

New Consideration of Electromagnetic Induction

- Faraday's Law of Electromagnetic Induction vs. Maxwell's Equation

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Abstract: Based on theoretical analysis and experimental verification, this study proves that the electromotive force truly expressed by Faraday's law of electromagnetic induction is the open-circuit electromotive force of a metal loop. Faraday's law of electromagnetic induction should be revised to Faraday's law of open-circuit electromagnetic induction. The electromotive force of a closed metal loop is equal to zero, that is, the line integral of the electric field intensity along a closed metal loop must be equal to zero. Therefore, Maxwell's mathematical expression of electromagnetic induction is inconsistent with Faraday's law of electromagnetic induction. This study further proposes a symmetrical metal closed loop - an equipotential metal current ring. The charges in the equipotential metal current ring are not affected by an electric field force, and are only affected by the Lorentz magnetic field force. Theoretical and experimental verification prove that the electric potential and electric field intensity in an equipotential metal current ring are equal to zero everywhere, and a changing magnetic field cannot induce an electric field in the equipotential metal current ring. Expanding the equipotential metal current ring to the vacuum, it can be concluded that in the vacuum, a changing magnetic field cannot induce an electric field, which is a great challenge to Maxwell's "electromagnetic waves" theory.

Keywords: Faraday's law of open-circuit electromagnetic induction, Equipotential metal current ring, Lorentz's magnetic field force theorem, Helmholtz coil, Maxwell's equations.

1. Introduction

In 1831, Faraday revealed for the first time through experiments that the change of magnetic flux in a metal coil could produce induced current and voltage in the metal coil.

As shown in Figure 1.1, an n-turn metal coil is connected in series with voltmeter V. When magnetic rod B is inserted or pulled out of metal coil C, coil C generates an electromotive force, and the pointer of voltmeter V deflects. The faster the insertion or withdrawal, the greater the electromotive force generated by coil C. When the magnetic rod stops in the coil, there is no electromotive force in the coil and the pointer of the voltmeter V does not deflect.

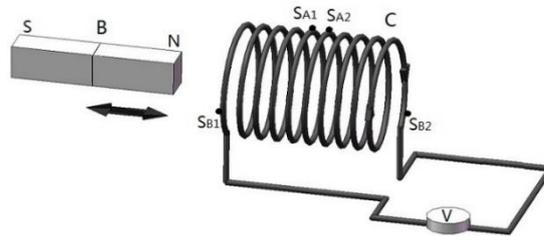


Figure 1.1 Electromagnetic induction of a metal coil

With a large number of experiments^{[1] [2] [3]}, Faraday revealed that the induction electromotive force generated in a metal coil was proportional to the change rate of the magnetic flux passing through the coil. This conclusion is called Faraday's law of electromagnetic induction. The induction electromotive force is expressed in Faraday's law as:

$$\varepsilon_1 = - d\Phi/dt \tag{1-1}$$

where "-" indicates the direction of the induction electromotive force, which is determined by Lenz's law.

The electromotive force in formula (1-1) is that of the one-turn coil, which is the induction electromotive force between points SA₁ and SA₂. For the n-turn metal coil C, the induction electromotive force between the two endpoints SB₁ and SB₂ of the coil is:

$$\varepsilon_n = - n d\Phi/dt \tag{1-2}$$

Faraday's law of electromagnetic induction states that whenever the magnetic flux passing through the coil changes, an electromotive force is generated in the coil. There are two ways the magnetic flux can change. The first is when the magnetic field intensity remains constant, and the whole or part of the metal loop moves in the magnetic field. The electromotive force generated in this way is called the motional electromotive force. The second is when no part of the metal loop moves while the space magnetic field changes. The electromotive force generated in this way is called the induced electromotive force.

In Figure 1.1 above, the electromotive force generated by coil C is the induced electromotive force. Below is an example to further analyze and illustrate the motional electromotive force.

As shown in Figure 1.2, which is cited from physics textbooks [4] [5]. In uniform magnetic field B , metal wireframe ABCD is placed. The wireframe has two parts. The fixed U-shaped part of the wireframe is composed of metal wires AD, AB and BC. The metal wire CD can slide left and right, and its length is L_{CD}

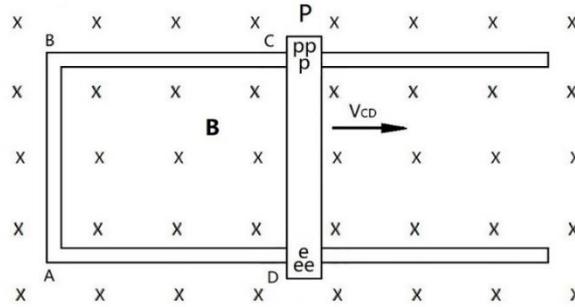


Figure 1.2 Motional electromotive force of a metal wire

When metal wire CD is not in contact with the fixed U-shaped part of the wireframe, that is, the circuit is open, and metal wire CD moves to the right at speed V_{CD} , the magnetic flux increased in wireframe ABCD within dt time is:

$$d\Phi = V_{CD} L_{CD} B dt \quad (1-3)$$

Then, the change rate of the magnetic flux passing through wireframe ABCD is:

$$d\Phi/dt = V_{CD} B L_{CD} \quad (1-4)$$

According to equations (1-1) and (1-4), the open-circuit electromotive force of metal wire CD is:

$$\varepsilon_{CDO} = V_{CD} B L_{CD} \quad (1-5)$$

The metal wire CD is equivalent to a battery, where the C end is positive and the D end is negative.

According to the definitions of electric potential and electric field, Maxwell expressed the electromotive force as the line integral of the electric field intensity along a closed metal loop. Maxwell integral type Faraday's law of electromagnetic induction is:

$$\oint_L E dl = - d\Phi/dt \quad (1-6)$$

In Figure 1.1, voltmeter V and metal coil C are connected in series to form a closed circuit, and the internal resistance of voltmeter V is much greater than that of metal coil C . The induced electromotive force ε_n measured by the voltmeter V is the open-circuit electromotive force between end S_{B1} and end S_{B2} . Similarly, end S_{A1} and end S_{A2} are not in contact with each other, and ε_1 is the open-circuit induced electromotive force of the one-turn coil.

