

An Interesting Case of Twins Paradox - Twins Approaching Each Other

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Abstract

In two recent papers the meaning of the Twins paradox is criticised interestingly and one of the configurations is described in both papers similarly. Therefore, it is easy to address some features of the papers simultaneously. The configuration is about the approaching of the twins in one inertial frame of reference that exhibit complete symmetry. We describe that configuration criticising the approach of both papers with the support of our recent work “The Resolution of the Twin Paradox with Three Frames of Reference – A Mathematical and Physical Report”, where we report how we can use a time dilation-like expression eliminating the twin paradox conundrum that standard formulation is unable to accomplish, since misleadingly attribute the meaning of a reciprocal relation to the ageing of the twins. The misinterpretation of the time-like dilation mathematical expression as the time dilation expression.

Introduction

In the following *Fig. of Section I*. O represent a point of an inertial frame S that we designate Einstein Frame (EF). The frame where the speed of light is isotropic. For frames moving with speed v in relation to EF the speed of light is anisotropic. Therefore, EF is the Preferred Frame [1]. Frames S'' and S' are the frames of twins A and B . If $O(S)$ is the EF , we have complete symmetry and therefore Einstein synchronization is effective as P. Mohazzabi and Q. Luo assume [2]. And more recently has been published by G. Alencar a similar configuration [3]. However, another symmetry can be accomplished with the adoption of Lorentzian coordinates, that standard interpretation avoid to consider although we publish papers about this matter several years ago and recently [4-13]. But this equivalence is restricted [7], and it is no more a complete equivalence. However, this solve the conundrum (see the solution in the general case [1]) and now we intend to clarify this matter with this simple configuration.

The standard formulation consider

$$dt_L'' = d\tau'' = dt_L' \sqrt{\left(1 - \frac{v_E'^2}{c^2}\right)} \quad (1)$$

the relation of the ageing of the twins [1, 2] – this is the origin of the conundrum: “The twin paradox is the consequence of the following thought experiment. System O is at rest and system O' is moving. Therefore, the clock in O' ticks slower than that in O . Thus, for example, if the two clocks are initially synchronized to read $t=t'=0$, after a while they may show $t=10$ (some arbitrary unit of time) but $t'=6$. Therefore, an observer moving with system O' will be younger than that in system O . However, as seen by the observer

in O' , she is at rest and system O is moving away from her. Therefore, according to the observer in O' , the observer in O should be younger. This is the foundation of the twin paradox, which is stated as follows: twin A is on Earth and twin B travels to a distant star with a speed close to the speed of light. Afterward, she returns to Earth with the same speed. “When they reunite, according to twin A , twin B must be younger, but according to twin B , twin A must be younger” [2]. Of course, twin A cannot be simultaneously younger and older than twin B [1]. And of course either, twin B cannot be simultaneously younger and older than twin A [1].

I. Twin Paradox as a Misconception

If S is the EF , Einstein synchronization is effective and simultaneity has the current meaning of the same instant with synchronized clocks and not the meaning of Einstein simultaneity [13] with the same instant with desynchronized clocks, Lorentzian clocks. Therefore, we approach the configuration with this presuppose to impose a complete symmetry. Therefore, the twins A and B moves from the initial positions with equal distance L to O at a simultaneous instant $t=0$, the same instant (when the frames initiated the acceleration reset the clock to zero simultaneously with the clocks of S marking zero).

The configuration referred is “ **Twins Approaching Each Other**” [2]:

“Consider twins, A and B , both initially at rest with respect to an inertial frame and separated by distance d . They synchronize their clocks according to the following method. When the clock of twin A reads $t_A=0$, she sends a light signal towards twin B . This light signal takes a time d/c to reach twin B . So, when twin B receives the light signal, she sets her clock to $t_B=d/c$ [2]. Then at a time that the two twins had previously agreed upon, they start moving towards each other with equal accelerations relative to an inertial frame O at their midpoint. The accelerations are very large but take place in a very short time (essentially a Dirac δ function) resulting in relativistic speeds. The two twins then start moving towards each other, each with a constant speed v relative to the other, as shown in [Figure 2](#).

