Number Notation for Operations and Hyperoperations.

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0- Abstract:

In this paper I will apply some of notation tools in hyperoperators to the understanding of a deep and developed theory in notation. Using [a] or [-a] for an a Natural we will see the richness of this algebraic writing form.

1- Introduction.

Hyperoperators are the operators which will go beyond exponent operator, some examples of this kind of algebra are Tetration and Pentation, which represent the iteration of exponentiation and the iteration of tetration. I have noticed that in the actual bibliography of hyperoperations (1)(2)(3), there are only accepted and assumed the positive operators, namely the sum, the product, the exponent, etc. There is true that when we think about hyperoperators we just think in the cases when we use it to iterate operation and not the anti-operations (or negative operators), namely the subtraction, the division, the root, etc.

2- Number notation.

We will denote for [a], where a is an integer except 0. The number a will in every moment the weight of our operation. The [a] will be positive when it indicates the positive operation, and it will be [-a] (in a negative form or with a minus sing) when it indicates the anti-operation.

2.1- Summation.

We will use [1] for sum, being in our notation $+=[1]$. For example, 2[1]3=5.

2.2- Anti-summation or subtraction.

We are now using [-1] to the subtraction, being the contrast operation of sum, but with very near properties. So we will write $- = [-1]$. And we will put as example: $2[-1]2[-1]3 = -3$.

2.3- Product.

We set [2] as the product operation. So $\cdot = [2]$. And as example we can set $3 \cdot 4 = 3[2]4 = 12$.

2.4- Anti-product or division.

The second of our negative operators will be division. We establish the next equality true $\div = [-2]$; Ex. $12 \div 3 = 12[-2]3 = 4$.

2.5- Exponents.

Following the logic we will assign to exponent the notation [3] so, in a parallelism with arrow notation introduced by Knuth (5), we have the next: $\uparrow = [3]$. And making it simple any exponent we will be represented with [3], for example: $4^3 = 4[3]3 = 64$.

2.6- Anti-exponents or roots.

We will use [-3] for roots, being in the left part of the numeric operator the basis of the root and in the right part the index of the root. $\sqrt[n]{k} = k[-3]n$. For example in a cubic root of five: $\sqrt[3]{5} = 5[-3]3$.

2.7- Tetration.

Following again up arrow notation we can consider the iteration of exponentiation as tetration. So we will equal $\uparrow\uparrow = [4]$. A short example will be $2\uparrow\uparrow 3 = 2[4]3 = 16$.

2.8- Anti-tetration.

I want to define in easy words anti-tetration as root-of-roots, we can just link the conclusion of tetration as a group of exponent with this part. Anti-tetration is a group of roots. I will not show here a full explanation of this operational tool, I will just give some of my own notation and an easy example. $\downarrow\downarrow=[-4]$; $65536\downarrow\downarrow 4=65536[-4]4=2$.

2.9- Pentation.

We define pentantion as iteration of tetrations. In Knuth notation we will set it as $\uparrow\uparrow\uparrow$, in this new notation it will be: $\uparrow\uparrow\uparrow=[5]$.

2.10- Anti-pentation.

We will define anti-pentation as an iteration of anti-tetration, obviously it will have the pair of notations: $\downarrow\downarrow\downarrow=[-5]$.

2.11- k-ation.

In k-ation we finally can see the advance of our numeric notation, in Knuth notation we have the following in the k-ation between 2 numbers or variables a and b:

$$a\underbrace{\uparrow\uparrow\uparrow\ldots\uparrow\uparrow\uparrow}_{(k-2)arrows}b \quad .$$

In numeric notation we have simply: $a[k]b$.

2.12- Anti-k-ation.

And lastly, in anti-k-ation we can write:

$$a\underbrace{\downarrow\downarrow\downarrow\ldots\downarrow\downarrow\downarrow}_{(k-2)arrows}b \quad .$$

Or simply: $a[-k]b$.

3. Priority number notation theorem.

Following PEMDAS rule (4): Parentheses, Exponents, Multiplication, and Division (from left to right), Addition and Subtraction (from left to right), we will assume Brackets in front of it and we will assume that could be more complex operations (hyperoperations), so we can show:

Theorem 1: The order of resolution of the algebraic structures will be: in the first place brackets, in the second place parenthesis, and finally from highest numeric operations to lowest numeric operations between numbers or variables, first positive numeric operations, then negative.

4. Algebraic notation examples.

Using the previous content we can apply to polynomials some equations changing the notation:

4.1. $(3+2^2-x\cdot\sqrt[3]{2})\div 2=0 \quad \rightarrow \quad (3[1]2[3]2[-1]x[2]2[-3]3)[-2]2=0$

4.2 $(x\uparrow\uparrow 2+\{5^{(-7)}+\sqrt[5]{39}\})-57\div y\downarrow\downarrow 3=0 \quad \rightarrow$

$(x[4]2[1]\{5[3]-7[1]39[-3]5\})[-1]57[-2]y[-4]3=0$

4.3 $(6+2)\uparrow\uparrow\uparrow 5-(x-2)+(9\uparrow\uparrow 3)\downarrow\downarrow\downarrow 4=0 \quad \rightarrow$

$(6[1]2)[5]5[-1](x[-1]2)[1](9[4]3)[-5]4=0$

5. Conclusions.

As you read this notation is an alternative to classical operation notation. It not for daily use, it is a bit advanced form to clarify some mathematical expressions but it is not my intention to substitute the actual notation. In fact, it will be maybe necessary in the future to develop a unique notation just for tetration, anti-tetration, pentation and anti-pentation.

6. Bibliography.

1) Patrick St-Amant- Number Theories. (2010) https://arxiv.org/pdf/1003.3509.pdf

2) Robbins, A. J. (November 2005). "Home of Tetration"

3) For the basic hyperoperation notation: https://en.wikipedia.org/wiki/Hyperoperation

4) Order of operations: https://en.wikipedia.org/wiki/Order_of_operations

5) Donald E. Knuth - Mathematics and Computer Science: Coping with Finiteness - “Science”, 17 December 1976, vol. 194, n. 4271, pp. 1235-1242