From Coulomb's force to magnetic force and experiments that show parallel-to-current magnetic force (letter)

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Abstract: It is known that the Lorentz forces between two current elements do not respect the Newton's third law. This discrepancy hides an unknown property of electromagnetism. For solving this problem, we will derive a new law of magnetic force which respects the Newton's third law. This new law reveals the mechanism that transforms Coulomb's force into magnetic force. We will also present experimental evidence that supports this new law.

1. Introduction

The Lorentz force law is a fundamental law that defines magnetic force. However, this law violates the Newton's third law. Let $d\boldsymbol{I}_a$ and $d\boldsymbol{I}_b$ be two current elements and $d\boldsymbol{F}_a$ and $d\boldsymbol{F}_b$ the magnetic forces they mutually act on one another. According to the Lorentz force law $d\boldsymbol{F}_a$ is perpendicular to $d\boldsymbol{I}_a$ and $d\boldsymbol{F}_b$ is perpendicular to $d\boldsymbol{I}_b$. The Figure 1 shows a case where $d\boldsymbol{F}_a$ is perpendicular to $d\boldsymbol{F}_b$. We see that $d\boldsymbol{F}_a + d\boldsymbol{F}_b \neq 0$ which violates the Newton's third law.

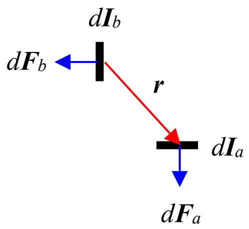

Figure 1

For justifying this discrepancy physicists argue that the Lorentz forces that two coils act on each other do satisfy the Newton's third law. Nevertheless, breaking the Newton's third law does not fit scientific standard. We will solve this problem with a new magnetic force law that respects the Newton's third law. The new law is derived from the Coulomb's law which is :

$$\boldsymbol{F} = \frac{q_1 q_2 \boldsymbol{r}}{4\pi\varepsilon_0 |\boldsymbol{r}|^3} \quad (1)$$

with q_1 and q_2 being two fixed electric charges, $\boldsymbol{r}$ the vector radius distance between them, ε_0 the permittivity of free space.

We have found that magnetic force is in fact Coulomb's force that is modified by two relativistic effects: the relativistic dynamic effect and the changing distance effect.

2. Relativistic dynamic effect

a) Couple of charges

Magnetic force is acted on currents that flow in neutral wires which we represent as an assembly of couples of charges. Let ⊖ represent a free electron and ⊕ a fixed positive charge which is a proton in the nucleus of an atom. A free electron and a fixed positive charge form a neutral couple of charges which is represented by ⊕⊖. When electrons move in the wires the Coulomb's force that two couples of charges act on one another is modified by the relativistic dynamic effect.

Let $\oplus_a\ominus_a$ and $\oplus_b\ominus_b$ be two couples of charges. There are 4 interactions between them : $\oplus_b \to \oplus_a$, $\ominus_b \to \oplus_a$, $\oplus_b \to \ominus_a$ and $\ominus_b \to \ominus_a$, which are shown in the Figure 2 (b) and (c). The distance between $\oplus_a\ominus_a$ and $\oplus_b\ominus_b$ is the vector $\boldsymbol{r}$, see Figure 2 (a). We suppose that $\ominus_a$ is at the same location as $\oplus_a$ and $\ominus_b$ is at the same location as $\oplus_b$.

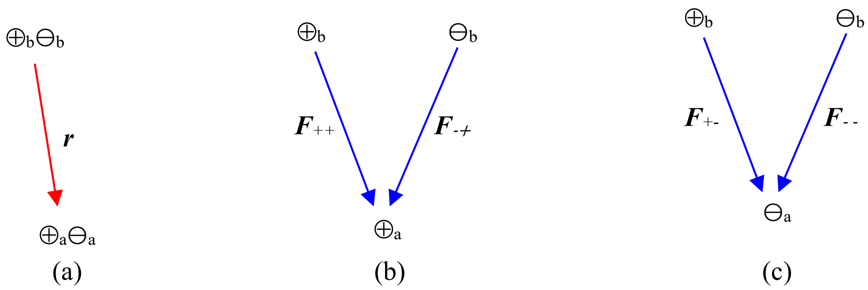

Figure 2

The electric charge of ⊕ is e and that of ⊖ is $-e$, then the products of charges $q_1 \cdot q_2$ for the 4 interactions are e^2, $-e^2$, $-e^2$ and e^2 respectively, see the line 2 of the Table 1. Applying e^2, $-e^2$, $-e^2$ and e^2 to the Coulomb's law (1) we get the 4 Coulomb's forces for the 4 interactions which are labeled as $\boldsymbol{F}_{++}$, $\boldsymbol{F}_{-+}$, $\boldsymbol{F}_{+-}$ and $\boldsymbol{F}_{--}$ and expressed in the line 3 of the Table 1. The sum of these 4 forces equals zero: $\boldsymbol{F}_{++} + \boldsymbol{F}_{-+} + \boldsymbol{F}_{+-} + \boldsymbol{F}_{--} = 0$.

1	$\oplus_b \to \oplus_a$	$\ominus_b \to \oplus_a$	$\oplus_b \to \ominus_a$	$\ominus_b \to \ominus_a$
2	$q_1 \cdot q_2 = e^2$	$q_1 \cdot q_2 = -e^2$	$q_1 \cdot q_2 = -e^2$	$q_1 \cdot q_2 = e^2$
3	$\boldsymbol{F}_{++} = \frac{e^2}{4\pi\varepsilon_0}\frac{\boldsymbol{r}}{\lvert\boldsymbol{r}\rvert^3}$	$\boldsymbol{F}_{-+} = -\frac{e^2}{4\pi\varepsilon_0}\frac{\boldsymbol{r}}{\lvert\boldsymbol{r}\rvert^3}$	$\boldsymbol{F}_{+-} = -\frac{e^2}{4\pi\varepsilon_0}\frac{\boldsymbol{r}}{\lvert\boldsymbol{r}\rvert^3}$	$\boldsymbol{F}_{--} = \frac{e^2}{4\pi\varepsilon_0}\frac{\boldsymbol{r}}{\lvert\boldsymbol{r}\rvert^3}$

Table 1 Coulomb's forces between the charges

b) Dynamic force across moving frames

How the relativistic dynamic effect modifies these 4 forces? Relativistic effect shows itself between two relatively moving frames. The Figure 3 shows a stationary body b_1 and a moving body b_2. The frame 1 is attached to b_1 and the frame 2 to b_2. The velocity of b_2 relative to b_1 is $\boldsymbol{v}$, so the frame 2 moves at the velocity $\boldsymbol{v}$ in the frame 1.

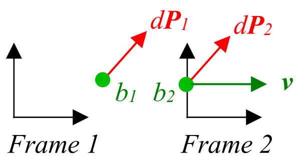

Figure 3

The momentum and time in the frame 1 are labeled as $\boldsymbol{P}_1$ and t_1. The force on b_1 equals the time derivative of $\boldsymbol{P}_1$:

$$\boldsymbol{F}_1 = \frac{d\boldsymbol{P}_1}{dt_1} \quad (2)$$

Let $\boldsymbol{F}_1$ be the Coulomb's force that two fixed charges exert on one another.

In the frame 2 the momentum and time are labeled as $\boldsymbol{P}_2$ and t_2, the force on b_2 equals the time derivative of $\boldsymbol{P}_2$:

$$\boldsymbol{F}_2 = \frac{d\boldsymbol{P}_2}{dt_2} \quad (3)$$

Let us express $\boldsymbol{F}_2$, the force in the frame 2, with the time of the frame 1, t_1 :

$$\boldsymbol{F}_2 = \frac{d\boldsymbol{P}_2}{dt_2} = \frac{d\boldsymbol{P}_2}{dt_1}\frac{dt_1}{dt_2} \quad (4)$$

Suppose that a differential momentum $d\boldsymbol{P}_1$ is transferred to the body b_1 and $d\boldsymbol{P}_2$ is transferred to the body b_2. Suppose that the body b_2 is in fact the body b_1 in motion. Then, $d\boldsymbol{P}_1$ and $d\boldsymbol{P}_2$ are in fact the same differential momentum transferred to the same body. So, $d\boldsymbol{P}_1$ equals $d\boldsymbol{P}_2$:

$$dP_1 = dP_2 \quad (5)$$

Note : For a more rigorous demonstration of this equality please see the equation (10) in « Relativistic dynamics: force, mass, kinetic energy, gravitation and dark matter » [1].

Then, we have :

$$\frac{d\boldsymbol{P_2}}{dt_1} = \frac{d\boldsymbol{P_1}}{dt_1} = \boldsymbol{F_1} \quad (6)$$

The time t_1 and t_2 are in 2 frames that are in relative motion. According to special relativity, t_1 is converted into t_2 with the Lorentz transformation below :

$$\frac{dt_1}{dt_2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (7)$$

Introducing (6) and (7) into (4) , the force $\boldsymbol{F_2}$ is expressed with the force $\boldsymbol{F_1}$:

$$\boldsymbol{F_2} = \frac{\boldsymbol{F_1}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (8)$$

Because b_2 is moving, $\boldsymbol{F_2}$ is the Coulomb's force acted on a charge moving at the velocity $\boldsymbol{v}$. Equation (8) converts the Coulomb's force acted on a fixed charge $\boldsymbol{F_1}$ into that on a moving charge $\boldsymbol{F_2}$.

c) Relative velocity between charges

Let $\boldsymbol{v}_a$ be the average velocity of the free electrons in the current element $d\boldsymbol{I}_a$, and $\boldsymbol{v}_b$ that in $d\boldsymbol{I}_b$. Let $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ be the velocities of two charges with respect to the stationary positive charges $\oplus_a$ and $\oplus_b$. The relative velocity between the two charges is labeled as $\boldsymbol{v}$ and equals $\boldsymbol{v}_1 - \boldsymbol{v}_2$. We compute the $\boldsymbol{v}$ for the 4 interactions $\oplus_b \rightarrow \oplus_a$, $\ominus_b \rightarrow \oplus_a$, $\oplus_b \rightarrow \ominus_a$, and $\ominus_b \rightarrow \ominus_a$ in Table 2.

