Cantor Dust and the Gravitational Wave Background

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Abstract

It was recently conjectured that Dark Matter consists of *Cantor Dust,* a relic cosmic web structure formed by condensation of *continuous spacetime dimensions* far above the Fermi scale. In this brief analysis we speculate that the Cantor Dust equations may generate the gravitational background recently reported by several international collaborations.

Key words: Fuzzy Dark Matter, Cantor Dust, topological condensation, continuous dimensions, stochastic gravitational wave background (SGWB).

1. Introduction

A few weeks ago, several collaborations reported evidence for continuous low frequency gravitational waves (GW) [1-2, 12]. Unlike the high-frequency

GW's seen by ground-based instruments like the Laser Interferometer Gravitational-wave Observatory (LIGO), this low-frequency signal forms a *stochastic gravitational wave background (SGWB)*, which can only be detected over long times using Pulsar Timing Arrays. The current belief is that there are several possible astrophysical sources of SGWB, besides supermassive Black Holes binaries that were abundant in the early Universe [12].

The goal of this brief report is to suggest an unconventional explanation of SGWB based on the hypothesis of *Cantor Dust*. According to this hypothesis, Cantor Dust is a relic cosmic web formed by the condensation of *continuous spacetime dimensions* in the deep ultraviolet (UV) sector of field theory. As argued in [8-10], there are two key points to consider here, namely,

1) The formation of Cantor Dust follows from the nonintegrability of UV dynamics and the onset of *fractal spacetime.*
2) Adequate modeling of this regime requires the tools of *fractional differential and integral operators*.

Our paper proceeds from these premises and is partitioned in the following way: Section 2 lists the main working assumptions; the putative duality of fractal spacetime and classical gravitation is discussed in section 3; the possible path leading from Cantor Dust to SGWB is detailed in the last section.

We caution upfront that ideas presented here are controversial. Readers are encouraged to keep an open mind and recall that our analysis is entirely provisional. Researchers unfamiliar with the topic are urged to carefully study the references prior to drawing premature conclusions.

2. Working assumptions

A1) To enable a meaningful analogy with the metric tensor, dimensional deviations (13) are configured as *tensor-like entities*. This ansatz implies that dimensional deviations are considered locally dependent on all four spacetime coordinates.

A2) In general, dimensional deviations are assumed to be *anisotropic* and have a preferential orientation in four-dimensional spacetime.

3. Duality of fractal spacetime and classical gravitation

To begin with, consider the fractional analog of a free particle Hamiltonian given by [4]

$$H = \frac{1}{2m} p^{2\alpha} \tag{1}$$

in which α denotes the order of fractional integration. If $\alpha = 1 - \varepsilon$, with $\varepsilon << 1$ (1) approximates the classical non-relativistic Hamiltonian in the limit $\varepsilon = 0$ namely,

$$H = \frac{1}{2m} p^{2(1-\varepsilon)} \approx \frac{1}{2m} p^2 \tag{2}$$

Refer now to the action of a free non-relativistic particle in a weak gravitational field,

$$S = \frac{1}{2m} \int dx \sqrt{-g}\, g^{\mu\nu} p^2 \tag{3a}$$

where $\eta^{\mu\nu}$ is the Minkowski metric, $g=\det(g_{\mu\nu})$ and

$$g^{\mu\nu}=\eta^{\mu\nu}+h^{\mu\nu}\,;\ \ \left|h^{\mu\nu}\right|<<1 \tag{3b}$$

Side-by-side comparison of (2) and (3) suggests the following analogy between the dimensional deviation ε and gravitational metric,

$$\boxed{p^{2(1-\varepsilon)}\Leftrightarrow\sqrt{-g}\,g^{\mu\nu}p^{2}} \tag{4}$$

Consider next a slightly different context involving the action functional for a classical field in $d=D+1$ spacetime dimensions [6],

$$S\,[\varphi]=\int dt\,d^{D}x\,L(\varphi,\partial_{\mu}\varphi)=\int dt\int d\mu(x)\;L(\varphi,\partial_{\mu}\varphi) \tag{5}$$

Here, the differential measure is

$$d\mu(x)=\prod_{i=1}^{3}d\mu_{i}(x^{i}) \tag{6}$$

where,

$$d\mu_{i}(x^{i})\propto\frac{1}{\Gamma(\alpha_{i})}\left|x^{i}\right|^{\alpha_{i}-1}dx^{i} \tag{7}$$

For the sake of convenience and simplicity, we take all fractional orders and all coordinates to be equal in magnitude, that is, $\alpha_i = \alpha$, $i = 1,2,3$ and $x = x^{\mu}$, $\mu = 0,1,2,3$. As a result, (6) assumes the form,

$$d\mu \Rightarrow d\mu_{\alpha} = \frac{|x|^{3(\alpha-1)} dx}{[\Gamma(\alpha)]^3} \tag{8}$$

If the spacetime is endowed with minimal fractality, (8) turns into [4-5]

$$d\mu_{1-\varepsilon} = \frac{|x|^{-3\varepsilon} dx}{[\Gamma(1-\varepsilon)]^3} \approx |x|^{-3\varepsilon} dx \tag{9}$$

where α depends on the one-dimensional deviation $\varepsilon << 1$ via

$$\alpha = 1 - \varepsilon \tag{10}$$

Next, insert (9) into (5) and compare (5) with the action functional of a free massless scalar field in curved spacetime,

$$S[\varphi] = \int dx \sqrt{-g}\, \frac{1}{2} [g^{\mu\nu} \partial_{\mu}\varphi\, \partial_{\nu}\varphi] \tag{11}$$

Recalling that, by construction, all fractional orders and coordinates are set to be equal in magnitude, leads to an analogy similar to (4), that is,

$$|x|^{-3\varepsilon} \Leftrightarrow \sqrt{-g}\, g^{\mu\mu} \tag{12}$$

By assumption A1), (12) links the dimensional deviation ε to an analogue gravitational metric, which is symbolically presented as

$$\boxed{\varepsilon^{\mu\mu} \Leftrightarrow -\frac{\log(\sqrt{-g}\, g^{\mu\mu})}{3\log|x|}} \tag{13}$$

The notation $\varepsilon \Rightarrow \varepsilon^{\mu\mu}$ highlights the fact that only the diagonal elements of the corresponding dimensional deviation tensor $\varepsilon^{\mu\nu}$ are considered in (13).