In Figure 1.2, metal wire CD is not in contact with the fixed U-shaped part of the wireframe, so the motional electromotive force ε_{CDO} by Formula (1-5) is the open-circuit electromotive force of metal wire CD.

In summary, whether it is a motional electromotive force or an induced electromotive force, the electromotive force expressed by Faraday's law of electromagnetic induction is the open-circuit electromotive force of a metal loop. Therefore, Faraday's law of electromagnetic induction should be revised to Faraday's law of open-circuit electromagnetic induction.

2. Motional electromotive force of a metal wire

Referring to Figure 1.2, below, we further analyze the motional electromotive force with specific examples. For intuitive understanding, it is assumed that the magnetic induction intensity B of a uniform magnetic field is 10^{-3} Tesla, the length of metal wires AB and CD is $L_{AB}=L_{CD}=0.1$ m. Wire CD can move left and right, and its speed $V_{CD}=1$ m/s. The length of wire BC is L_{BC} , and the length of wire DA is L_{DA} , L_{BC} and L_{DA} vary between 0.1 m and 0.2 m. The resistance value of metal wire 0.1 m is 0.1 ohms, the resistance values R_{AB} and R_{CD} of metal wires AB and CD are $R_{AB}=R_{CD}=0.1$ ohms, and the resistance values R_{BC} and R_{DA} of metal wires BC and DA vary between 0.1 ohms and 0.2 ohms.

When metal wire CD and the fixed U-shaped part of the wireframe are not in contact, that is, the circuit is open, and metal wire CD moves to the right at speed $V_{CD}=1$ m/s. According to Formula (1-5) of Faraday's law of electromagnetic induction, the open-circuit electromotive force of wire CD is:

$$\begin{aligned} \varepsilon_{CDO} &= V_{CD} B L_{CD} = 1 \times 10^{-3} \times 0.1 \\ \varepsilon_{CDO} &= 0.0001 \text{ V} \end{aligned} \quad (2-1)$$

When metal wire CD moves right from the P position to the P1 position, $L_{BC}=L_{DA}=0.12$ m. The open-circuit motional electromotive force of wire CD remains unchanged, $\varepsilon_{CDO} = 0.0001$ V, as shown in Figure 2.1

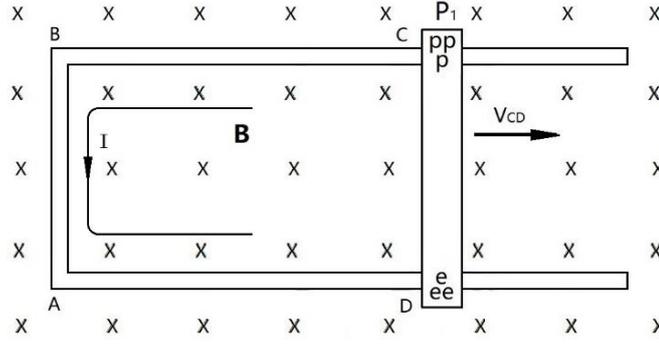


Figure 2.1 Motional electromotive force of wire CD at P₁

If metal wire CD moves right from position P to position P₁, metal wire CD and the fixed U-shaped part of the wireframe is connected, that is, the circuit is closed. Metal wire CD is equivalent to a battery with a voltage of 0.0001V, where the C end is positive and the D end is negative. Internal resistance $R_{CD} = 0.1$ ohms, and the fixed U-shaped part of the wire is equivalent to the load resistance:

$$R_L = R_{BC} + R_{AB} + R_{DA} = 0.12 + 0.10 + 0.12$$

$$R_L = 0.34 \text{ ohms}$$

The current value in the DCBAD closed loop is:

$$I = U_{CDO} / (R_L + R_{CD})$$

$$I = 0.00022727 \text{ A}$$

In the closed loop at the P₁ position, the closed-circuit electromotive force of wire CD is:

$$\epsilon_{CDC} = U_{CDO} - I R_{CD} = 0.0001 - 0.00022727 \times 0.1$$

$$\epsilon_{CDC} = 0.00007727 \text{ V} \quad (2-2)$$

In the closed loop at the P₁ position, the voltages of metal wires BC, AB and DA are:

$$\epsilon_{BCC} = I R_{BC} = -0.00002727 \text{ V}$$

$$\epsilon_{ABC} = I R_{AB} = -0.00002273 \text{ V}$$

$$\epsilon_{DAC} = I R_{DA} = -0.00002727 \text{ V}$$

In the closed loop at the P₁ position, according to Formula (1-6), the line integral of the electric field intensity along the closed path of DCBAD is:

$$\oint_L E \, dl = \int_{CD} E_{CD} \, dl + \int_{BC} E_{BC} \, dl + \int_{AB} E_{AB} \, dl + \int_{DA} E_{DA} \, dl$$

$$= \epsilon_{CDC} + \epsilon_{BCC} + \epsilon_{ABC} + \epsilon_{DAC}$$

$$= 0.00007727 - 0.00002727 - 0.00002273 - 0.00002727$$

$$= 0.00007727 - 0.00007727$$

$$\oint_L E \, dl = 0.00000000 \text{ V} \quad (2-3)$$

Formula (2-3) indicates that at position P₁, the line integral of the electric field intensity E along the closed path is equal to zero, that is, the closed-circuit electromotive force of the DCBAD metal loop is equal to zero. When metal wire CD moves right at speed V_{CD}, load resistance R_L changes, and electric field intensity E in the loop also changes simultaneously.