According to twin A , twin B is moving with speed v . Therefore, when they reach each other at the midpoint O , the clock of B should show a shorter time than the clock of A , *i.e.*, $t_B < t_A$. On the other hand, according to twin B , twin A is moving with speed v . Therefore, when they reach each other, the clock of A should show a shorter time than the clock of B , *i.e.*, $t_A < t_B$. In this situation, the system is completely symmetric.

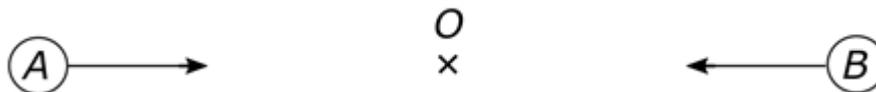


Figure 2. Twins A and B approaching each other with relative speed v . Neither twin leaves her reference frame, and both have the same initial acceleration. Therefore, none of the suggested explanations can resolve the paradox in this case.”

Of course, those affirmations are not completely accurate. Moahazzabi and Luo seems to accept the veracity of what they think is time dilation and therefore accept the impossible affirmation of the reciprocity of ageing. When it is possible that the ageing of the twins are equal and therefore both twins has the same ageing, the same age when arrive at O and are younger than the twin at O . Simple. Without any paradox. Indeed, because both twins moves with the same speed in relation to S (EF) the time dilation for each twin is equal between the events departure and meeting since

$$d\tau' = dt \sqrt{1 - \frac{v_1^2}{c^2}} \quad (2)$$

$$d\tau'' = dt \sqrt{1 - \frac{v_2^2}{c^2}} \quad (3)$$

where $v_1 = v$ and $v_2 = -v$, and $d\tau'$ and $d\tau''$ are respectively the differential of proper times of S' and S'' in relation to the preferred frame with $dt = d\tau$ since for the preferred frame there is no intrinsic desynchronization when $v_1 = 0$ [4-13]

$$t'_L = t' - \frac{v_1}{c^2} x' = t' = t \Rightarrow dt = d\tau \quad (4)$$

This is eventually a remarkable result that standard formulation is not aware [1, 2] originating the Twin Paradox conundrum.

The time dilation means that a clock moving in relation to the Preferred Frame has time dilated [1] (see also Pirooz Mohazabbi and Qinghua Luo article [2]).

But from (2) and (3)

$$d\tau'' = d\tau' \frac{\sqrt{1 - \frac{v_2^2}{c^2}}}{\sqrt{1 - \frac{v_1^2}{c^2}}} \quad (5)$$

Therefore, the relation of proper times show that for a generic frame moving in relation to EF we can have the ageing contracted or dilated or equal and not only dilated as standard interpretation induce to think [1]. Because what standard approach consider is the time-like dilation has we designate it [1].

Therefore, because both twins A and B move with equal speeds in relation to S , EF , both twins are ageing equally and therefore when reach O both are equally younger that twin O , both have the same age although the time dilation-like expression continue valid but without the meaning of ageing less than twins located in EF . This is a result of using Lorentz coordinates as we explain previously [1]. Therefore, we have a misconception in the origin of the paradox.

If we consider O (S) located at EF the analysis is simple. Twins A and B (Bob and John in Alencar article [3]) ageing less than O (or any twin located at S as Alice in Alencar article). A twin moving from Alice to Air and returning to Earth is younger and twins A and B meet at the same instant at O since

$$\tau'' = \tau' = \frac{L}{v} \sqrt{\left(1 - \frac{v^2}{c^2}\right)} \quad (6)$$

The frame of A is S'' with $v_2 = v$ and the frame of B is S' with $v_1 = -v$ [1].

