Mass	Secondary Mass	Remnant Mass	Lost Mass Observed	Ratio Lost Mass Theoretical/Observed
31	GW190421	40.6	31.4	68.6	3.4	0.84243
32	GW190910	43.9	35.6	75.8	3.7	0.843042
33	GW200220	87.	61.	141.	7.	0.860975
34	GW190719	37.	20.8	55.	2.8	0.864057
35	GW190731	41.5	28.8	67.	3.3	0.869275
36	GW191216	12.1	7.7	18.87	0.93	0.881222
37	GW190413	45.4	30.9	72.8	3.5	0.892817
38	GW200209	35.6	27.1	59.9	2.8	0.894614
39	GW190803	37.3	27.3	61.7	2.9	0.898656
40	GW191109	65.	47.	107.	5.	0.906182
41	GW190527	36.5	22.6	56.4	2.7	0.909079
42	GW191204	11.9	8.2	19.21	0.89	0.922816
43	GW190413	33.4	23.4	54.3	2.5	0.925346
44	GW170809	35.2	23.8	56.4	2.6	0.930461
45	GW191105	10.7	7.7	17.6	0.8	0.931449
46	GW200208	37.8	27.5	62.5	2.8	0.942148
47	GW190708	17.6	13.2	29.5	1.3	0.950151
48	GW190503	43.3	28.4	68.6	3.1	0.953161
49	GW190408	24.5	18.3	41.	1.8	0.954554
50	GW200220	38.9	27.9	64.	2.8	0.966873
51	GW190924	8.9	5.	13.3	0.6	0.969727
52	GW190915	35.3	24.4	57.2	2.5	0.975181
53	GW151226	13.7	7.7	20.5	0.9	0.995287
54	GW170818	35.5	26.8	59.8	2.5	0.997725
55	GW191113	29.	5.9	34.	0.9	1.00198
56	GW190630	35.1	23.7	56.4	2.4	1.00482
57	GW190707	11.6	8.4	19.2	0.8	1.01103
58	GW191126	12.1	8.3	19.6	0.8	1.04285
59	GW200302	37.8	20.	55.5	2.3	1.04602
60	GW190513	35.7	18.	51.6	2.1	1.04815

Enumeration	GW Event Name	Primary Mass	Secondary Mass	Remnant Mass	Lost Mass Observed	Ratio Lost Mass Theoretical/Observed
61	GW190728	12.3	8.1	19.6	0.8	1.05021
62	GW170104	31.	20.1	49.1	2.	1.05476
63	GW190512	23.3	12.6	34.5	1.4	1.07443
64	GW170608	10.9	7.6	17.8	0.7	1.07743
65	GW191215	24.9	18.1	41.4	1.6	1.08591
66	GW190412	29.7	8.4	37.	1.1	1.09495
67	GW190828	24.1	10.2	33.1	1.2	1.09854
68	GW200202	10.1	7.3	16.76	0.64	1.09996
69	GW190720	13.4	7.8	20.4	0.8	1.10692
70	GW200225	19.3	14.	32.1	1.2	1.1218
71	GW200216	51.	30.	78.	3.	1.12692
72	GW190514	39.	28.4	65.	2.4	1.13427
73	GW190929	81.	24.	102.	3.	1.13517
74	GW191103	11.8	7.9	19.	0.7	1.15593
75	GW191204	27.3	19.3	45.	1.6	1.18401
76	GW190930	12.3	7.8	19.4	0.7	1.18906
77	GW191129	10.7	6.7	16.8	0.6	1.20281
78	GW200316	13.1	7.8	20.2	0.7	1.24491
79	GW200306	28.3	14.8	41.7	1.4	1.27679
80	GW151012	23.3	13.6	35.7	1.2	1.28419
81	GW190909	46.	28.	72.	2.	1.53951
82	GW200210	24.1	2.83	26.7	0.23	2.02542
83	GW190814	23.2	2.59	25.6	0.19	2.25558
84	GW191219	31.1	1.17	32.2	0.07	2.96295
85	GW191127	53.	24.	76.	1.	3.03859

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