Furthermore, when metal wire CD moves right from position P₁ to position P₂, metal wire CD and the fixed U-shaped part of the wireframe are not connected, as shown in Figure 2.2. $L_{BC} = L_{DA} = 0.15$ m, the open-circuit electromotive force of wire CD remains unchanged and it is:

$$\epsilon_{CDO} = 0.0001 \text{ V} \quad (2-4)$$

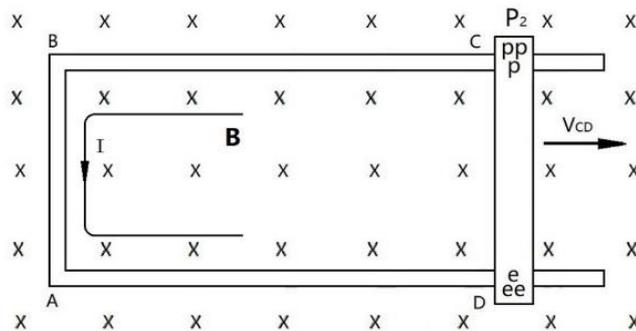


Figure 2.2 Motional electromotive force of wire CD at P₂

If metal wire CD moves right from position P₁ to position P₂, metal wire CD and the fixed U-shaped part of the wireframe are connected, forming a closed loop. Internal resistance R_{CD} = 0.1 ohms, and the fixed U-shaped part of the wire is equivalent to the load resistance, which is:

$$\begin{aligned} R_L &= R_{BC} + R_{AB} + R_{DA} \\ &= 0.15 + 0.10 + 0.15 \\ R_L &= 0.40 \text{ ohms} \end{aligned}$$

The current value in the DCBAD closed loop is:

$$\begin{aligned} I &= U_{CDO} / (R_L + R_{CD}) \\ I &= 0.0002 \text{ A} \end{aligned}$$

In the closed loop at the P₂ position, the closed-circuit electromotive force of wire CD is:

$$\begin{aligned} \varepsilon_{CDC} &= U_{CDO} - I R_{CD} \\ &= 0.0001 - 0.0002 \times 0.1 \\ \varepsilon_{CDC} &= 0.00008 \text{ V} \end{aligned} \quad (2-5)$$

In the closed loop at the P₂ position, the voltages of metal wires BC, AB and DA are:

$$\begin{aligned} \varepsilon_{BCC} &= I R_{BC} = -0.00003 \text{ V} \\ \varepsilon_{ABC} &= I R_{AB} = -0.00002 \text{ V} \\ \varepsilon_{DAC} &= I R_{DA} = -0.00003 \text{ V} \end{aligned}$$

In the closed loop at the P₂ position, according to Formula (1-6), the line integral of the electric field intensity E along the closed path of DCBAD:

$$\begin{aligned} \oint_L E \, dl &= \int_{CD} E_{CD} \, dl + \int_{BC} E_{BC} \, dl + \int_{AB} E_{AB} \, dl + \int_{DA} E_{DA} \, dl \\ &= \varepsilon_{CDC} + \varepsilon_{BCC} + \varepsilon_{ABC} + \varepsilon_{DAC} \\ &= 0.00008 - 0.00003 - 0.00002 - 0.00002 \\ &= 0.00008 - 0.00008 \\ \oint_L E \, dl &= 0.00000 \text{ V} \end{aligned} \quad (2-6)$$

Formula (2-6) indicates that at position P₂, the line integral of the electric field intensity E along a closed path is equal to zero, that is, the closed-circuit electromotive force of the DCBAD metal loop is equal to zero. When metal wire CD moves right at speed V_{CD}, load resistance R_L changes, and electric field intensity E in the loop also changes simultaneously.

Based on the theoretical analysis and calculation of the motional electromotive force in the metal loop, the following conclusions are drawn:

- 1) According to Formulas (2-1) and (2-4), the open-circuit electromotive forces of metal wire CD at different positions P₁ and P₂ are equal, that is, ε_{CDO} is 0.0001 V, which is equal to the electromotive force from Faraday's law of electromagnetic induction with Formula (1-5).
- 2) According to Formulas (2-2) and (2-5), the closed-circuit electromotive forces of metal wire CD at different positions P₁ and P₂ are not equal. The closed-circuit electromotive forces of metal wire CD at positions P₁ and P₂ are 0.00007727 V and 0.00008 V, which are not equal to the electromotive force from Faraday's electromagnetic induction law with Formula (1-5).
- 3) According to Formulas (2-3) and (2-6), metal wire CD at different positions P₁ and P₂, even if electric field intensity E in the closed loop is time-varying, the line integral of the electric field intensity E along the closed path is equal to zero, that is, the closed-circuit electromotive force of the DCBAD metal loop is equal to zero. Therefore, Maxwell's mathematical expression of electromagnetic induction is inconsistent with Faraday's law of electromagnetic induction.

The microscopic physical essence of electromagnetic induction is that the charges within the metal wire move relative to a magnetic field, and the charges are affected by the Lorentz magnetic field force. Based on Lorentz's magnetic field force theorem, we derive the motional electromotive force of metal wire CD below.

When metal wire CD is not in contact with the fixed U-shaped part of the wireframe and moves right at speed V_{CD}, the free electrons e in metal wire CD move along the wire from the C end to the D end under the action of the Lorentz magnetic field force such that the negative electrons e accumulate at D end and the positive charges p accumulate at the C end, thus generating electromotive force ε_{CDO} from the C end to D end. The charges on metal wire CD are affected by both the Lorentz magnetic field force and the electric field force. When the Lorentz magnetic field force and the electric field force are balanced, the accumulation of charges stops, as shown in Figure 2.3.