We have

$$V'_E = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}} = \frac{2v}{1 + \frac{v^2}{c^2}} \quad (7)$$

$$d\tau' = dt \sqrt{1 - \frac{v_1^2}{c^2}} = dt \sqrt{1 - \frac{v^2}{c^2}} \quad (8)$$

$$d\tau'' = dt \sqrt{1 - \frac{v_2^2}{c^2}} = dt \sqrt{1 - \frac{v^2}{c^2}} \quad (9)$$

From (8) and (9)

$$d\tau'' = d\tau' \quad (10)$$

$$\tau'' = \tau' = \frac{L}{v} \sqrt{1 - \frac{v^2}{c^2}} \quad (11)$$

From (5) and (7) eliminating v_2 we obtain

$$d\tau'' = \frac{d\tau'}{1 + \frac{v_1}{c^2} V'_E} \sqrt{\left(1 - \frac{V'^2}{c^2}\right)} \quad (12)$$

and from [7] since $v_1 = -v$

$$1 + \frac{v_1}{c^2} V'_E = 1 - v \frac{2v}{1 + \frac{v^2}{c^2}} \frac{1}{c^2} = \frac{1 - \frac{v^2}{c^2}}{1 + \frac{v^2}{c^2}} \quad (13)$$

Since

$$V_E' = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}} = \frac{2v}{1 + \frac{v^2}{c^2}} \quad (14)$$

we obtain,

$$\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} = \frac{1 - \frac{v^2}{c^2}}{1 + \frac{v^2}{c^2}} \quad (15)$$

From Lorentz transformation [1]

$$dt_L'' = d\tau'' = \frac{dt_L' - \frac{V_E'}{c^2} dt_L'}{\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} = \frac{dt_L' \left(1 - \frac{V_E'^2}{c^2}\right)}{\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} = dt_L' \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \quad (16)$$

From (6), (15) and (16)

$$\begin{aligned} \tau'' = \tau' &= \frac{L}{v} \sqrt{1 - \frac{v^2}{c^2}} \\ \tau'' = \tau' &= \Delta t_L' \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \end{aligned} \quad (17)$$

we obtain,

$$\Delta t_L' = \frac{L}{v} \sqrt{\left(1 - \frac{v^2}{c^2}\right)} \frac{1 + \frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} = \frac{L}{v} \frac{1 + \frac{v^2}{c^2}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \quad (18)$$

Since

$$\Delta x' = V_E' \Delta t_L' \quad (19)$$

$$V_E' = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}} = \frac{2v}{1 + \frac{v^2}{c^2}} \quad (20)$$

therefore

$$\Delta x' = V_E' \Delta t_L' = \frac{2v}{1 + \frac{v^2}{c^2}} \frac{L}{v} \frac{1 + \frac{v^2}{c^2}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = \frac{2L}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \quad (21)$$

We see the consistency of time dilation and time dilation-like with eq. (17) without the meaning that standard interpretation attribute to time between the events arrival and departure with Lorentzian clocks. Only at EF Lorentz clocks are synchronized clocks as eq. (4) reveal. If we consider a twin of Alice located at planet “Fire” also located at frame S (let’s call Mary to this twin) we can use the time dilation-like equation for the ageing of Mary when Bob departure from Alice and arrive at Mary and this ageing is also the ageing of Alice, τ , between the same events and also the same time for the events departure and arrival of John trip from “Fire” to “Earth” (note that $|V'_E| = v$ for this configuration)

$$\tau = \Delta t'_L \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} = \frac{2L}{\sqrt{1 - \frac{v^2}{c^2}}} \frac{1}{v} \sqrt{1 - \frac{v^2}{c^2}} = \frac{2L}{v} \quad (22)$$

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Appendix

From (11)

$$dt_L'' = d\tau'' = \frac{dt_L' - \frac{V_E'}{c^2} V_E' dt_L'}{\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} = \frac{dt_L' \left(1 - \frac{V_E'^2}{c^2}\right)}{\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} = dt_L' \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \quad (17)$$

This seems time dilation (eq. (2)) but it is not (eq. (2), (3) and (5) and the following (eq. (19)).

Notice

$$t_L' = t' - \frac{v_1}{c^2} x'$$

$$dt_L' = dt' - \frac{v_1}{c^2} V_E' dt_L'$$

$$dt_L' \left(1 + \frac{v_1}{c^2} V_E'\right) = dt' = d\tau' \quad (18)$$

We assume as obvious $dt' = d\tau'$. The assumption of standard relativity is similar but with Lorentzian clocks (eq. (17), with $dt_L' = dt' = d\tau'$ and of course this standard assumption it is not correct). This standard assumption originates the conundrum.