4. From Cantor Dust to SGWB

In what follows, we take the diagonal elements $\varepsilon^{\mu\mu}$ to represent a single-variable function as in

$$\varepsilon^{\mu\mu} = \varepsilon(\mu) \tag{14}$$

By assumption A2), (14) can be configured as a vector-like entity having both amplitude and phase. Presented in complex form, (14) turns into,

$$\varepsilon(\mu) = \varepsilon_0(\mu)\exp[i\theta(\mu)] \tag{15}$$

From these observations, the action functional of Cantor Dust may be presented as,

$$S[\varepsilon, \varepsilon^*] = \int d^4x\sqrt{-g}\,[g^{\mu\mu}\partial_\mu\varepsilon^*\partial_\mu\varepsilon + V(\varepsilon^*\varepsilon)] \tag{16}$$

Here, $V(\varepsilon^*\varepsilon)$ stands for the self-interaction potential of the field. Equation (16) underlines the description Cantor Dust as a classical complex scalar field placed in a background metric $g_{\mu\nu}$. As a side note, it is instructive to point out that (16) models superfluidity in curved spacetime, as well as the behavior of Higgs-like fields over cosmological scales [7].

The equation of motion derived from (16) reads,

$$[\nabla^\mu\nabla_\mu - \frac{\partial V}{\partial(\varepsilon^*\varepsilon)}]\varepsilon = 0 \tag{17}$$

where ∇_μ represents the covariant derivative and,

$$\nabla^{\mu}\nabla_{\mu}\varepsilon = \frac{1}{\sqrt{-g}}\partial_{\mu}(\sqrt{-g}\,g^{\mu\mu}\partial_{\mu}\varepsilon) \tag{18}$$

The vacuum solution of (16) corresponds to the minimum of the potential, in which case (17) reduces to the classical wave equation in free space,

$$\boxed{\Box\varepsilon = \nabla^{\mu}\nabla_{\mu}\varepsilon = 0} \tag{19}$$

Taking the real and imaginary parts of (15) yields two equations describing, respectively, the *relativistic Euler fluid* and the *continuity equation*, [7]

$$\boxed{\frac{1}{\varepsilon_0}(\nabla^{\mu}\nabla_{\mu}\varepsilon_0) - \nabla^{\mu}\theta\nabla_{\mu}\theta = 0} \tag{20}$$

$$\nabla^{\mu}j_{\mu} = 0 \tag{21}$$

Here, the four-dimensional current takes the form,

$$j_{\mu} \equiv \varepsilon_0\partial_{\mu}\theta \tag{22}$$

and the oscillation frequency is given by,

$$\omega = 2\pi\nu = \partial_0\theta \tag{23}$$

There are two independent scenarios related to (20):

a) $\varepsilon(\mu)=\varepsilon_0(\mu)$ is a pure scalar and the phase term drops out [$\theta(\mu)=0$]. In this case, dimensional deviations are isotropic and (20) is identical to (19).

b) the amplitude of the dimensional deviation (ε_0) is *coordinate independent* and the first term of (20) drops out.

In either one of these cases, (20) echoes the equation of plane gravitational waves in empty space [13]

$$\boxed{\Box h_{\mu\nu}=0} \tag{24}$$

It follows that, *at least in principle*, appealing to the (13), (19) - (20) and (24), hints that Cantor Dust equations can *mimic the behavior* of SGWB.

In closing, we point out that there are currently many studies on modeling scalar Dark Matter through a variety of equations (such as Schrödinger-Poisson, Vlasov-Boltzmann, anyon wave equations, complex Ginzburg-Landau, time dependent Ginzburg-Landau, fractional wave equations, stochastic equations and so on – see e.g. [11]). According to these models, Dark Matter consists of exotic objects such as (but not limited to) scalars, complex scalars, superfluids, axions, gravitons, glueballs and q-bosons. It is conceivable that either one of these proposals may be used for building alternative scenarios regarding the physics of SGWB.

References

1. https://iopscience.iop.org/article/10.3847/2041-8213/acdac6/pdf

2. https://nanograv.org/news/15yrRelease

4. https://arxiv.org/pdf/cond-mat/0505720.pdf, equation (41).

5. https://arxiv.org/pdf/nlin/0602062.pdf

6. https://arxiv.org/pdf/2109.02855.pdf

7. https://arxiv.org/pdf/1408.0779.pdf

8. https://www.researchgate.net/publication/369584882

9. https://www.researchgate.net/publication/369814408

10. Available at the following sites:

https://www.researchgate.net/publication/363456790

https://www.researchgate.net/publication/358486078

https://www.researchgate.net/publication/343426172

11. https://arxiv.org/pdf/2203.09396.pdf

12. https://arxiv.org/pdf/2306.17827.pdf

13. Landau L. D. and Lifschitz E. M., The Classical Theory of Fields, Butterworth-Heineman, 1980.