The simple structure of prime numbers

Ihsan Raja Muda Nasution

August 21, 2023

Abstract

The prime numbers have a pseudo-random structure. And this structure is not simple. In this paper, we analyze the behavior of prime numbers. And we diagnose the inner body of the prime numbers.

MSC: 11A41
Keywords: prime numbers, distribution of primes, critical line, structure of primes

1 Introduction

In 1859, Riemann [Rie59] showed that the location of zeros on the critical line implies the distribution of prime numbers. Our goal is to obtain the fundamental structure of primes.

2 The pattern of prime numbers

These below are several patterns of prime numbers.

Theorem 2.1 (Fundamental Theorem of Arithmetic). *Every integer $n > 1$ can be expressed as a product of primes; this representation is unique, apart from the order in which the factors occur [Bur02].*

Theorem 2.2 (Euclid). *There are infinitely many primes [Bur02].*

Theorem 2.3 (Dirichlet). *If a and b are coprime, then the arithmetic progression*

$$a, a+b, a+2b, a+3b, \ldots$$

contains infinitely many primes [Bur02].

Theorem 2.4 (Prime Number Thorem). *Let $x \in \mathbf{R}$, then*

$$\lim_{x\to\infty} \frac{\pi(x)}{x/\log x} = 1,$$

where $\pi(x) = \sum_{p\leq x} 1$ [Bur02].

Theorem 2.5 (Bertrand's Postulate). *For every integer $n > 1$, there is a prime p such that $n < p < 2n$ [Ros05].*

The *Goldbach's conjecture* asserts that every even integer greater than 2 can be written as the sum of two primes [Ros05].

Twin prime conjecture asserts that there are infinitely many pairs of primes p and $p + 2$ [Ros05].

3 The axiomatic approach

In this section, we propose the simple characteristics of prime numbers. The set of prime numbers obeys several basic assumptions. Given the number 0,1, a prime number p, and the set $\mathbf{N}$. Then

Postulate 3.1. $p \neq 0, 1$.

Postulate 3.1 shows the existence of 0 and 1 implicitly. This postulate says that any prime number p does not equal 0 and 1.

Postulate 3.2. $p^0 = 1$.

Postulate 3.2 shows the connection between the prime number p and 1. The unit 1 is generated by a prime number p over the number 0.

Postulate 3.3. $1 \mid p$.

Postulate 3.3 expresses the divisibility of primes over the unit 1. Postulate 3.2 deduces Postulate 3.3; i.e., $(p^0 = 1) \mid p$.

Postulate 3.4. $(-1)^p = \pm 1$.

Postulate 3.4 shows that the number ± 1 depends of the number -1 over any prime number p. Postulate 3.1 deduces Postulate 3.4 by using 0 and 1; i.e., given 0 and 1, then $(0 - 1)^p = (-1)^p = \pm 1$.

Postulate 3.5. $p < p + 1$.

Postulate 3.5 shows the ordered structure of prime numbers. Postulate 3.1 deduces Postulate 3.5; i.e., given 0 and 1, then $p + 0 < p + 1$ if and only if $p < p + 1$. We see that $p + 1$ is the successor of p.

Postulate 3.6. $0^p \in \mathbf{N}$.

First postulate of Peano Postulate says that $0 \in \mathbf{N}$. Postulate 3.6 can deduce first postulate of Peano Postulate; i.e., $0^p = 0 \in \mathbf{N}$.

References

[Bur02] D. M. Burton. *Elementary Number Theory.* McGraw-Hill, New York, 5th edition, 2002.

[Rie59] B. Riemann. Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse. *Monatsber. Akad. Berlin*, pages 671–680, 1859.

[Ros05] K. H. Rosen. *Elementary Number Theory and Its Applications.* Addison-Wesley, Boston, 5th edition, 2005.