Therefore from (17) and (18)

$$d\tau'' = \frac{d\tau'}{1 + \frac{v_1}{c^2} V_E'} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \quad (19)$$

And from (5)

$$d\tau'' = d\tau' \frac{\sqrt{1 - \frac{v_2^2}{c^2}}}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{d\tau'}{1 + \frac{v_1}{c^2} V_E'} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \quad (20)$$

$$\frac{\sqrt{1 - \frac{v_2^2}{c^2}}}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{1}{1 + \frac{v_1}{c^2} V_E'} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} = \frac{d\tau''}{d\tau'} \quad (21)$$

Therefore if

$$[v_2] > [v_1] \quad d\tau'' < d\tau' \quad (23)$$

$$[v_2] < [v_1] \quad d\tau'' > d\tau' \quad (24)$$

$$v_2 = -v_1 \quad d\tau'' = d\tau' \quad (25)$$

In previous works [1-17] particularly in “The physical meaning of synchronization and simultaneity in Special Relativity” [1] it is criticized the approach of Einstein [18] based on the postulates of the isotropy of speed light in every frame and the equivalence of every frame. Several works, some very recent, point out the importance of this discussion about the foundations of Mathematics, Philosophy, Relativity, Quantum Mechanics, Cosmology and Biophysics [19-104]. The consequent Principle of Relativity has been also considered in the articles “On the Consistency between the Assumption of a Special System of Reference and Special Relativity” [10] and “The Principle of Relativity and the Indeterminacy of Special Relativity” [12] and “Special Relativity as a simple geometry problem” introducing “Feynman clock” associated to a preferred frame, Einstein Frame, with time dilation [13]. In a more recent work “Speakable and Unspeakable in Special Relativity: time readings and clock rhythms” [14] it is referred the consequences of these analysis particularly the physical meaning of time dilation and Lorentz-FitzGerald contraction mathematical expressions.

Twin A'' is moving through the x' axis of S' with Einstein speed $|V_E'|$. At $x' = l_1$ the twin return with speed $|V_E'|$ to the origin of S' . The proper times of the twin A'' , τ'' for the trip to and τ'' for the trip from, are calculated. The proper times of the twins located at S' , τ' , is also calculated between the same events. We show how the standard formulation misinterpret the relation of proper times, the ageing of the twins at S'' and S' . For that we calculate through the time dilation – like equation the proper times τ'' with the Lorentzian times. It is easy to show the misinterpretation of the standard formulation

through the equality of the two-way trip result, that is consistent with the one-way results, as expected.

Consider S with a rod with length l between O and x of the x axis.

Also consider S' and S'' moving with v_1 and v_2 through the direction of the x axis (Fig. 1).

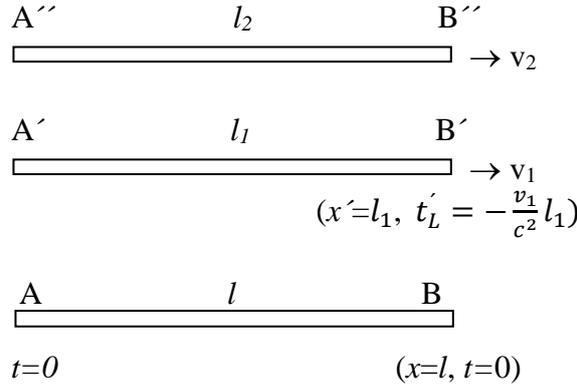


Fig. 1 Frame S' represented by a rod with length l_1 is moving with speed v_1 in relation to frame S , EF, rod with length l . The extremities of the rods coincide simultaneously and therefore, can synchronize clocks at A, A', A'' and B, B', B'' [49, 50]. A twin located at A' we designate by twin A' . The same rule for the other positions.