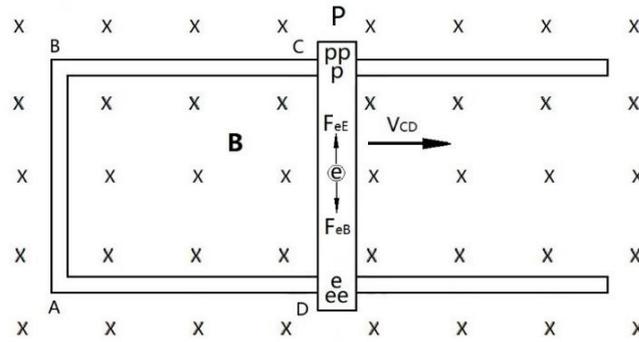


Figure 2.3 Force analysis of electrons on metal wire CD

Let the open-circuit motional electromotive force between the two ends of wire CD be ε_{CDO} , the electric field intensity is:

$$E_{CD} = \varepsilon_{CDO} / L_{CD}$$

The electric field force on a free electron e is:

$$F_{eE} = e E_{CD}$$

$$F_{eE} = e \varepsilon_{CDO} / L_{CD} \quad (2-7)$$

The electrons e within wire CD move together at V_{CD} relative to magnetic field B. According to Lorentz's magnetic field force theorem, the Lorentz magnetic field force on the free electrons e is:

$$F_{eB} = e V_{CD} B \quad (2-8)$$

The electric field force F_{eE} and the Lorentz magnetic field force F_{eB} are equal in magnitude and opposite in direction. From Formulas (2-7) and (2-8), the following can be obtained:

$$e \varepsilon_{CDO} / L_{CD} = e V_{CD} B$$

Then, the open-circuit motional electromotive force of metal wire CD is:

$$\varepsilon_{CDO} = B L_{CD} V_{CD} \quad (2-9)$$

Formulas (2-9) and (1-5) are the same. Formula (2-9) is derived based on Lorentz's magnetic field force theorem, and Formula (1-5) is derived based on Faraday's law of electromagnetic induction. Lorentz's magnetic field force theorem is the microscopic physical essence of electromagnetic induction, while Faraday's law of electromagnetic induction is the macroscopic physical manifestation of electromagnetic induction.

In summary, Faraday's law of electromagnetic induction should be revised to Faraday's law of open-circuit electromagnetic induction: The electromotive force of an open metal coil is proportional to the change rate of magnetic flux passing through the metal coil. The line integral of the electric field intensity along a closed metal loop is equal to zero, that is, the closed-circuit electromotive force of a closed metal loop is equal to zero. Maxwell's mathematical expression of electromagnetic induction is inconsistent with Faraday's law of electromagnetic induction.

3. Equipotential metal current ring

Compared to the motional electromotive force, the induced electromotive force is more complex. A recent study^[6] proposed a new structure of electromagnetic Induction - equipotential metal current ring. As shown in Figure 3.1.

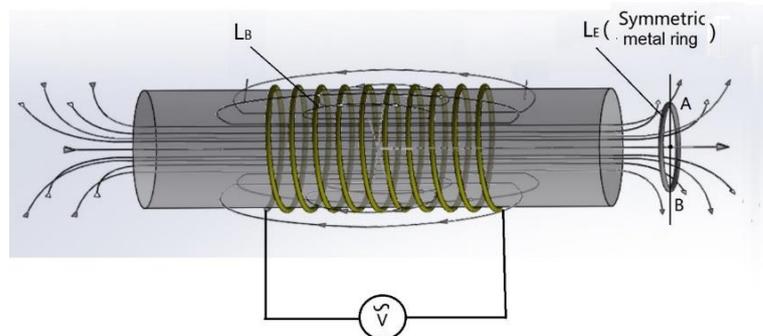


Figure 3.1 Equipotential metal current ring

With an input alternating current, a symmetrical cylindrical electromagnetic coil L_B generates a symmetrical alternating magnetic field. Place a closed metal ring L_E in the symmetrical alternating magnetic field, and the radius of the ring is r . The center of the closed metal ring L_E is located on the central axis of the electromagnetic coil L_B , and the metal ring L_E is perpendicular to the central axis of the electromagnetic coil L_B . Since the alternating magnetic field generated by the cylindrical electromagnetic coil L_B is completely symmetric with respect to the closed metal ring L_E , the electric field intensity E_L at any point on closed metal ring L_E must be equal.

On closed metal ring L_E , take any point A and symmetrically take point B on the ring with respect to point A, and AB is the diameter of metal ring L_E . The electric potential at point B relative to point A is:

$$\varepsilon_{BA} = \pi r E_L \tag{3-1}$$

Similarly, according to the principle of symmetry, the electric potential at points A and B must be equal, which is defined as:

$$\varepsilon_{BA} = 0 \tag{3-2}$$

According to Formulas (3-1) and (3-2), the electric field intensity is:

$$E_L = 0$$

From the above analysis, we conclude that on the closed metal ring L_E , the electric potential and electric field intensity are equal and equal to zero everywhere, and a changing magnetic field cannot induce an electric field. However, according to the Lenz metal ring experiments, there must be an alternating current in closed metal ring L_E , and the alternating magnetic field of L_E is repelled by the alternating magnetic field of L_B . The closed metal ring L_E is defined **an equipotential metal current ring**.

At the microscope level, the equipotential metal current ring is caused by the alternating magnetic field waves generated by L_B , which causes the free electrons in the metal ring L_E to be affected by the Lorentz magnetic field force. Due to symmetry, the Lorentz magnetic field force affected on any free electron is equal. During the flow process, electrons maintain a uniform distribution in metal ring L_E , so the electric potential and electric field intensity are equal everywhere, as shown in Figure 3.2.

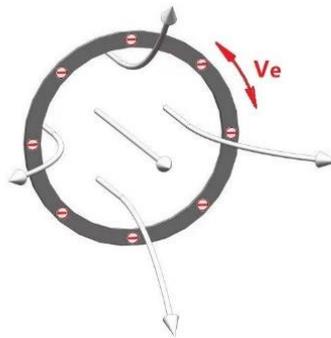


Figure 3.2 Electrons maintain a uniform distribution in an equipotential metal current ring

Because the electric potential and electric field intensity are equal to zero everywhere, the free electrons in the equipotential metal current ring are not affected by the electric field force, and are only affected by the Lorentz magnetic field force.