$\oplus_a$	$\boldsymbol{v}_1 = 0$	$\oplus_b$	$\boldsymbol{v}_2 = 0$	$\boldsymbol{v}_1 - \boldsymbol{v}_2 = 0$	$\boldsymbol{v} = 0$
$\oplus_a$	$\boldsymbol{v}_1 = 0$	$\ominus_b$	$\boldsymbol{v}_2 = \boldsymbol{v}_b$	$\boldsymbol{v}_1 - \boldsymbol{v}_2 = 0 - \boldsymbol{v}_b = - \boldsymbol{v}_b$	$\boldsymbol{v} = - \boldsymbol{v}_b$
$\ominus_a$	$\boldsymbol{v}_1 = \boldsymbol{v}_a$	$\oplus_b$	$\boldsymbol{v}_2 = 0$	$\boldsymbol{v}_1 - \boldsymbol{v}_2 = \boldsymbol{v}_a - 0 = \boldsymbol{v}_a$	$\boldsymbol{v} = \boldsymbol{v}_a$
$\ominus_a$	$\boldsymbol{v}_1 = \boldsymbol{v}_a$	$\ominus_b$	$\boldsymbol{v}_2 = \boldsymbol{v}_b$	$\boldsymbol{v}_1 - \boldsymbol{v}_2 = \boldsymbol{v}_a - \boldsymbol{v}_b$	$\boldsymbol{v} = \boldsymbol{v}_a - \boldsymbol{v}_b$

Table 2 The relative velocities between the charges

d) Relativistic dynamic effect

The relativistic relation (8) defines the relativistic dynamic effect and will be applied to the 4 forces $\boldsymbol{F}_{++}$, $\boldsymbol{F}_{-+}$, $\boldsymbol{F}_{+-}$ and $\boldsymbol{F}_{--}$ given in the Table 1 with the corresponding $\boldsymbol{v}$ given in Table 2 to get the modified forces that we denote as $\boldsymbol{F'}_{++}$, $\boldsymbol{F'}_{-+}$, $\boldsymbol{F'}_{+-}$ and $\boldsymbol{F'}_{--}$.

For $\boldsymbol{F'}_{++}$, $\boldsymbol{v} = 0$:

$$\boldsymbol{F'}_{++} = \frac{\boldsymbol{F}_{++}}{\sqrt{1 - \frac{0^2}{c^2}}} = \frac{e^2}{4\pi\varepsilon_0} \frac{\boldsymbol{r}}{|\boldsymbol{r}|^3} \quad (9)$$

For $\boldsymbol{F'}_{-+}$, $\boldsymbol{v} = - \boldsymbol{v}_b$:

$$\boldsymbol{F'}_{-+} = \frac{\boldsymbol{F}_{-+}}{\sqrt{1 - \frac{\boldsymbol{v}_b{}^2}{c^2}}} = -\frac{e^2}{4\pi\varepsilon_0} \frac{\boldsymbol{r}}{|\boldsymbol{r}|^3} \frac{1}{\sqrt{1 - \frac{\boldsymbol{v}_b{}^2}{c^2}}} \quad (10)$$

[1] Kuan Peng, 2021, « Relativistic dynamics: force, mass, kinetic energy, gravitation and dark matter », https://www.academia.edu/49921891/Relativistic_dynamics_force_mass_kinetic_energy_gravitation_and_dark_matter

For $\boldsymbol{F'}_{+-}$, $\boldsymbol{v} = \boldsymbol{v}_a$:

$$\boldsymbol{F'}_{+-} = \frac{\boldsymbol{F}_{+-}}{\sqrt{1-\frac{\boldsymbol{v}_a{}^2}{c^2}}} = -\frac{e^2}{4\pi\varepsilon_0}\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}\frac{1}{\sqrt{1-\frac{\boldsymbol{v}_a{}^2}{c^2}}} \quad (11)$$

For $\boldsymbol{F'}_{--}$, $\boldsymbol{v} = \boldsymbol{v}_a - \boldsymbol{v}_b$:

$$\boldsymbol{F'}_{--} = \frac{\boldsymbol{F}_{--}}{\sqrt{1-\frac{(\boldsymbol{v}_a-\boldsymbol{v}_b)^2}{c^2}}} = \frac{e^2}{4\pi\varepsilon_0}\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}\frac{1}{\sqrt{1-\frac{(\boldsymbol{v}_a-\boldsymbol{v}_b)^2}{c^2}}} \quad (12)$$

As $\boldsymbol{v}_a$ and $\boldsymbol{v}_b$ are very small before the speed of light c, we can do the linear expansion :

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 1 + \frac{1}{2}\frac{v^2}{c^2} \quad (13)$$

Then, $\boldsymbol{F'}_{-+}$, $\boldsymbol{F'}_{+-}$ and $\boldsymbol{F'}_{--}$ become:

$$\boldsymbol{F'}_{-+} \approx -\frac{e^2}{4\pi\varepsilon_0}\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}\left(1+\frac{1}{2}\frac{\boldsymbol{v}_b{}^2}{c^2}\right) \quad (14)$$

$$\boldsymbol{F'}_{+-} \approx -\frac{e^2}{4\pi\varepsilon_0}\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}\left(1+\frac{1}{2}\frac{\boldsymbol{v}_a{}^2}{c^2}\right) \quad (15)$$

$$\boldsymbol{F'}_{--} \approx \frac{e^2}{4\pi\varepsilon_0}\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}\left(1+\frac{1}{2}\frac{(\boldsymbol{v}_a-\boldsymbol{v}_b)^2}{c^2}\right) \quad (16)$$

The velocity squared $\boldsymbol{v}^2$ is the vector $\boldsymbol{v}$ dotted by itself :

$$\boldsymbol{v}^2 = \boldsymbol{v}\cdot\boldsymbol{v} \quad (17)$$

We develop $(\boldsymbol{v}_a - \boldsymbol{v}_b)^2$:

$$(\boldsymbol{v}_a-\boldsymbol{v}_b)^2 = (\boldsymbol{v}_a-\boldsymbol{v}_b)\cdot(\boldsymbol{v}_a-\boldsymbol{v}_b) = \boldsymbol{v}_a{}^2 - 2\boldsymbol{v}_a\cdot\boldsymbol{v}_b + \boldsymbol{v}_b{}^2 \quad (18)$$

Introducing (18) into (16) gives :

$$\boldsymbol{F'}_{--} \approx \frac{e^2}{4\pi\varepsilon_0}\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}\left(1+\frac{\boldsymbol{v}_a{}^2 - 2\boldsymbol{v}_a\cdot\boldsymbol{v}_b + \boldsymbol{v}_b{}^2}{2c^2}\right) \quad (19)$$

The sum $\boldsymbol{F'}_{++} + \boldsymbol{F'}_{-+} + \boldsymbol{F'}_{+-} + \boldsymbol{F'}_{--}$ is the sum of (14), (15), (16) and (19) :

$$\boldsymbol{F'}_{ba} \approx \frac{e^2}{4\pi\varepsilon_0}\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}\left(1-\left(1+\frac{\boldsymbol{v}_b{}^2}{2c^2}\right)-\left(1+\frac{\boldsymbol{v}_a{}^2}{2c^2}\right)+\left(1+\frac{\boldsymbol{v}_a{}^2 - 2\boldsymbol{v}_a\cdot\boldsymbol{v}_b + \boldsymbol{v}_b{}^2}{2c^2}\right)\right) \quad (20)$$

The terms $\left(1+\frac{v_b{}^2}{2c^2}\right)$ and $\left(1+\frac{v_a{}^2}{2c^2}\right)$ are canceled and the force $\boldsymbol{F'}_{ba}$ equals :

$$\boldsymbol{F'}_{ba} \approx -\frac{e^2}{4\pi\varepsilon_0 c^2}\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}(\boldsymbol{v}_a\cdot\boldsymbol{v}_b) \quad (21)$$

$\boldsymbol{F'}_{ba}$ is the resultant force that the couple of charges $\oplus_b\ominus_b$ exerts on $\oplus_a\ominus_a$ which is modified by the relativistic dynamic effect. I have obtained this expression in «Length-contraction-magnetic-force between arbitrary currents»[2] using the length-contraction effect of relativity. But deriving from couples of charges is a more fundamental approach because the forces are on individual electric charges.

3. Changing distance effect

a) Distance change

The second relativistic effect is caused by the change of the electrons' position. The Figure 4 shows a positive charge ⊕ and an electron that moves from the position ⊖ to the position ⊖'. Because the distance between ⊖ and ⊕ varies from $\boldsymbol{r_0}$ to $\boldsymbol{r}$ the Coulomb's force the electron acts on ⊕ varies from $\boldsymbol{F_0}$ to $\boldsymbol{F}$. In the same way, the resultant Coulomb's force that $⊕_b⊖_b$ acts on $⊕_a⊖_a$ varies because the electrons $⊖_a$ and $⊖_b$ change positions. We call this variation the "Changing distance effect".