S is Einstein Frame (EF) as we previously designate it [8], the frame where the one-way speed of light is isotropic (in vacuum) with the value c , the two-way speed of light experimentally measured. It is also assumed that the speed of light in S is independent of the movement of the source. With these assumptions a Lorentz transformation has been obtained by us introducing an intrinsic desynchronization, as we designate it, in the IST transformation [1, 6, 8]

$$t'_L = t' - \frac{v_1}{c^2} x' \quad (1)$$

where t'_L is the Lorentzian time and t' is the synchronized time. Also by Georgy I. Burde more recently [37]. This has been achieved previously (2002-EPS12 Trends in Physics) [1-3] and the published results about the time dilation meaning and also the meaning of the Lorentz-FitzGerald contraction are also referred by by Zbigniew Oziewicz in relation to the resolution of the twin paradox conundrum in several works, particularly [33]. In relation to the Preferred Frame the time dilation formula has a clear mathematical and physical meaning. Indeed

$$d\tau' = dt \sqrt{1 - \frac{v_1^2}{c^2}} \quad (2)$$

$$d\tau'' = dt \sqrt{1 - \frac{v_2^2}{c^2}} \quad (3)$$

where $d\tau'$ and $d\tau''$ are the differential of proper times of S' and S'' in relation to the preferred frame with $dt = d\tau$ since for the preferred frame there is no intrinsic desynchronization when $v_1 = 0$

$$t'_L = t' - \frac{v_1}{c^2} x' = t' = t \Rightarrow dt = d\tau \quad (4)$$

This is eventually a remarkable result that standard formulation is not aware [97, 98] originating the Twin Paradox conundrum.

The time dilation means that a clock moving in relation to the Preferred Frame has time dilated (see Mohazabbi and Luo article [97, 98, 33]).

But from (2) and (3)

$$d\tau'' = d\tau' \frac{\sqrt{1 - \frac{v_2^2}{c^2}}}{\sqrt{1 - \frac{v_1^2}{c^2}}} \quad (5)$$

The Lorentz Transformation between S , S' and S'' is

$$x' = \frac{x - v_1 t}{\sqrt{\left(1 - \frac{v_1^2}{c^2}\right)}} \quad (6)$$

$$t'_L = \frac{t - \frac{v_1}{c^2} x}{\sqrt{\left(1 - \frac{v_1^2}{c^2}\right)}} \quad (7)$$

$$x'' = \frac{x - v_2 t}{\sqrt{\left(1 - \frac{v_2^2}{c^2}\right)}} \quad (8)$$

$$t''_L = \frac{t - \frac{v_2}{c^2} x}{\sqrt{\left(1 - \frac{v_2^2}{c^2}\right)}} \quad (9)$$

From (6), (7), (8) and (9) [6, 8, 99]

$$x'' = \frac{x' - V'_E t'_L}{\sqrt{\left(1 - \frac{V'^2_E}{c^2}\right)}} \quad (10)$$

$$t''_L = \frac{t'_L - \frac{V'_E}{c^2} x'}{\sqrt{\left(1 - \frac{V'^2_E}{c^2}\right)}} \quad (11)$$

with Einstein velocity [6, 8, 33, 99]

$$V'_E = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}} \quad (12)$$

but with a meaning that standard approach cannot accomplished, naturally.

Indeed, from (6) and (7) for $x=l$ and $t=0$

$$x' = l_1 = \frac{l - v_1 \times 0}{\sqrt{\left(1 - \frac{v_1^2}{c^2}\right)}} \quad (13)$$

$$x'' = l_2 = \frac{l - v_2 \times 0}{\sqrt{\left(1 - \frac{v_2^2}{c^2}\right)}} \quad (14)$$

Therefore,

$$\frac{l_2}{l_1} = \frac{\sqrt{\left(1 - \frac{v_1^2}{c^2}\right)}}{\sqrt{\left(1 - \frac{v_2^2}{c^2}\right)}} \quad (15)$$