Below, we use experiments to further analyze and explore the equipotential metal current ring and the induced electromotive force of metal coils. To achieve the physical structure of the equipotential metal current ring, the experiment is based on a Helmholtz coil. A Helmholtz coil can form a uniform magnetic field with equal magnetic induction intensity in its central region. The Helmholtz coil in this experiment has two coils on the left and right, with an inner diameter of 180 mm and an outer diameter of 286 mm. A uniform magnetic field area of 50 mm x 50 mm x 50 mm in the center of the Helmholtz coil can be formed, and its magnetic field error is less than 1%. Moreover, in the central region, the magnetic induction intensity has a linear relationship with the input current of the coil, as shown in Figure 3.2a. In the experiment, the induction coil C_E has a symmetrical round structure, as shown in Figure 3.2b

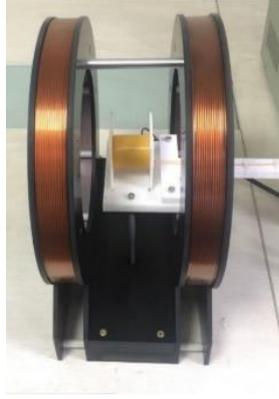


Figure 3.2a Photo of Helmholtz coil

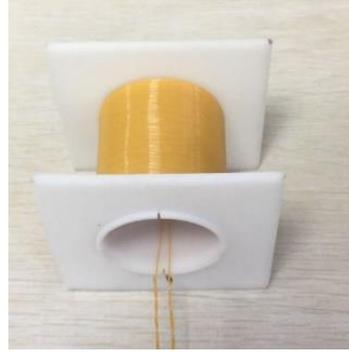


Figure 3.2b Photo of induction coil C_E

With calibration of the Helmholtz coil, the relationship between magnetic induction intensity B and current I is as follows:

$$B = 0.00114 I \quad (3-3)$$

The units of magnetic induction intensity B are Tesla (T), and the units of current are amperes (A).

When the Helmholtz coil inputs a sinusoidal alternating current with a frequency of f , an approximately equal intensity sinusoidal alternating magnetic field will be formed in the central area of the Helmholtz coil, and its sinusoidal alternating magnetic induction intensity is:

$$B(t) = B_{\max} \sin(2\pi f) t \quad (3-4)$$

where B_{\max} is the maximum magnetic induction intensity. The unit of frequency f is Hz.

We place the round induction coil C_E in the center area of the Helmholtz coil. The sectional area of the induction coil C_E is set as S , and the number of turns is set as n , then its magnetic flux is:

$$\Phi(t) = n S B(t) \quad (3-5)$$

According to Formulas (3-4) and (3-5), the magnetic flux of the induction coil C_E is:

$$\Phi(t) = n S B_{\max} \sin(2\pi f) t \quad (3-6)$$

According to Faraday's law of electromagnetic induction, the induced electromotive force of the coil is:

$$\varepsilon = - d\Phi(t)/dt \quad (3-7)$$

According to Formulas (3-6) and (3-7), the induced electromotive force is:

$$\varepsilon = -n S (2\pi f) B_{\max} \cos(2\pi f) t \quad (3-8)$$

According to Formula (3-4), the magnetic induction intensity B is sinusoidal. According to Formula (3-8), the induced electromotive force ε is also sinusoidal. Based on the effective value, Formula (3-8) is simplified as follows:

$$\varepsilon_{\text{rms}} = n S (2\pi f) B_{\text{rms}} \quad (3-9)$$

where $B_{\text{rms}} = 0.707B_{\max}$ is the effective value of the magnetic induction intensity. $\varepsilon_{\text{rms}} = 0.707\varepsilon_{\max}$ is the effective value of the induced electromotive force.

In this experiment, the round induction coil C_E has a height of 40 mm and is wound in three layers with 0.5 mm magnetic wires. The total number of coil turns is $n=240$. The radius of the first layer of coils $r_1=17.5$ mm, cross-sectional area $S_1=962.1$ mm², and the turn number is $n_1=80$; The radius of the second layer of coils $r_2=17.9$ mm, cross-sectional area $S_2=1006.6$ mm², and the turn number is $n_2=80$; The radius of the third layer of coils $r_3=18.3$ mm, cross-sectional area $S_3=1052.1$ mm², and the turn number is $n_3=80$. According to Formula (3-9), the induced electromotive force of the round induction coil C_E is:

$$\varepsilon_{\text{rms}} = n_1 S_1 (2\pi f) B_{\text{rms}} + n_2 S_2 (2\pi f) B_{\text{rms}} + n_3 S_3 (2\pi f) B_{\text{rms}} \quad (3-10)$$

Formula (3-10) is the theoretical calculation formula of the induced electromotive force on the induction coil C_E derived from Faraday's Law of Electromagnetic Induction.

The internal resistance of the above induction coil C_E is $R_{\text{rEnd}} = 7.8 \Omega$. Two intermediate metal wires are led out at the starting point and ending point of the second layer, the turn number of the second layer $n_2=80$, the internal resistance of the second layer $R_{\text{rMid}}=2.6 \Omega$, an intermediate load resistance $R_{\text{LMid}}=220 \text{ k}\Omega$ is connected in series between the two intermediate metal wires, and a voltmeter V_{Mid} is connected in parallel with the intermediate load resistance R_{LMid} . When a terminal load resistance $R_{\text{LEnd}}=220 \text{ k}\Omega$ is connected in series between two ends of coil C_E , a voltmeter V_{End} is connected in parallel with R_{LEnd} . The terminal load resistance $R_{\text{LEnd}}=220 \text{ k}\Omega$ is far greater than the internal resistance R_{rEnd} , the induction coil C_E is open-circuit, as shown in Figure 3.3a. When

the terminal load resistance $R_{LEnd}=0 \Omega$, the induction coil C_E is short-circuit, coil C_E is approximately an equipotential metal current ring, as shown in Figure 3.3b.