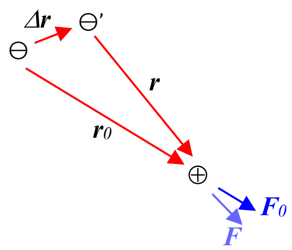

Figure 4

Let us compute the Coulomb's force $\boldsymbol{F}$ that the electron in Figure 4 exerts on ⊕ :

$$\boldsymbol{F} = -\frac{e^2}{4\pi\varepsilon_0}\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3} \quad (22)$$

Because $\boldsymbol{r}$ varies, we express $\boldsymbol{r} = \boldsymbol{r_0} + \Delta\boldsymbol{r}$. Then $\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}$ is expressed with $\Delta\boldsymbol{r}$:

$$\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3} = (\boldsymbol{r_0} + \Delta\boldsymbol{r})|\boldsymbol{r_0} + \Delta\boldsymbol{r}|^{-3} \quad (23)$$

We consider the case where $\Delta\boldsymbol{r}$ is very small before $\boldsymbol{r_0}$ and do the linear expansion **:**

$$\begin{aligned} |\boldsymbol{r_0} + \Delta\boldsymbol{r}|^{-3} &= ((\boldsymbol{r_0} + \Delta\boldsymbol{r})^2)^{-\frac{3}{2}} \\ &= (\boldsymbol{r_0}^2 + 2\boldsymbol{r_0}\cdot\Delta\boldsymbol{r} + \Delta\boldsymbol{r}^2)^{-\frac{3}{2}} \\ &\approx |\boldsymbol{r_0}|^{-3}\left(1 - 3\frac{\boldsymbol{r_0}\cdot\Delta\boldsymbol{r}}{|\boldsymbol{r_0}|^2}\right) \end{aligned} \quad (24)$$

Then $\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}$ becomes :

$$\begin{aligned} \frac{\boldsymbol{r}}{|\boldsymbol{r}|^3} &\approx (\boldsymbol{r_0} + \Delta\boldsymbol{r})|\boldsymbol{r_0}|^{-3}\left(1 - 3\frac{\boldsymbol{r_0}\cdot\Delta\boldsymbol{r}}{|\boldsymbol{r_0}|^2}\right) \\ &= |\boldsymbol{r_0}|^{-3}\left(\boldsymbol{r_0} + \Delta\boldsymbol{r} - 3\boldsymbol{r_0}\frac{\boldsymbol{r_0}\cdot\Delta\boldsymbol{r}}{|\boldsymbol{r_0}|^2} - 3\Delta\boldsymbol{r}\frac{\boldsymbol{r_0}\cdot\Delta\boldsymbol{r}}{|\boldsymbol{r_0}|^2}\right) \end{aligned} \quad (25)$$

Let $\boldsymbol{v}$ be the velocity of the electron and $\Delta\boldsymbol{r}$ be the distance traveled by the electron in time t, then $\Delta\boldsymbol{r}$ equals :

$$\Delta\boldsymbol{r} = \boldsymbol{v}t \quad (26)$$

We replace $\Delta\boldsymbol{r}$ with $\boldsymbol{v}t$ in (25) and $\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}$ becomes :

[2] Kuan Peng, 2017, «Length-contraction-magnetic-force between arbitrary currents», https://www.academia.edu/32815401/Length-contraction-magnetic-force_between_arbitrary_currents

$$\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3} \approx |\boldsymbol{r_0}|^{-3}\left(\boldsymbol{r_0} + \boldsymbol{v}t - 3\boldsymbol{r_0}\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}t}{|\boldsymbol{r_0}|^2} - 3\boldsymbol{v}t\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}t}{|\boldsymbol{r_0}|^2}\right) \quad (27)$$

b) Changing distance effect

The 4 Coulomb's forces modified by the changing distance effect are labeled as $\boldsymbol{F''}_{++}$, $\boldsymbol{F''}_{-+}$, $\boldsymbol{F''}_{+-}$ and, $\boldsymbol{F''}_{--}$ and are computed by introducing (27) into the expressions of $\boldsymbol{F}_{++}$, $\boldsymbol{F}_{-+}$, $\boldsymbol{F}_{+-}$ and $\boldsymbol{F}_{--}$ given in Table 1. The relative velocities are given in Table 2.

For $\boldsymbol{F''}_{++}$, $\boldsymbol{v} = 0$:

$$\boldsymbol{F''}_{++} = \frac{e^2}{4\pi\varepsilon_0}\frac{\boldsymbol{r}_0}{|\boldsymbol{r_0}|^3} \quad (28)$$

For $\boldsymbol{F''}_{-+}$, $\boldsymbol{v} = -\boldsymbol{v}_b$:

$$\boldsymbol{F''}_{-+} \approx -\frac{e^2}{4\pi\varepsilon_0}|\boldsymbol{r_0}|^{-3}\left(\boldsymbol{r_0} + (-\boldsymbol{v}_b)t - 3\boldsymbol{r_0}\frac{\boldsymbol{r_0}\cdot(-\boldsymbol{v}_b)t}{|\boldsymbol{r_0}|^2} - 3(-\boldsymbol{v}_b)t\frac{\boldsymbol{r_0}\cdot(-\boldsymbol{v}_b)t}{|\boldsymbol{r_0}|^2}\right) \quad (29)$$

For $\boldsymbol{F''}_{+-}$, $\boldsymbol{v} = \boldsymbol{v}_a$:

$$\boldsymbol{F''}_{+-} \approx -\frac{e^2}{4\pi\varepsilon_0}|\boldsymbol{r_0}|^{-3}\left(\boldsymbol{r_0} + \boldsymbol{v}_a t - 3\boldsymbol{r_0}\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}_a t}{|\boldsymbol{r_0}|^2} - 3\boldsymbol{v}_a\Delta t\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}_a t}{|\boldsymbol{r_0}|^2}\right) \quad (30)$$

For $\boldsymbol{F''}_{--}$, $\boldsymbol{v} = \boldsymbol{v}_a - \boldsymbol{v}_b$:

$$\boldsymbol{F''}_{--} \approx \frac{e^2}{4\pi\varepsilon_0}|\boldsymbol{r_0}|^{-3}\left(\boldsymbol{r_0} + (\boldsymbol{v}_a - \boldsymbol{v}_b)t - 3\boldsymbol{r_0}\frac{\boldsymbol{r_0}\cdot(\boldsymbol{v}_a - \boldsymbol{v}_b)t}{|\boldsymbol{r_0}|^2} - 3(\boldsymbol{v}_a - \boldsymbol{v}_b)t\frac{\boldsymbol{r_0}\cdot(\boldsymbol{v}_a - \boldsymbol{v}_b)t}{|\boldsymbol{r_0}|^2}\right) \quad (31)$$

The sum $\boldsymbol{F''}_{++} + \boldsymbol{F''}_{-+} + \boldsymbol{F''}_{-+} + \boldsymbol{F''}_{-+}$ is the sum of (28), (29), (30) and (31) :

$$\boldsymbol{F''}_t = \frac{e^2}{4\pi\varepsilon_0}|\boldsymbol{r_0}|^{-3}\begin{pmatrix} \boldsymbol{r_0} \\ -\left(\boldsymbol{r_0} + (-\boldsymbol{v}_b)t - 3\boldsymbol{r_0}\frac{\boldsymbol{r_0}\cdot(-\boldsymbol{v}_b)t}{|\boldsymbol{r_0}|^2} - 3(-\boldsymbol{v}_b)t\frac{\boldsymbol{r_0}\cdot(-\boldsymbol{v}_b)t}{|\boldsymbol{r_0}|^2}\right) \\ -\left(\boldsymbol{r_0} + \boldsymbol{v}_a t - 3\boldsymbol{r_0}\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}_a t}{|\boldsymbol{r_0}|^2} - 3\boldsymbol{v}_a t\frac{\boldsymbol{r_0}\cdot\boldsymbol{v}_a t}{|\boldsymbol{r_0}|^2}\right) \\ +\left(\boldsymbol{r_0} + (\boldsymbol{v}_a - \boldsymbol{v}_b)t - 3\,\boldsymbol{r_0}\frac{\boldsymbol{r_0}\cdot(\boldsymbol{v}_a - \boldsymbol{v}_b)t}{|\boldsymbol{r_0}|^2} - 3(\boldsymbol{v}_a - \boldsymbol{v}_b)t\frac{\boldsymbol{r_0}\cdot(\boldsymbol{v}_a - \boldsymbol{v}_b)t}{|\boldsymbol{r_0}|^2}\right) \end{pmatrix} \quad (32)$$

The term $\boldsymbol{r_0} + (\boldsymbol{v}_a - \boldsymbol{v}_b)t - 3\boldsymbol{r_0}\frac{\boldsymbol{r_0}\cdot(\boldsymbol{v}_a - \boldsymbol{v}_b)t}{|\boldsymbol{r_0}|^2}$ is canceled and $\boldsymbol{F''}_t$ becomes:

$$\boldsymbol{F''}_t = 3t^2\frac{e^2}{4\pi\varepsilon_0}|\boldsymbol{r_0}|^{-5}\left(\boldsymbol{v}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_b) + \boldsymbol{v}_a(\boldsymbol{r_0}\cdot\boldsymbol{v}_a) - (\boldsymbol{v}_a - \boldsymbol{v}_b)\big(\boldsymbol{r_0}\cdot(\boldsymbol{v}_a - \boldsymbol{v}_b)\big)\right) \quad (33)$$