From (10) for $(x' = l_1, t'_L = -\frac{v_1}{c^2} l_1)$ (see Fig. 1)

$$l_2 = \frac{l_1 + V'_E \frac{v_1 l_1}{c^2}}{\sqrt{\left(1 - \frac{V'^2_E}{c^2}\right)}} = \frac{l_1}{\sqrt{\left(1 - \frac{V'^2_E}{c^2}\right)}} \left(1 + V'_E \frac{v_1}{c^2}\right) \quad (16)$$

From (11)

$$dt_L'' = d\tau'' = \frac{dt_L' - \frac{V_E'}{c^2} V_E' dt_L'}{\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} = \frac{dt_L' \left(1 - \frac{V_E'^2}{c^2}\right)}{\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} = dt_L' \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \quad (17)$$

This seems time dilation (eq. (2)) but it is not (eq. (2), (3) and (5) and the following (eq. (19))).

Notice

$$\begin{aligned} t_L' &= t' - \frac{v_1}{c^2} x' \\ dt_L' &= dt' - \frac{v_1}{c^2} V_E' dt_L' \\ dt_L' \left(1 + \frac{v_1}{c^2} V_E'\right) &= dt' = d\tau' \quad (18) \end{aligned}$$

We assume as obvious $dt' = d\tau'$. The assumption of standard relativity is similar but with Lorentzian clocks (eq. (17), with $dt_L' = dt' = d\tau'$ and of course this standard assumption it is not correct). This standard assumption originates the conundrum.

Therefore from (17) and (18)

$$d\tau'' = \frac{d\tau'}{1 + \frac{v_1}{c^2} V_E'} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \quad (19)$$

And from (5)

$$d\tau'' = d\tau' \frac{\sqrt{1 - \frac{v_2^2}{c^2}}}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{d\tau'}{1 + \frac{v_1}{c^2} V_E'} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \quad (20)$$

$$\frac{\sqrt{1 - \frac{v_2^2}{c^2}}}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{1}{1 + \frac{v_1}{c^2} V_E'} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} = \frac{d\tau''}{d\tau'} \quad (21)$$

Therefore if

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$$v_2 = -v_1 \quad d\tau'' = d\tau' \quad (25)$$

Pirooz Mohazzabi and Qinghua Luo are partially right. Standard relativity seems ... does not resolve the twin paradox. The ageing of the twins is given by (20) and the statement of reciprocity based on the time dilation-like expression given by (17) is inaccurate based on the error of the synchronization of Lorentzian clocks.

Indeed, if $v_2 > v_1$ and twin A'' is moving from A' to B' with

$$V_E' = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}} > 0 \quad (26)$$

When A'' pass by A', B'' pass by B' and all twins has proper times zero. When twin A'' arrive at twin B' the ageing of twin A'' is τ''

$$\tau'' = \frac{l_1}{V_E'} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} = \Delta t_L' \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \quad (27)$$

and B' simultaneously moves through S'' from B'' to A'' ageing, using the time dilation-like expression (17),

$$\tau' = \frac{l_2}{V_E'} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} = \Delta t_L'' \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \quad (28)$$

From (16)

$$\tau' = \frac{l_2}{V_E'} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} = \frac{l_1 \left(1 + V_E' \frac{v_1}{c^2}\right)}{V_E' \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \quad (29)$$

and from (27)

$$\tau' = \tau'' \frac{\left(1 + V_E' \frac{v_1}{c^2}\right)}{\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} \quad (30)$$

Consistently with

$$\tau'' = \tau' \frac{\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}}{\left(1 + V_E' \frac{v_1}{c^2}\right)} = \Delta t_L' \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \quad (31)$$

The Lorentzian times $\Delta t_L''$ and $\Delta t_L'$, $\frac{l_2}{V_E}$ and $\frac{l_1}{V_E}$ are not the proper times τ' and τ'' . This is what standard interpretation assume and Pirooz Mohazzabi and Qinghua Luo also affirm, questioning however the complete accuracy of the standard interpretation.