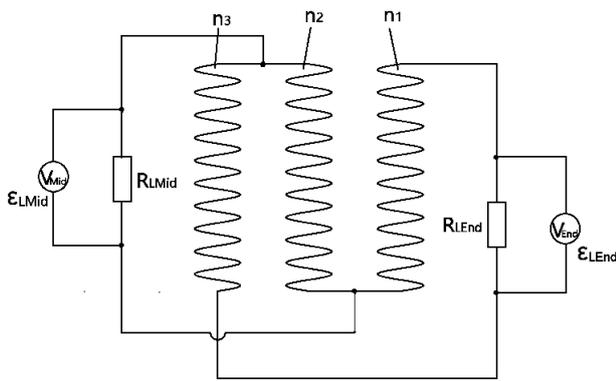


Figure 3.3a Coil C_E is open-circuit

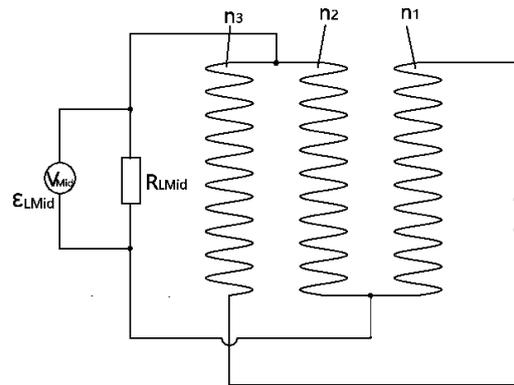


Figure 3.3b Coil C_E is short-circuit

In this experiment, a sine AC power supply is used to supply power to the Helmholtz coil. The frequency is set to 50 Hz, and the effective value of the input current is set to 2.00 A and 4.00 A. The round induction coil C_E is placed in the center area of the Helmholtz coil, and the central axis of the induction coil C_E coincides with the central axis of the Helmholtz coil. Experimental photos are shown in Figure 3.4a and 3.4b.



Figure 3.4a Electromotive force of the middle layer ϵ_{LMid} (rms) = 0.0028 V when coil C_E is short-circuit ($R_{LEnd} = 0 \Omega$)

As shown in Figure 3.4a, the input current of the Helmholtz coil is 50 Hz / 4.0 A, and the terminal load resistance $R_{LEnd}=0 \Omega$, the induction coil C_E is short-circuit, coil C_E is approximately an equipotential metal current ring, the connection of coil C_E is as shown Figure 3.3b. With voltmeter V_{Mid} , we get the induced electromotive force of the middle layer of coil C_E ϵ_{LMid} is 0.0028 V.

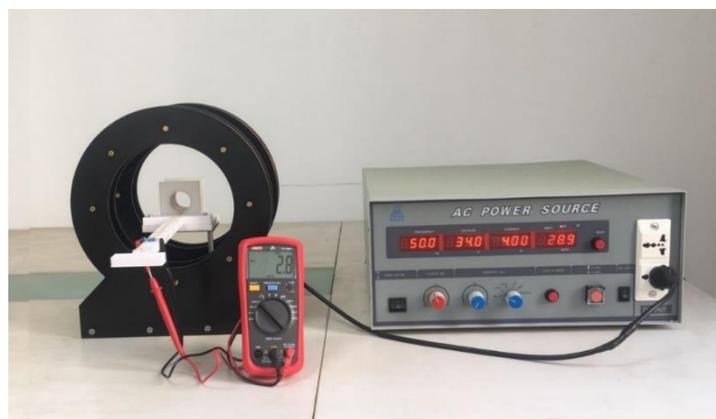


Figure 3.4b Electromotive force of the middle layer ϵ_{LMid} (rms) = 0.1184 V when coil C_E is open-circuit ($R_{LEnd}=220 \text{ k}\Omega$)

As shown in Figure 3.4b, the input current of the Helmholtz coil is 50 Hz / 4.0 A, and the terminal load resistance

$R_{LEnd}=220\text{ k}\Omega$, the induction coil C_E is open-circuit, the connection of coil C_E is as shown Figure 3.3a. With voltmeter V_{End} , we get the induced electromotive force of the coil C_E ϵ_{LEnd} is 0.3515 V, and with voltmeter V_{Mid} , we get the induced electromotive force of the middle layer of coil C_E ϵ_{LMid} is 0.1184 V.

Table 3.1 shows the induced electromotive force values calculated theoretically from Formula (3-10) and the induced electromotive force values measured experimentally as described above.

Table 3.1 Induced electromotive force (rms) by theoretical calculation and experiments

Frequency /Current (rms) /Magnetic induction intensity (rms)*	Calculation according to Formula (3-10) Induced electromotive force (rms)	$R_{LEnd}=220\text{ k}\Omega$, $R_{LMid}=220\text{ k}\Omega$ (Coil C_E is open-circuit)		$R_{LEnd}=0.0\ \Omega$, $R_{LMid}=220\text{ k}\Omega$ (Coil C_E is closed, approximately equipotential metal current ring)	
		Induced electromotive force $\epsilon_{LEnd}(\text{rms})$	Induced electromotive force $\epsilon_{LMid}(\text{rms})$	Induced electromotive force $\epsilon_{LEnd}(\text{rms})$	Induced electromotive force $\epsilon_{LMid}(\text{rms})$
50 Hz/2.00 A/0.00228 T	0.173 V	0.1756 V	0.0592 V	0.000 V	0.0015 V
50 Hz/4.00 A/0.00456 T	0.346 V	0.3515 V	0.1184 V	0.000 V	0.0028 V

* Magnetic induction intensity (rms) is calculated according to Formula (3-1) based on current

From Table 3.1, when the input current of the Helmholtz coil is 50 Hz /4.0 A, and the magnetic induction intensity is 0.00456 T, the following conclusions are drawn:

- 1) When $R_{LEnd} = 220\text{ k}\Omega$, the induction coil C_E is open-circuit, the induced electromotive force of the middle layer ($n_2=80$) ϵ_{LMid} is 0.1184 V, that is, the induced electromotive force of one turn is 1.48 mV. When $R_{LEnd} = 0\ \Omega$, the induction coil C_E is short-circuit, the induced electromotive force of the middle layer ($n_2=80$) ϵ_{LMid} is 0.0028 V, that is, the induced electromotive force of one turn is 0.035 mV. The ratio of 0.035 mV to 1.48 mV is 2.36%. Within the experimental error range, 0.035 mV can be considered equal to zero relative to 1.48 mV. Therefore, this experiment proves that when $R_{LEnd} = 0\ \Omega$, that is, the induction coil C_E is short-circuit, the induction coil C_E is approximately an equipotential metal current ring, and its electric field intensity and electric potential are equal to zero everywhere.
- 2) When $R_{LEnd}=220\text{ k}\Omega$, the induction coil C_E is open-circuit, the induced electromotive force of the coil C_E is 0.3515 V, which is the experimental measurement value. While the induced electromotive force of the coil C_E is 0.346 V, which is calculated by Faraday's law of electromagnetic induction with equation (2-10). Within the experimental error range, 0.3516 V and 0.346 V can be considered equal. Therefore, the electromotive force truly expressed by Faraday's law of electromagnetic induction is the open-circuit electromotive force.
- 3) When $R_{LEnd} = 0.0\ \Omega$, the induction coil C_E is short-circuit, the induced electromotive force of one turn is 0.035 mV. The induction coil C_E is 240 turns, so its experimental induced electromotive force is 0.0084 V, which is the induced electromotive force expressed by Maxwell's equation (1-6). While the induced electromotive force, calculated by Faraday's law of electromagnetic induction with equation (2-10) is 0.346 V. Therefore, Maxwell's mathematical expression of electromagnetic induction is inconsistent with Faraday's law of electromagnetic induction. 0.0084V is approximately equal to zero relative to 0.346V. This experiment proves that the line integral of the electric field intensity along a closed loop must be equal to zero, that is, the left term of Maxwell's equation (1-6) is equal to zero.

In summary, theoretical and experimental verification prove that the electric potential and electric field intensity in an equipotential metal current ring are equal to zero everywhere, the charges in the equipotential metal current ring are not affected by an electric field force, and are only affected by the Lorentz magnetic field force. The electromotive force expressed by Faraday's law of electromagnetic induction is the open-circuit electromotive force. Faraday's law of electromagnetic induction should be revised to Faraday's law of open-circuit electromagnetic induction. The line integral of the electric field intensity along a closed loop must be equal to zero, that is, the left term of Maxwell equation (1-6) must be equal to zero. Maxwell's mathematical expression of electromagnetic induction is inconsistent with Faraday's law of electromagnetic induction.

4. Expansion of equipotential metal current ring in a vacuum

In an alternating magnetic field, the electric potential and electric field intensity in an equipotential metal current

ring are equal to zero everywhere. In other words, in an equipotential metal current ring, a changing magnetic field cannot induce an electric field. This leads to the question. Can the equipotential metal current ring be expanded to the vacuum? The following theoretical analysis proves that in a vacuum, a changing magnetic field cannot induce an electric field.

Referring to Figure 3.1 above, we replace the metal ring with a vacuum ring, as shown in Figure 4.1.

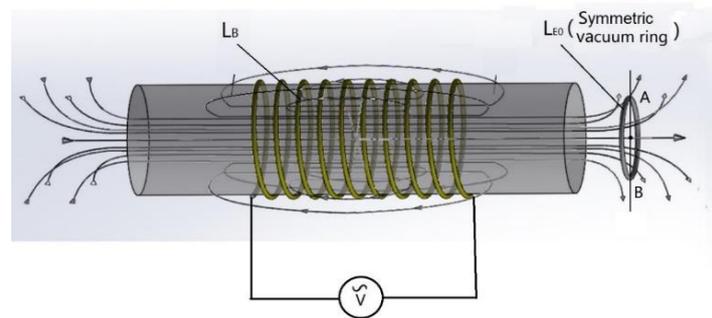


Figure 4.1 Electromagnetic induction of a symmetrical vacuum ring

With an input alternating current, a symmetrical cylindrical electromagnetic coil L_B generates a symmetrical alternating magnetic field in a vacuum (air). Take any point A in the vacuum of the alternating magnetic field generated by L_B , and select a closed vacuum ring L_{E0} through point A with a radius of r_0 . The center of the vacuum ring L_{E0} is located on the central axis of the cylindrical electromagnetic coil L_B and perpendicular to the central axis of the electromagnetic coil L_B . From formula (3-3) above, we similarly obtain that in the closed vacuum ring L_{E0} , the electric potential and electric field intensity are equal to zero everywhere. Therefore, in a vacuum of the alternating magnetic field, the electric field intensity E_{A0} at any point A is equal to zero.

In summary, in a vacuum, a changing magnetic field cannot induce an electric field. This is a huge challenge to Maxwell's "electromagnetic waves" theory

5. Conclusion and prospects

As a great experimental physicist, Faraday had little knowledge of mathematics, and his three-volume monograph "Experimental Researches in Electricity^[1]" did not include any mathematical formulas. As a mathematical genius, Maxwell had little knowledge of physics and physical experiments. These factors resulted in Maxwell's mathematical equations not fully capturing Faraday's electromagnetism experiments.

Based on theoretical analysis and experimental verification, this study reveals that the electromotive force expressed by Faraday's law of electromagnetic induction is the open-circuit electromotive force. Faraday's law of electromagnetic induction should be revised to Faraday's law of open-circuit electromagnetic induction. This study further proposes a symmetrical metal closed loop - an equipotential metal current ring. The charges in the equipotential metal current ring are not affected by an electric field force, and only by a Lorentz magnetic field force. Theoretical and experimental verification prove that a changing magnetic field cannot induce an electric field in the equipotential metal current ring, and the electric potential and electric field intensity in an equipotential metal current ring are equal to zero everywhere.

With the participation of charges, the electromotive force of a closed metal loop must be equal to zero, that is, the line integral of the electric field intensity along a closed metal loop must be equal to zero. Therefore, Maxwell's mathematical expression of electromagnetic induction is inconsistent with Faraday's law of electromagnetic induction. Expanding the equipotential metal current ring to the vacuum, we conclude that without the participation of charges, a changing magnetic field also cannot induce an electric field in a vacuum, which is a huge challenge to Maxwell's "electromagnetic waves" theory.