We develop the term $(\boldsymbol{v}_a - \boldsymbol{v}_b)\big(\boldsymbol{r_0}\cdot(\boldsymbol{v}_a - \boldsymbol{v}_b)\big)$ **:**

$$(\boldsymbol{v}_a - \boldsymbol{v}_b)\big(\boldsymbol{r_0}\cdot(\boldsymbol{v}_a - \boldsymbol{v}_b)\big) = \boldsymbol{v}_a(\boldsymbol{r_0}\cdot\boldsymbol{v}_a) - \boldsymbol{v}_a(\boldsymbol{r_0}\cdot\boldsymbol{v}_b) - \boldsymbol{v}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_a) + \boldsymbol{v}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_b) \quad (34)$$

and introduce it into (33). Then the term $\boldsymbol{v}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_b) + \boldsymbol{v}_a(\boldsymbol{r_0}\cdot\boldsymbol{v}_a)$ is canceled and $\boldsymbol{F''}_t$ becomes :

$$\boldsymbol{F''}_t = 3t^2\frac{e^2}{4\pi\varepsilon_0}|\boldsymbol{r_0}|^{-5}\big(\boldsymbol{v}_a(\boldsymbol{r_0}\cdot\boldsymbol{v}_b) + \boldsymbol{v}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_a)\big) \quad (35)$$

Because $\boldsymbol{F''}_t$ varies with time, we take its average over the time period from $t = 0$ to t_e. The average value of $\boldsymbol{F''}_t$ equals the time integral of $\boldsymbol{F''}_t$ divided by t_e :

$$F''_{ba} = \frac{1}{t_e}\int_0^{te} F''_t dt \quad (36)$$

Using (35), (36) becomes :

$$\begin{aligned} F''_{ba} &= \frac{1}{t_e}\int_0^{t_e} 3t^2 \frac{e^2|\boldsymbol{r_0}|^{-5}}{4\pi\varepsilon_0}\left(\boldsymbol{v}_a(\boldsymbol{r_0}\cdot\boldsymbol{v}_b)+\boldsymbol{v}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_a)\right)dt \\ &= t_e{}^2\frac{e^2|\boldsymbol{r_0}|^{-5}}{4\pi\varepsilon_0}\left(\boldsymbol{v}_a(\boldsymbol{r_0}\cdot\boldsymbol{v}_b)+\boldsymbol{v}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_a)\right) \end{aligned} \quad (37)$$

c) What is t_e?

What is the value of t_e in (37) ? The answer is in the trajectory of electrons. Let us see the Figure 5 in which the trajectory of an electron is represented by the red arrows. The electron passes through the nodes $\ominus_0$, $\ominus_1$, $\ominus_2$, $\ominus_3$ and $\ominus_4$ … . At a distance from the trajectory is a charge q which receives the Coulomb's force from the moving electron.

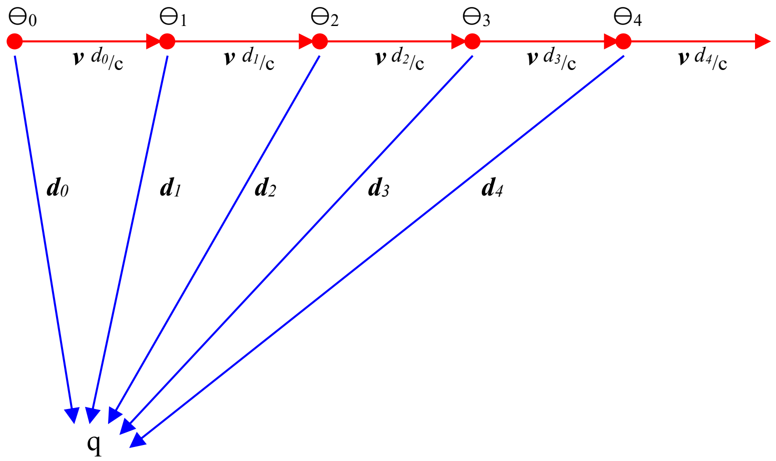

Figure 5

At the time $t = 0$, the electron is at the node $\ominus_0$ and emits the momentum $d\boldsymbol{P}_0$ to q. According to relativity electromagnetic signals travel at the speed of light c. Carried by electric field the momentum $d\boldsymbol{P}_0$ travels at c. The radial distance between $\ominus_0$ and q is $\boldsymbol{d}_0$. So, $d\boldsymbol{P}_0$ arrives at q at the time $t = {}^{|\boldsymbol{d}_0|}/c$. At this time the electron arrives at the node $\ominus_1$ and emits the momentum $d\boldsymbol{P}_1$ to q. The radial distance between $\ominus_1$ and q is $\boldsymbol{d}_1$ and $d\boldsymbol{P}_1$ arrives at q at the time $t = {}^{|\boldsymbol{d}_0|}/c + {}^{|\boldsymbol{d}_1|}/c$. The time period between the arrival of $d\boldsymbol{P}_0$ and $d\boldsymbol{P}_1$ equals :

$$t_0 = \frac{|\boldsymbol{d_0}|}{\boldsymbol{c}} + \frac{|\boldsymbol{d_1}|}{\boldsymbol{c}} - \frac{|\boldsymbol{d_0}|}{\boldsymbol{c}} = \frac{|\boldsymbol{d_1}|}{\boldsymbol{c}} \quad (38)$$

So, the first arrow is the trajectory of the electron during the time t_0. The momentum $d\boldsymbol{P}$ received by q in the time period dt equals :

$$d\boldsymbol{P} = \boldsymbol{F}dt \quad (39)$$

with $\boldsymbol{F}$ being the Coulomb's force on q.

The total momentum emitted by the electron between the node $\ominus_0$ and $\ominus_1$ and received by the charge q equals the integral of the momentum $\boldsymbol{P}$ when the electron is between $\ominus_0$ and $\ominus_1$:

$$\Delta\boldsymbol{P} = \int_{\ominus_0}^{\ominus_1} d\boldsymbol{P} = \int_0^{t_0} \boldsymbol{F}dt \quad (40)$$

So, the average Coulomb's force on q in the time period t_0 is :

$$\boldsymbol{F} = \frac{\Delta \boldsymbol{P}}{t_0} = \frac{1}{t_0}\int_0^{t_0} \boldsymbol{F} dt \quad (41)$$

Let the arrow between $\ominus_0$ and $\ominus_1$ be a trajectory element. Because the speed of light c is much bigger than the velocity of the electron, the radial distance between this trajectory element and the charge q equals the average vector $\boldsymbol{d}_a$:

$$\boldsymbol{d}_a = \frac{\boldsymbol{d}_0 + \boldsymbol{d}_1}{2} \approx \boldsymbol{d}_1 \quad (42)$$

Then, the time period for the integration of ***P*** given by (38) equals :

$$t_0 \approx \frac{|\boldsymbol{d}_a|}{\boldsymbol{c}} \quad (43)$$

The general trajectory element is the i^{th} trajectory element between $\ominus_i$ and $\ominus_{i+1}$, the radial distance between this trajectory element and the charge q is labeled as ***d*** and equals :

$$\boldsymbol{d} = \frac{\boldsymbol{d}_i + \boldsymbol{d}_{i+1}}{2} \quad (44)$$

Then, the time period for the integration of ***P*** for the general trajectory element is t_e and is obtained with (43) in which $\boldsymbol{d}_a$ is replaced by ***d*** and t_0 by t_e :

$$t_e \approx \frac{|\boldsymbol{d}|}{\boldsymbol{c}} \quad (45)$$

By combining (37) and (45) the average Coulomb's force on q from an arbitrary trajectory element is :

$$\boldsymbol{F}''_{ba} = \left(\frac{|\boldsymbol{d}|}{\boldsymbol{c}}\right)^2 \frac{e^2 |\boldsymbol{r_0}|^{-5}}{4\pi\varepsilon_0} \left(\boldsymbol{v}_a(\boldsymbol{r_0}\cdot\boldsymbol{v}_b) + \boldsymbol{v}_b(\boldsymbol{r_0}\cdot\boldsymbol{v}_a)\right) \quad (46)$$

Let us rename ***d*** and $\boldsymbol{r_0}$ as ***r*** :

$$\boldsymbol{r} = \boldsymbol{d} = \boldsymbol{r_0} \quad (47)$$

Then (46) becomes :

$$\boldsymbol{F}''_{ba} = \frac{e^2}{4\pi\varepsilon_0} \frac{1}{c^2 |\boldsymbol{r}|^3} \left(\boldsymbol{v}_a(\boldsymbol{r}\cdot\boldsymbol{v}_b) + \boldsymbol{v}_b(\boldsymbol{r}\cdot\boldsymbol{v}_a)\right) \quad (48)$$

One may ask : Since $\boldsymbol{F}''_{ba}$ is an averaged force, why cannot t_e be bigger or smaller than $|\boldsymbol{d}|/c$?