If we consider the “returning twin” with equal $[V_E']$ but with, from (24)

$$[v_2] < [v_1] \quad d\tau'' > d\tau' \quad (32)$$

This can be described with B'' moving with $V_E' < 0$ from B' to A' (Fig.1) with

$$\tau'' = \frac{l_1}{[V_E']} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} = \Delta t_L' \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \quad (33)$$

and simultaneously A' moving from A'' to B'' (Fig. 1) with

$$\tau' = \frac{l_2}{[V_E']} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} = \frac{l_1 \left(1 - [V_E'] \frac{v_1}{c^2}\right)}{V_E' \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)} \quad (34)$$

$$\tau' = \tau'' \frac{\left(1 - [V_E'] \frac{v_1}{c^2}\right)}{\sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}} \quad (35)$$

Therefore, we obtain as expected for the two-way proper times τ'' and τ' (the addition of proper times for the trip to and fro)

$$\tau'' = \tau' \sqrt{1 - \frac{V_E'^2}{c^2}} = \frac{2l_1}{|V_E'|} \sqrt{1 - \frac{V_E'^2}{c^2}} \quad (36)$$

The standard formulation consider (17)

$$d\tau'' = dt_L' \sqrt{\left(1 - \frac{V_E'^2}{c^2}\right)}$$

the relation of the ageing of the twins [97] – this is the origin of the conundrum:

“The twin paradox is the consequence of the following thought experiment. System O is at rest and system O' is moving. Therefore, the clock in O' ticks slower than that in O . Thus, for example, if the two clocks are initially synchronized to read $t=t'=0$, after a while they may show $t=10$ (some arbitrary unit of time) but $t'=6$. Therefore, an observer moving with system O' will be younger than that in system O . However, as seen by the observer in O' , she is at rest and system O is moving away from her. Therefore, according to the observer in O' , the observer in O should be younger. This is the foundation of the twin paradox, which is stated as follows: Twin A is on Earth and twin B travels to a distant star with a speed close to the speed of light. Afterward, she returns to Earth with the same speed. When they reunite, according to twin A , twin B must be younger, but according to twin B , twin A must be younger “[97]. This is not so. The time dilation-like (time dilation is valid in relation to a preferred frame) and considered by the standard formulation the time dilation expression, valid reciprocally. This conundrum has been eliminated. We have shown how we can use this time dilation-like expression eliminating the twin paradox conundrum that standard formulation is unable to accomplish, since erroneously attribute the meaning of a reciprocal relation to the ageing of the twins. Therefore, we calculate the classic example of the twins whatever the frames considered. The twin that returns is the younger because the cumulative effect of the ageing is not reciprocal. Since the time dilation-like exist and can be used, originating the idea of “seeing the other twin ageing slower”- the origin of the conundrum. This cannot subsist because the relation between ageing is a relation between proper times. The time dilation-like expression is a relation between proper times only for the preferred frame.

Another example of the conundrum is “ **Twins Approaching Each Other**“ [97]:

“Consider twins, A and B , both initially at rest with respect to an inertial frame and separated by distance d . They synchronize their clocks according to the following method. When the clock of twin A reads $t_A=0$, she sends a light signal towards twin B . This light signal takes a time d/c to reach twin B . So, when twin B receives the light signal, she sets her clock to $t_B=d/c$ [2]. Then at a time that the two twins had previously agreed upon, they start moving towards each other with equal accelerations relative to an

inertial frame O at their midpoint. The accelerations are very large but take place in a very short time (essentially a Dirac δ function) resulting in relativistic speeds. The two twins then start moving towards each other, each with a constant speed v relative to the other, as shown in Figure 2.

According to twin A , twin B is moving with speed v . Therefore, when they reach each other at the midpoint O , the clock of B should show a shorter time than the clock of A , *i.e.*, $t_B < t_A$. On the other hand, according to twin B , twin A is moving with speed v . Therefore, when they reach each other, the clock of A should show a shorter time than the clock of B , *i.e.*, $t_A < t_B$. In this situation, the system is completely symmetric;



Figure 2. Twins A and B approaching each other with relative speed v . Neither twin leaves her reference frame, and both have the same initial acceleration. Therefore, none of the suggested explanations can resolve the paradox in this case”

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