Maxwell introduced the "displacement current" hypothesis in 1865 and predicted that a changing electric field could induce a changing magnetic field in a vacuum. Furthermore, Maxwell extended Faraday's law of electromagnetic induction from a metal circuit to a vacuum, and predicted that a changing magnetic field could induce an electric field in a vacuum. Thus, Maxwell theoretically predicted the existence of "electromagnetic waves", which could achieve wireless communication. Maxwell's "electromagnetic waves" theory was not accepted by most physicists at that time. More than 20 years later, in 1887, Hertz achieved wireless communication through experiments^[3], and Maxwell's "electromagnetic waves" theory began to be accepted by people.

Based on theoretical analysis and experiments, this study proves that without the participation of charges, a changing magnetic field cannot induce an electric field in a vacuum. Recent research^[6] revealed that without

the participation of charges, a changing electric field could not induce a magnetic field in a vacuum. Maxwell's "displacement current" hypothesis was not true, and the "electromagnetic waves", which were theoretically predicted by Maxwell, did not exist in the real physical world.

In fact, Hertz's experiments in 1887, did not prove the existence of "electromagnetic waves", but rather proved that it was "electric field waves" that achieved wireless communication. The experimental setup of Hertz is shown in Figure 5.1.

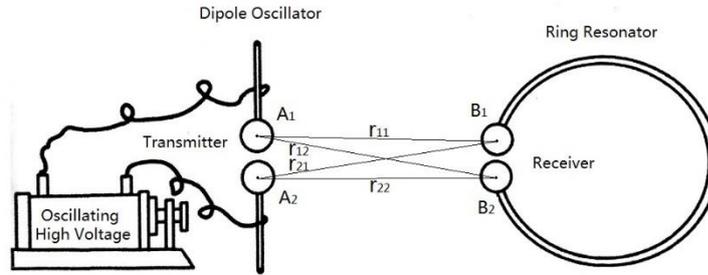


Figure 5.1 Hertz's experimental device

The experimental device mainly composed an electric field wave transmitter and an electric field wave receiver. The electric wave transmitter used a dipole oscillator, which composed two metal rods that were equipped with copper balls A_1 and A_2 , and there was a gap between the two copper balls. The receiver used a ring resonator, and the two ends of the ring were also two copper balls B_1 and B_2 with a gap.

Powered by the alternating oscillating high voltage, the amount of charge $Q_{A1}(t)$ and $Q_{A2}(t)$ on the two brass balls A_1 and A_2 of the emitter varied periodically with time. Then $Q_{A1}(t)$ and $Q_{A2}(t)$ generated a periodic alternating high voltage between two brass balls A_1 and A_2 , producing a discharge spark. The distances from brass ball A_1 to the two brass balls B_1 and B_2 of the receiver were r_{11} and r_{12} , respectively, corresponding to their unit vectors \mathbf{r}_{11} and \mathbf{r}_{12} . The distances from brass ball A_2 to brass balls B_1 and B_2 were r_{21} and r_{22} , respectively, and their unit vectors were \mathbf{r}_{21} and \mathbf{r}_{22} .

According to Coulomb's law, the electric field intensity on brass ball B_1 was:

$$\mathbf{E}_{B1}(t) = k \left(\frac{Q_{A1}(t_1)}{r_{11}^2} \mathbf{r}_{11} + \frac{Q_{A2}(t_1)}{r_{21}^2} \mathbf{r}_{21} \right) \quad (5-1)$$

The electric field intensity on brass ball B_2 was:

$$\mathbf{E}_{B2}(t) = k \left(\frac{Q_{A1}(t_1)}{r_{12}^2} \mathbf{r}_{12} + \frac{Q_{A2}(t_1)}{r_{22}^2} \mathbf{r}_{22} \right) \quad (5-2)$$

where $k = 1/4\pi\epsilon_0$, $t_1 = t - r_{11}/c$, and c is the speed of the electric field, which is equal to the speed of light.

Electric field intensities $\mathbf{E}_{B1}(t)$ and $\mathbf{E}_{B2}(t)$ induced charges $Q_{B1}(t)$ and $Q_{B2}(t)$ on the brass balls B_1 and B_2 of the receiver. The charges $Q_{B1}(t)$ and $Q_{B2}(t)$ also varied periodically with time, and their frequencies and charge changes were synchronized with $Q_{A1}(t)$ and $Q_{A2}(t)$. By adjusting the structural parameters, the emitter and receiver could form resonance. Thus, a sufficient amount of charges $Q_{B1}(t)$ and $Q_{B2}(t)$ accumulated on the two copper balls B_1 and B_2 of the receiver, and a periodic alternating high voltage also formed between two copper balls B_1 and B_2 to generate discharge sparks.

At the time of Hertz's experiment in 1887, general knowledge of wireless communication was almost zero. Today, one and a half centuries later, we re-examine the above experiment. In Equations (5-1) and (5-2), there is only a charge and an electric field, and there is no magnetic field. This illustrates the fact that the electric field waves can be generated by the change of the charge at the emitter side, the electric field waves can propagate independently in a vacuum (or air), and the electric field waves can be received independently at the receiver side. The generation, propagation, and reception of electric field waves can be completed independently by the electric field itself without the participation of the magnetic field. Therefore, Hertz's experiment did not prove the existence of "electromagnetic waves", but rather proved that it was "electric field waves" that independently achieved wireless communication. Without introducing new theoretical assumptions, based on the most basic

principle of electromagnetism, Coulomb's law, it can be strictly derived in theory that the propagation of electric field waves in the air can achieve wireless communication.

For wired communication through metal wires, the flow speed of electrons in the wire is extremely slow, approximately 10^{-6} meters per second. Wired communication is also based on the propagation of electric field waves in metal wires. Therefore, both wired and wireless communication are based on the propagation of electric field waves.

Electromagnetism was the greatest scientific achievement of physics since Newton. It is the technical basis of the second industrial revolution characterized by electrification in the 19th century and the modern society characterized by microelectronics informatization. The new consideration of Faraday's law of electromagnetic induction and Maxwell's equations is a huge challenge and opportunity for modern scientific discoveries and technological development.

Availability of Data and Materials:

All data generated or analysed during this study are included in this published article.

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