The trajectories of electrons are continuous lines. In order to form a continuous line, all the arrows of the trajectory elements must join end to end like the arrows in the Figure 5. Each arrow is the trajectory element for the time period for integration $t_e = |\boldsymbol{d}|/c$. If t_e is bigger than $|\boldsymbol{d}|/c$, the arrows overlap on one another making the integrated force bigger than the force from the continuous line trajectory, see Figure 6. If t_e is smaller than $|\boldsymbol{d}|/c$, the arrows do not join end to end making the integrated force smaller than the force from the continuous line trajectory, see the Figure 7.

Figure 6 $t_e > |\boldsymbol{d}|/c$

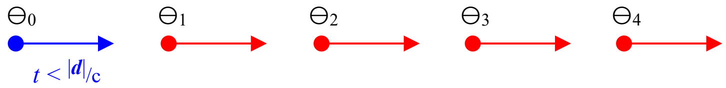

Figure 7 $t_e < {}^{|\boldsymbol{d}|}/_c$

In consequence, for an arbitrary trajectory element the time period for the integration of $\boldsymbol{P}$ is the t_e given by (45) and the average Coulomb's force that an electron exerts on the charge q is given by the equation (48).

The $\boldsymbol{F''}_{ba}$ given by (48) is the magnetic force due to the changing distance effect. This formula was already derived in «Changing distance effect»[3]. But the derivation of the changing distance effect and the explanation were different.

4. Complete magnetic force

a) Magnetic force on couples of charges

We have derived the Coulomb's forces that the couple of charge $\oplus_b\ominus_b$ exerts on $\oplus_a\ominus_a$ which are modified by the relativistic dynamic effect and the changing distance effect. The complete magnetic force equals the sum of the two modified forces which is the sum of (21) and (48) :

$$\boldsymbol{F}_{ba} = \boldsymbol{F'}_{ba} + \boldsymbol{F''}_{ba} \approx -\frac{e^2}{4\pi\varepsilon_0 c^2}\frac{\boldsymbol{r}}{|\boldsymbol{r}|^3}(\boldsymbol{v}_a\cdot\boldsymbol{v}_b) + \frac{e^2}{4\pi\varepsilon_0 c^2|\boldsymbol{r}|^3}\big(\boldsymbol{v}_a(\boldsymbol{r}\cdot\boldsymbol{v}_b) + \boldsymbol{v}_b(\boldsymbol{r}\cdot\boldsymbol{v}_a)\big) \quad (49)$$

which we rearrange into :

$$\boldsymbol{F}_{ba} \approx \frac{e^2}{4\pi\varepsilon_0 c^2|\boldsymbol{r}|^3}\big(-\boldsymbol{r}(\boldsymbol{v}_a\cdot\boldsymbol{v}_b) + \boldsymbol{v}_a(\boldsymbol{r}\cdot\boldsymbol{v}_b) + \boldsymbol{v}_b(\boldsymbol{r}\cdot\boldsymbol{v}_a)\big) \quad (50)$$

b) Magnetic force on current elements

A current element $d\boldsymbol{I}$ is an assembly of couple of charges $\oplus\ominus$. Let $\oplus_a\ominus_a$ be one couple of charges in the current element $d\boldsymbol{I}_a$ and $\oplus_b\ominus_b$ one in $d\boldsymbol{I}_b$. Let m be the number of $\oplus_a\ominus_a$ in $d\boldsymbol{I}_a$ and n be that of $\oplus_b\ominus_b$ in $d\boldsymbol{I}_b$. One $\oplus_b\ominus_b$ exerts the magnetic force $\boldsymbol{F}_{ba}$ on one $\oplus_a\ominus_a$. Then, n $\oplus_b\ominus_b$ exert the magnetic force $n\cdot\boldsymbol{F}_{ba}$ on one $\oplus_a\ominus_a$. Because $d\boldsymbol{I}_a$ contains m $\oplus_a\ominus_a$ and each $\oplus_a\ominus_a$ feels $n\cdot\boldsymbol{F}_{ba}$ from $d\boldsymbol{I}_b$, the total magnetic force that $d\boldsymbol{I}_b$ exerts on $d\boldsymbol{I}_a$ equals $n\cdot m\cdot\boldsymbol{F}_{ba}$.

Multiplying (50) with $n\cdot m$ we obtain the total magnetic force $\boldsymbol{F}_{Iba}$ that $d\boldsymbol{I}_b$ exerts on $d\boldsymbol{I}_a$:

$$\begin{aligned}\boldsymbol{F}_{Iba} &= m\cdot n\cdot\boldsymbol{F}_{ba}\\ &= \frac{1}{4\pi\varepsilon_0 c^2|\boldsymbol{r}|^3}\big(-\boldsymbol{r}(em\boldsymbol{v}_a\cdot en\boldsymbol{v}_b) + em\boldsymbol{v}_a(\boldsymbol{r}\cdot en\boldsymbol{v}_b) + en\boldsymbol{v}_b(\boldsymbol{r}\cdot em\boldsymbol{v}_a)\big)\end{aligned} \quad (51)$$

The charge of m electrons is $-m\cdot e$. Let $\boldsymbol{v}_a$ be the mean velocity of the free electrons in $d\boldsymbol{I}_a$, then the current element $d\boldsymbol{I}_a$ equals $-m\cdot e$ times $\boldsymbol{v}_a$:

$$d\boldsymbol{I}_a = -m\cdot e\boldsymbol{v}_a \quad (52)$$

In the same way, $d\boldsymbol{I}_b$ has n free electrons, the mean velocity of which is $\boldsymbol{v}_b$. Then the current element $d\boldsymbol{I}_b$ equals :

$$d\boldsymbol{I}_b = -n\cdot e\boldsymbol{v}_b \quad (53)$$

In (51) we replace $m\cdot e\cdot\boldsymbol{v}_a$ with $-d\boldsymbol{I}_a$ and $n\cdot e\cdot\boldsymbol{v}_b$ with $-d\boldsymbol{I}_b$. Then, $\boldsymbol{F}_{Iba}$ is expressed with $d\boldsymbol{I}_a$ and $d\boldsymbol{I}_b$:

$$\boldsymbol{F}_{Iba} = \frac{1}{4\pi\varepsilon_0 c^2|\boldsymbol{r}|^3}\big(-\boldsymbol{r}(d\boldsymbol{I}_a\cdot d\boldsymbol{I}_b) + d\boldsymbol{I}_a(\boldsymbol{r}\cdot d\boldsymbol{I}_b) + d\boldsymbol{I}_b(\boldsymbol{r}\cdot d\boldsymbol{I}_a)\big) \quad (54)$$

[3] Kuan Peng, 2018, «Changing distance effect», https://www.academia.edu/36272940/Changing_distance_effect

$\boldsymbol{F}_{Iba}$ is the elementary magnetic force that the current element $d\boldsymbol{I}_b$ exerts on $d\boldsymbol{I}_a$. Because this magnetic force is derived from the Coulomb's law, we call it "Coulomb magnetic force". I have already obtained this expression in «Coulomb magnetic force»[4].

Let us transform (54) with the double vector product identity below :

$$\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = -\boldsymbol{C}(\boldsymbol{A} \cdot \boldsymbol{B}) + \boldsymbol{B}(\boldsymbol{A} \cdot \boldsymbol{C}) \quad (55)$$

which we introduce into (54) with :

$$\boldsymbol{A} = d\boldsymbol{I}_a, \quad \boldsymbol{B} = d\boldsymbol{I}_b, \quad \boldsymbol{C} = \boldsymbol{r} \quad (56)$$

Then, the elementary magnetic force $\boldsymbol{F}_{Iba}$ is expressed with double vector product:

$$\boldsymbol{F}_{Iba} = \frac{1}{4\pi\varepsilon_0 c^2} \frac{d\boldsymbol{I}_a \times (d\boldsymbol{I}_b \times \boldsymbol{r}) + d\boldsymbol{I}_a(\boldsymbol{r} \cdot d\boldsymbol{I}_b)}{|\boldsymbol{r}|^3} \quad (57)$$

c) Force a coil exerts on a current element

Let $d\boldsymbol{I}_b$ be a current element of a coil, $\boldsymbol{F}_{coil}$ be the magnetic force the coil exerts on $d\boldsymbol{I}_a$. Then $\boldsymbol{F}_{coil}$ equals the closed line integral of $\boldsymbol{F}_{Iba}$ over the coil, with $d\boldsymbol{I}_a$ constant, see Figure 8 :

$$\boldsymbol{F}_{\text{Coil}} = \oint \frac{1}{4\pi\varepsilon_0 c^2} \left(\frac{d\boldsymbol{I}_a \times (d\boldsymbol{I}_b \times \boldsymbol{r})}{|\boldsymbol{r}|^3} + \frac{d\boldsymbol{I}_a(\boldsymbol{r} \cdot d\boldsymbol{I}_b)}{|\boldsymbol{r}|^3} \right) \quad (58)$$

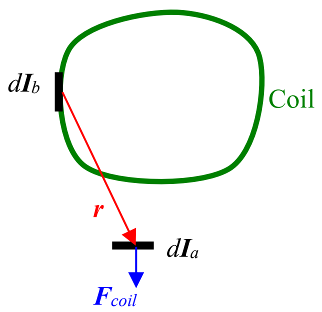

Figure 8

The closed line integral of the last term of (58) equals zero :

$$\oint \frac{d\boldsymbol{I}_a(\boldsymbol{r} \cdot d\boldsymbol{I}_b)}{|\boldsymbol{r}|^3} = d\boldsymbol{I}_a \oint \frac{\boldsymbol{r} \cdot d\boldsymbol{I}_b}{|\boldsymbol{r}|^3} = 0 \quad (59)$$

Then, the magnetic force the coil exerts on the current element $d\boldsymbol{I}_a$ equals :

$$\boldsymbol{F}_{\text{Coil}} = \oint d\boldsymbol{F} = \oint \frac{1}{4\pi\varepsilon_0 c^2} \frac{d\boldsymbol{I}_a \times (d\boldsymbol{I}_b \times \boldsymbol{r})}{|\boldsymbol{r}|^3} \quad (60)$$

We notice that $d\boldsymbol{F}$ the integrand of (60) equals the elementary Lorentz force that $d\boldsymbol{I}_b$ exerts on $d\boldsymbol{I}_a$:

$$\boldsymbol{F}_{\text{Lorentz}} = d\boldsymbol{F} = \frac{1}{4\pi\varepsilon_0 c^2} \frac{d\boldsymbol{I}_a \times (d\boldsymbol{I}_b \times \boldsymbol{r})}{|\boldsymbol{r}|^3} \quad (61)$$

[4] Kuan Peng, 2018, «Coulomb magnetic force», https://www.academia.edu/36278169/Coulomb_magnetic_force

5. Consequences

a) The relation $\mu_0\varepsilon_0 c^2 = 1$

Let $d\boldsymbol{I}_a$ be a current element placed in a magnetic field $\boldsymbol{B}$. The Lorentz force on $d\boldsymbol{I}_a$ equals :

$$\boldsymbol{F}_{\text{Lorentz}} = d\boldsymbol{I}_a \times \boldsymbol{B} \quad (62)$$

By comparing (62) with (61) we find that the magnetic field of $d\boldsymbol{I}_b$ derived in (61) equals :

$$\boldsymbol{B} = \frac{1}{\varepsilon_0 c^2}\frac{1}{4\pi}\frac{d\boldsymbol{I}_b \times \boldsymbol{r}}{|\boldsymbol{r}|^3} \quad (63)$$

The Biot–Savart law is :

$$\boldsymbol{B_b} = \mu_0 \frac{1}{4\pi}\frac{d\boldsymbol{I}_b \times \boldsymbol{r}}{|\boldsymbol{r}|^3} \quad (64)$$

By comparing (63) with (64) we find that the coefficient $\frac{1}{\varepsilon_0 c^2}$ that we have derived fully in theory equals the coefficient μ_0 of the Biot–Savart law :

$$\mu_0 = \frac{1}{\varepsilon_0 c^2} \quad (65)$$

Until now the relation $\mu_0\varepsilon_0 c^2 = 1$ is an empirical law because historically, the values of μ_0 , ε_0 and the speed of light c were measured experimentally. It was James Clerk Maxwell who noticed that c equals $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$. In our derivation above, this relation emerged naturally from both the relativistic dynamic effect and the changing distance effect. So, we have theoretically proven the relation $\mu_0\varepsilon_0 c^2 = 1$ making it a theoretical law.

b) Biot–Savart law

The Biot–Savart law (64) is identical to the equation (63) which is derived fully in theory. So, the Biot–Savart law becomes a theoretical law.

c) Lorentz force law

Because $\mu_0 = \frac{1}{\varepsilon_0 c^2}$, (61) can be written as :

$$\boldsymbol{F}_{\text{Lorentz}} = \frac{\mu_0}{4\pi}\frac{d\boldsymbol{I}_a \times (d\boldsymbol{I}_b \times \boldsymbol{r})}{|\boldsymbol{r}|^3} \quad (66)$$

(66) is identical to the elementary Lorentz force law for current elements $d\boldsymbol{I}_b$ and $d\boldsymbol{I}_a$, but it is derived fully in theory. So, the Lorentz force law becomes also a theoretical law.

d) Magnetic force vs. Newton's third law

- Elementary Lorentz force law

Let us compute the sum of the Lorentz force that $d\boldsymbol{I}_b$ exerts on $d\boldsymbol{I}_a$ and the back Lorentz force that $d\boldsymbol{I}_a$ exerts on $d\boldsymbol{I}_b$. The first force is given in (66) :

$$d\boldsymbol{F}_a = \frac{\mu_0}{4\pi|\boldsymbol{r}|^3} d\boldsymbol{I}_a \times (d\boldsymbol{I}_b \times \boldsymbol{r}) = \frac{\mu_0}{4\pi|\boldsymbol{r}|^3}\big(-\boldsymbol{r}(d\boldsymbol{I}_a \cdot d\boldsymbol{I}_b) + d\boldsymbol{I}_b(\boldsymbol{r} \cdot d\boldsymbol{I}_a)\big) \quad (67)$$

The back Lorentz force is obtained from (67) by replacing $\boldsymbol{r}_0$ with $-\boldsymbol{r}_0$, $d\boldsymbol{I}_a$ with $d\boldsymbol{I}_b$ and $d\boldsymbol{I}_b$ with $d\boldsymbol{I}_a$:

$$d\boldsymbol{F}_b = \frac{\mu_0}{4\pi|\boldsymbol{r}|^3} d\boldsymbol{I}_b \times \big(d\boldsymbol{I}_a \times (-\boldsymbol{r})\big) = \frac{\mu_0}{4\pi|\boldsymbol{r}|^3}\Big(-(-\boldsymbol{r})(d\boldsymbol{I}_b \cdot d\boldsymbol{I}_a) + d\boldsymbol{I}_a\big((-\boldsymbol{r}) \cdot d\boldsymbol{I}_b\big)\Big) \quad (68)$$

Adding (67) with (68) gives the sum $d\boldsymbol{F}_a + d\boldsymbol{F}_b$ which is not zero :

$$\begin{aligned} d\boldsymbol{F}_a + d\boldsymbol{F}_b &= \frac{\mu_0}{4\pi|\boldsymbol{r}|^3}\Big(-\boldsymbol{r}(d\boldsymbol{I}_a \cdot d\boldsymbol{I}_b) + d\boldsymbol{I}_b(\boldsymbol{r} \cdot d\boldsymbol{I}_a) - (-\boldsymbol{r})(d\boldsymbol{I}_b \cdot d\boldsymbol{I}_a) + d\boldsymbol{I}_a\big((-\boldsymbol{r}) \cdot d\boldsymbol{I}_b\big)\Big) \\ &= \frac{\mu_0}{4\pi|\boldsymbol{r}|^3}\big(d\boldsymbol{I}_b(\boldsymbol{r} \cdot d\boldsymbol{I}_a) - d\boldsymbol{I}_a(\boldsymbol{r} \cdot d\boldsymbol{I}_b)\big) \\ &\neq 0 \end{aligned} \quad (69)$$

So, the elementary Lorentz force law violates Newton's third law.

- New magnetic force law

The new magnetic force is given by (54) and its back force is obtained by replacing $\boldsymbol{r}_0$ with $-\boldsymbol{r}_0$, $d\boldsymbol{I}_b$ with $d\boldsymbol{I}_a$ and $d\boldsymbol{I}_a$ with $d\boldsymbol{I}_b$ in (54) :

$$\boldsymbol{F}_{Iab} = \frac{1}{4\pi\varepsilon_0 c^2|\boldsymbol{r}|^3}\Big(-(-\boldsymbol{r})(d\boldsymbol{I}_b \cdot d\boldsymbol{I}_a) + d\boldsymbol{I}_b\big((-\boldsymbol{r}) \cdot d\boldsymbol{I}_a\big) + d\boldsymbol{I}_a\big((-\boldsymbol{r}) \cdot d\boldsymbol{I}_b\big)\Big) \quad (70)$$

Adding (54) and (70) yields $\boldsymbol{F}_{Iba} + \boldsymbol{F}_{Iab} = 0$ because all the terms cancel out :

$$\boldsymbol{F}_{Iba} + \boldsymbol{F}_{Iab} = \frac{1}{4\pi\varepsilon_0 c^2|\boldsymbol{r}|^3}\begin{pmatrix} -\boldsymbol{r}(d\boldsymbol{I}_a \cdot d\boldsymbol{I}_b) + d\boldsymbol{I}_a(\boldsymbol{r} \cdot d\boldsymbol{I}_b) + d\boldsymbol{I}_b(\boldsymbol{r} \cdot d\boldsymbol{I}_a) \\ -(-\boldsymbol{r})(d\boldsymbol{I}_b \cdot d\boldsymbol{I}_a) + d\boldsymbol{I}_b\big((-\boldsymbol{r}) \cdot d\boldsymbol{I}_a\big) + d\boldsymbol{I}_a\big((-\boldsymbol{r}) \cdot d\boldsymbol{I}_b\big) \end{pmatrix} = 0 \quad (71)$$

So, the new magnetic force law (54) satisfies the Newton's third law.

- Comparison

Why the Lorentz force law violates the Newton's third law? By comparing (57) with (66) we find that (66) lacks the term $\frac{d\boldsymbol{I}_a(\boldsymbol{r}\cdot d\boldsymbol{I}_b)}{|\boldsymbol{r}|^3}$. Because all coils are closed loops, the magnetic force component corresponding to this term and from a coil equals the closed line integral of this term which equals zero, see (59).

When the Lorentz force law was established in the 19th century all experiments were done with coils. Because this magnetic force component is zero, it was not seen. This is why the Lorentz force law does not describe this magnetic force. Thanks to the fully theoretical derivation, the new magnetic force law (54) contains this term and satisfies Newton's third law, while the Lorentz force law does not.

6. Experimental evidence

e) My experiments

The magnetic force component corresponding to the term $\frac{d\boldsymbol{I}_a(\boldsymbol{r}\cdot d\boldsymbol{I}_b)}{|\boldsymbol{r}|^3}$ is parallel to the current element $d\boldsymbol{I}_a$, that is, this magnetic force component is parallel to the test current. So, the new magnetic force law describes parallel-to-current magnetic force which has never been detected. Because this magnetic force cannot be seen with coils, we have specially designed experiments with non-closed wire and have successfully shown parallel-to-current magnetic force.

The first experiment is «Continuous rotation of a circular coil experiment»[5]. The video of this experiment is: https://www.youtube.com/watch?v=9162Qw-wNow[6]. In this video we see a round coil rotating in its plane. Because the coil is round the driving force must be parallel to the wire, that is, to the current. This force cannot be Lorentz force which is perpendicular to the current. A detailed technical explanation is in the paper «Showing tangential magnetic force by experiment»[7] .

I have also made the experiment « Circular motor driven by tangential magnetic force »[8] . The video of this

[5] Kuan Peng, 2017, «Continuous rotation of a circular coil experiment», https://www.academia.edu/33604205/Continuous_rotation_of_a_circular_coil_experiment
[6] Kuan Peng, 2017, Video https://www.youtube.com/watch?v=9162Qw-wNow
[7] Kuan Peng, 2018, «Showing tangential magnetic force by experiment», https://www.academia.edu/36652163/Showing_tangential_magnetic_force_by_experiment
[8] Kuan Peng, 2014, « Circular motor driven by tangential magnetic force », https://www.academia.edu/6227926/Circular_motor_driven_by_tangential_magnetic_force

experiment is: https://www.youtube.com/watch?v=JkGUaJqa6nU&list=UUuJXMstqPh8VY4UYqDgwcvQ[9]. The technical details of this experiment is: « Detail of my circular motor using tangential force and the equivalence with homopolar motor » [10].

f) Experiment of wire fragmentation

In 1961, Jan Nasilowski in Poland has carried out an experiment consisting of passing a huge current in a thin wire which exploded the wire into small pieces. The interesting thing is that the wires were not melted but teared apart by mechanical force. The Figure 9 is a photograph of the exploded wires which shows that all the small pieces have approximately the same length. Jan Nasilowski has published his result in two papers [11, 12] which are cited by Lars Johansson in his Thesis[13] .

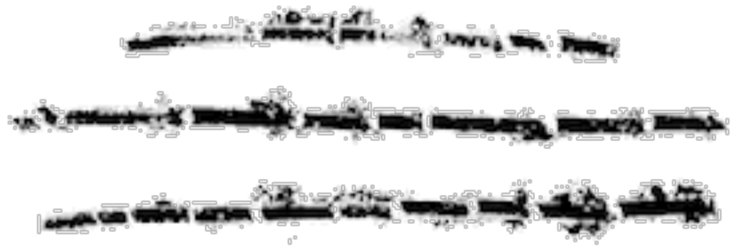

Figure 9

The magnetic force shown in this experiment is parallel to the current and strong enough to tear the wire apart. For explaining this experiment, let us compute the magnetic force in the wire using the new magnetic force law (54). Let $d\boldsymbol{I}_a$ and $d\boldsymbol{I}_b$ be two current elements of the wire and $\boldsymbol{r}$ the vector distance between them. Because $d\boldsymbol{I}_b \times \boldsymbol{r} = 0$, the magnetic force that $d\boldsymbol{I}_b$ exerts on $d\boldsymbol{I}_a$ is :

$$\boldsymbol{F}_{Iba} = \frac{1}{4\pi\varepsilon_0 c^2}\frac{d\boldsymbol{I}_a(\boldsymbol{r}\cdot d\boldsymbol{I}_b)}{|\boldsymbol{r}|^3} \quad (72)$$

The back magnetic force that $d\boldsymbol{I}_a$ exerts on $d\boldsymbol{I}_b$ is

$$\boldsymbol{F}_{Iab} = \frac{1}{4\pi\varepsilon_0 c^2}\frac{d\boldsymbol{I}_b(-\boldsymbol{r}\cdot d\boldsymbol{I}_a)}{|\boldsymbol{r}|^3} \quad (73)$$

$d\boldsymbol{I}_b$, $d\boldsymbol{I}_a$ and $\boldsymbol{r}$ are parallel, we write them in (74) with $\boldsymbol{e}_r$ their common unit vector :

$$d\boldsymbol{I}_a = I_a dl_a \boldsymbol{e}_r, \quad d\boldsymbol{I}_b = I_b dl_b \boldsymbol{e}_r, \quad \boldsymbol{r} = r\boldsymbol{e}_r \quad (74)$$

We introduce the expressions for $d\boldsymbol{I}_b$, $d\boldsymbol{I}_a$ and $\boldsymbol{r}$ into (72) and (73) and obtain :

$$\boldsymbol{F}_{Iba} = \frac{I_a dl_a \boldsymbol{e}_r(r\boldsymbol{e}_r \cdot I_b dl_b \boldsymbol{e}_r)}{4\pi\varepsilon_0 c^2 r^3} = \frac{rI_a dl_a I_b dl_b}{4\pi\varepsilon_0 c^2 r^3}\boldsymbol{e}_r \quad (75)$$

$$\boldsymbol{F}_{Iab} = \frac{I_b dl_b \boldsymbol{e}_r\big((-r\boldsymbol{e}_r)\cdot I_a dl_a \boldsymbol{e}_r\big)}{4\pi\varepsilon_0 c^2 r^3} = -\frac{rI_a dl_a I_b dl_b}{4\pi\varepsilon_0 c^2 r^3}\boldsymbol{e}_r \quad (76)$$

In Figure 10 we have plotted the current element $d\boldsymbol{I}_a$ with $\boldsymbol{F}_{Iba}$ the force on it and $d\boldsymbol{I}_b$ with $\boldsymbol{F}_{Iab}$ the force on it. Let S be a point of the wire. $d\boldsymbol{I}_a$ is on the right of S and $\boldsymbol{F}_{Iba}$ pulls $d\boldsymbol{I}_a$ to the right; $d\boldsymbol{I}_b$ is on the left of S and $\boldsymbol{F}_{Iab}$ pulls $d\boldsymbol{I}_b$ to the left. So, $\boldsymbol{F}_{Iba}$ and $\boldsymbol{F}_{Iab}$ create a tension that tears the point S. If the tension is strong enough, the wire

[9] Kuan Peng, 2014, Video https://www.youtube.com/watch?v=JkGUaJqa6nU&list=UUuJXMstqPh8VY4UYqDgwcvQ
[10] Kuan Peng, 2014, « Detail of my circular motor using tangential force and the equivalence with homopolar motor » , https://www.academia.edu/7879755/Detail_of_my_circular_motor_using_tangential_force_and_the_equivalence_with_homopolar_motor
[11] Jan Nasiowski Phenomena Connected with the Disintegration of Conductors Overloaded by Short-Circuit Current (in Polish) Przeglad Elektrotechniczny, 1961, p.397-403 52
[12] Jan Nasiowski Unduloids and striated Disintegration of Wires Exploding Wires, W.G. Chase, H.K. Moore Eds., Vol.3, Plenum, N.Y., 1964
[13] Lars Johansson, 1996, "Longitudinal electrodynamic forces | and their possible technological applications", https://deanostoybox.com/hot-streamer/temp/LongitudinalMSc.pdf

will break at S, which was the result of Jan Nasilowski's experiment. So, the parallel-to-current magnetic force predicted by the magnetic force law (54) explains well the breaking of the wire in Jan Nasiowski's experiment.

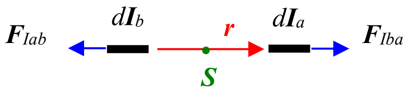

Figure 10

In the wire of Jan Nasilowski's experiment, the tension is created by the segments on either side of a point. Because the current is constant within the wire, the magnetic force per unit length is the same everywhere in the wire and segments of the same length create the same tension. Because the segments on either side of a breaking point should create the same tension, they should have the same length. This is why all the pieces of an exploded wire have approximately the same length. So, the parallel-to-current magnetic force predicted by the magnetic force law (54) explains well why all the pieces of an exploded wire have approximately the same length.

7. Result

1. We have derived fully in theory a new magnetic force law from Coulomb's law and relativity.
2. This law respects the Newton's third law and contains a component of magnetic force that is parallel to current. The Lorentz force law violates the Newton's third law because it lacks this component.
3. The Biot–Savart law and the Lorentz force law are derived fully in theory.
4. We have proven theoretically the experimental relation $\mu_0\varepsilon_0 c^2 = 1$ which shows that only ε_0 and c are fundamental constants, μ_0 is a secondary constant.
5. We have found two relativistic effects: the relativistic dynamic effect and the changing distance effect. It is these effects that transform electric force into magnetic force.
6. We have presented two of my experiments that show the action of parallel-to-current magnetic force.
7. Our new magnetic force law explains well the experiment of Jan Nasiowski : both for the breaking of the wires by mechanical force and the regularity of the small pieces of an exploded wire.
8. These experiments are strong evidence for the existence of parallel-to-current magnetic force.

For closed loop currents the new magnetic force law (54) gives the same prediction as the Lorentz force law, so it correctly describes the magnetic force for coil. The new parallel-to-current component of magnetic force is shown to be significant by experiments. It could be used to drive devices.

8. Discussion

The present electromagnetic theory is based on electric and magnetic fields which are defined by the Maxwell's equations. From magnetic field the Lorentz force law defines magnetic force. Let us call Maxwell's equations plus the Lorentz force law the Maxwell's theory.

The parallel-to-current magnetic force is not defined by magnetic field and thus, cannot be predict by the Lorentz force law. Because the new magnetic force law defines magnetic force without magnetic field and explains parallel-to-current magnetic force, it extends electromagnetism beyond the domain of the Maxwell's theory.

Physics is heavily based on the Maxwell's theory and has not had major breakthrough in the last century. It is possible that unknown phenomena such as the parallel-to-current magnetic force are the culprit for this stalemate. In fact, the Maxwell's theory is a mathematical formulation of the results of Ampère's and Faraday's experiments done in the 19th century which did not see the parallel-to-current magnetic force. It is normal that a theory does not predict phenomena that its founding experiments do not see and we should not be surprised if the Maxwell's theory does not predict some phenomena in particle physics which is well beyond the experiments of Ampère and Faraday. Without the guidance of a correct theory, how can particle physics make progress ?

One can argue that Quantum mechanics is extremely accurate at predicting subatomic phenomena, which proves that Maxwell's theory is correct. But, correct predictions do not prove that the used theory is correct. For example, Aristotle's theory about falling object predicts that lighter object falls slower than heavier object, and indeed, experiments in the air confirm his prediction. But we all know that his theory is wrong.

Every theory has a domain of validity within which the theory is correct. Outside this domain the theory could be wrong. We have found the parallel-to-current magnetic force that is out of the domain of validity of the Maxwell's theory, which shows that the Maxwell's theory does not predict all electromagnetic phenomena.

In consequence, we need a new electromagnetic theory that works beyond the domain of the experiments of the 19th century. The new electromagnetic theory will be constituted by the new magnetic force law derived above and a new law for the electromotive force induced by time-varying magnetic flux. This new electromagnetic theory will be a breakthrough in physics because a change of the Maxwell's theory will change large part of physics.

Appendices

1. Paradox of the angular coil

Let us see the Lorentz forces that the 2-parts coil shown in Figure 11 exerts on itself. The Lorentz force being perpendicular to current, the force on the current 1 is Force 1 and that on the current 2 is Force 2. Then the force on the angular part is Force 1 + Force 2. The Lorentz force on the circular part is Force on the circular part. When we compute the sum Force 1 + Force 2 + Force on the circular part we find that this sum is not zero. I call this sum the self-force of the coil, which is created from nothing and violates the Newton's third law. In the contrary, the new magnetic force law does not create the self-force and thus, respects the Newton's third law. Please see the computation of self-force in the article «Computation of the self force of a coil» [14].

For understanding this inconsistency, let us take the Force 1 and Force 2 as the Lorentz forces on two current elements near the vertex. If we move the two current elements towards the vertex, the distance between them tends to zero and Force 1 and Force 2 will become infinite. Since Force 1 and Force 2 do not cancel out, Force 1 + Force 2 becomes infinite which is inconsistent. See the analysis of self-force in the article «Lorentz force on open circuit»[15]. See also the list of my articles about this inconsistency below.

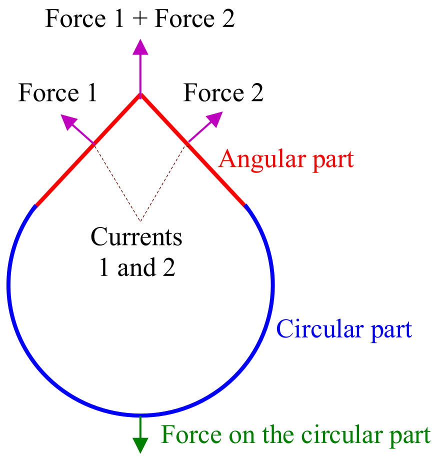

Figure 11 Coil with angular and circular parts

2. A list of my articles about the inconsistency of the Lorentz force law

a. Intuitive paradoxes

1. Analyze of the Lorentz forces internal to an equilateral triangle coil, html
http://pengkuanem.blogspot.com/2012/03/lorentz-forces-internal-to-equilateral.html

2 straight wires making an angle exert a force on each other and their sum lies on the bisector. The 2 bisector-lying forces on the lower corners of a triangular coil sum to a downward self force, violating Newton's third law.

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14 http://pengkuanem.blogspot.com/2013/05/computation-of-self-force-of-coil.html
http://www.academia.edu/3386868/Numerical_computation_of_the_Lorentz_force_internal_to_an_asymmetric_coil

15 http://pengkuanem.blogspot.com/2012/09/lorentz-force-on-open-circuit.html
http://www.academia.edu/1905835/Lorentz_force_on_open_circuit

2. Paradoxical Lorentz force internal to a triangle coil, html
http://pengkuanem.blogspot.com/2012/03/paradoxical-lorentz-force-internal-to.html

Place a wire inside a magnetic shield and it will not feel Lorentz force. Shield one side of a triangular coil, the non shielded sides will feel a Lorentz force which is a non-zero self force on the coil.

3. Lorentz forces internal to a polygon coil, analyze and computation, html
http://pengkuanem.blogspot.com/2012/03/lorentzforce-internal-to-coil-analyze.html

The pentagon coil has a sharp angular top and a high rectangular bottom. The top exerts on itself a force that depends on the angle, while the force on the rectangular bottom is constant. Varying independently, the forces on the top and the bottom do no cancel each other and make a non-zero self force.

4. Lorentz force on open circuit
http://pengkuanem.blogspot.com/2012/09/lorentz-force-on-open-circuit.html
http://www.academia.edu/1905835/Lorentz_force_on_open_circuit

The 2 ball capacitors enable an alternate current to circulate in the angular wire that creates a Lorentz force on the wire. Since there is not straight wire connecting the 2 capacitors there is no counter force and the self force is not zero.

5. Self force of a 3D coil
http://pengkuanem.blogspot.com/2014/11/self-force-of-3d-coil.html
https://www.academia.edu/9413326/Self_force_of_a_3D_coil

The 2 upward pointing corners of the 3D coil exert on themselves vertical forces that cannot be balanced by the forces on the 2 horizontal wires, violating Newton's third law.

b. Numerical computation

6. Numerical computation of the Lorentz force internal to an asymmetric coil
http://pengkuanem.blogspot.com/2013/04/numerical-computation-of-lorentz-force.html
http://www.academia.edu/3386868/Numerical_computation_of_the_Lorentz_force_internal_to_an_asymmetric_coil

Numerical computation of the self force of coils formed with 2 half ellipses of different major axis. The computed values of the self force are all non-zero.

7. Computation of the self force of a coil
http://pengkuanem.blogspot.com/2013/05/computation-of-self-force-of-coil.html
http://www.academia.edu/3386868/Numerical_computation_of_the_Lorentz_force_internal_to_an_asymmetric_coil

The self force of an asymmetric coil (2 different half ellipses) is computed using different number of discretization, which converges to a definite value showing the consistency of the computation.

8. Unhappiness of Newton with Lorentz and triangular coil experiment
http://pengkuanem.blogspot.com/2013/05/unhappiness-of-newton-with-lorentz-and.html
http://www.academia.edu/3500211/Unhappiness_of_Newton_with_Lorentz_and_triangular_coil_experiment
The self force of a triangular coil is computed using the Lorentz force law and the corrected magnetic force law, which is non-zero for the former and zero for the latter.

Letter to readers

Dear readers,

I have been working on new electromagnetism since year 1997 and written many articles which I sent to several journals of physics for publication. But all my articles were turned down. The main reason for the rejection is that I put the Maxwell's theory into question. Normal physicists believe that the Maxwell's theory is flawless and think that any challenge to it is absurd.

However, we have shown that the prediction of the Lorentz force law is wrong, which makes the energy and mass of high energy particles unreliable because derived with the Lorentz force law. So, refusing to solve the violation of the Newton's third law by the Lorentz force law harms the development of physics. Only a revision of electromagnetic theory can unblock the situation.

Revising the Maxwell's theory is such a radical change of paradigm that normal physicists will laugh at it. But few of them will understand and promote it. Later, in the same way as Niels Bohr, Werner Heisenberg, Max Born or Erwin Schrödinger were for Quantum mechanics, when the new theory is finally accepted, the early promoters will become the masters of the discipline.

Revision of Maxwell's theory is a type of revolution that occurs once in a century. You have the opportunity to be the early promoters of the new theory and become later great masters of physics. For promoting the new theory, you can carry out your own experiment, produce your own development of the theory. You can also spread the idea through your friends or colleagues who think they know all about electromagnetism, just challenge them to solve the paradox of the angular coil or those in the list above. These paradoxes are consequent of the inconsistency of the Maxwell's theory and cannot be solved. Beating them with these paradoxes will make them rethink about the electromagnetism they know and be interested in the new theory.

Because of the aversion to question the Maxwell's theory of the physical community, I got repeatedly rejected by mainstream journals which hurts me badly. So, I decide to publish my articles only online and count on the wise of you, my readers, to spread the new idea. Breakthroughs in science always fight against old convictions before being accepted, like when geocentrism was replaced by heliocentrism. Once accepted, it will be the happiest event for everyone who is involved. So, let us join our force to revolutionize physics.

Kuan Peng
9 December 2023