Symmetry groups in quantum relativistic dynamics.

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Chapters

1. Introduction

The transformations of relativistic dynamics in the Special Theory of Relativity and quantum relativistic dynamics (it's fashionable to say Quantum Theory of Relativity) are presented in "Unified Theory 2" in one mathematical truth. We are talking about dynamic space-matter, a special case of a zero or fixed angle of parallelism, there is the Euclidean axiomatics of space-time. The Special Theory of Relativity cannot describe space-time in quantum fields with their uncertainty principle. It is impossible to fix both the time and the coordinate at the same time. And there is no quantum relativistic dynamics in the gauge fields that follow from the Dirac equation. Relativistic dynamics is represented by the Lorentz group, and the Dirac equation invariance condition ($A_\mu(X) = \bar{A}_\mu(X) + i\frac{\partial a(X)}{\partial x_\mu}$) is represented by the condition ($\frac{\partial a(X)}{\partial x_\mu} \equiv f'(x) = 0$). But this is a constant extremal of a dynamic function $a(X) = f(x) \neq const$. In the Yang-Mills theory , the derivative of the scalar function is added to the potential, which does not change the potential itself, in the symmetry group:

$$A_\mu = \Omega(x)A_\mu(\Omega)^{-1}(x) + i\Omega(x)\partial_\mu(\Omega)^{-1}(x), \text{ where} \Omega(x) = e^{i\omega} ,$$

and ω- an element of any group A and (SU (N), SO (N), Sp (N), E $_6$, E $_7$, E $_8$, F $_4$, G $_2$), and $A_\mu \to A_\mu + \partial_\mu\omega$. In this case $U(1)$- describes the electromagnetic interaction, $SU(2)$- Weak Interactions and $SU(3)$- describes Strong Interactions, and so on. We will consider the conditions: $a(X) = f(x) \neq const$, and substantiation of symmetries in quantum relativistic dynamics (in the Quantum Theory of Relativity).

2. General representations.

Let's start the mathematical representation of symmetries with the simplest geometric figures. Regular figures on a plane retain their symmetry during rotations, inversions. For example:

2.1. the rectangle is symmetrical when rotated by 180 ^{0}, and when rotated by 0 ^{0} does not change.

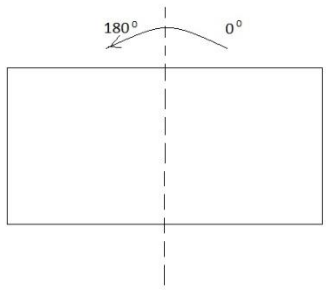


Figure 2.1.

We have two operations. Rotate 0 ^{0} as $R_0 = I$, and rotate 180 ^{0} as R_{180}. They can be multiplied by first $R_0 * R_{180}$ turning by 0 ^{0}, then by 180 ^{0}, or vice versa: $R_{180} * R_0$, in the Cayley table.

C2	I	R_{180}
I	I	R_{180}
R_{180}	R_{180}	I

$R_0 * R_0 = R_0 = I$, $\quad$ $R_0 * R_{180} = R_{180}$. $\quad$ $R_{180} * R_0 = R_{180}$. $\quad$ $R_{180} * R_{180} = R_{360} = R_0 = I$

The operation $R_0 = I$, does not change anything, is called the identity element of the given group. The group is defined by properties.

1). A group operation is defined, here is a turn.

2). The presence of a single element, $R_0 = I$,

3). closedness, when an operation in a group gives an element that does not leave the group,

4). the presence of an inverse element $I^{-1} = I$, or $R_{180}^{-1} = R_{180}$. This is the element that undoes the previous operation of each group element.

5).associativity property: A(BC)=(AB)C. This group is called C2.

2.2. an example of an equilateral triangle, with rotations of 0 ^{0} , 120 ^{0} , and 240 ^{0} .

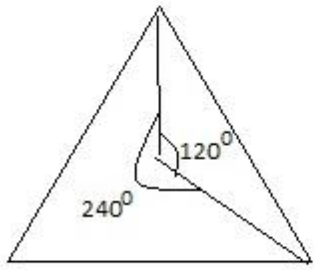


Figure 2.2

The multiplication table of a given group of rotations is compiled in exactly the same way.

C3	I	R_{120}	R_{240}
I	I	R_{120}	R_{240}
R_{120}	R_{120}	R_{240}	I
R_{240}	R_{240}	I	R_{120}

All possible elements of a group, when multiplied, give the elements of the same group. The group is closed. For each element there is an inverse element and also in the group. $R_{120}^{-1} = R_{240}$, $R_{240}^{-1} = R_{120}$.

Not only turns of figures give a group. The numbers (+1) and (-1) also form group .

	1	-1
1	1	-1
-1	-1	1

The group operation is multiplication. The identity element is 1. Inverse element: $-1^{-1} = -1$. All conditions for the group are met. This group is identical to the C 2 group . They are called isomorphic. There are also other isomorphic groups. For example, during the operation of reflection $\sigma$of the considered rectangle.

S2 _	I	σ
I	I	σ
σ	σ	I

If we reflect the rectangle twice around the axis, we get the original object, a group with all the properties.
Such a group is called S 2 and is isomorphic to the group C 2 . Multiplying the coordinates of the vector (2,1) by (-1), leads to the reflection of the coordinates relative to the origin. Therefore, the group of numbers (1) and (-1) is also isomorphic.

2.3 Abelian and non-Abelian groups and subgroups. The considered groups of rotations C3 and reflections S 3

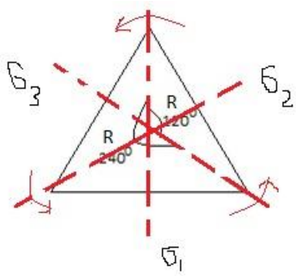


Figure 2.3.

Such rotations and reflections also do not change the triangle, and form a group. Let's write a table for it.

D3	I	R_{120}	R_{240}	σ_1	σ_2	σ_3
I	I	R_{120}	R_{240}	σ_1	σ_2	σ_3
R_{120}	R_{120}	R_{240}	I	σ_2	σ_3	σ_1
R_{240}	R_{240}	I	R_{120}	σ_3	σ_1	σ_2
σ_1	σ_1	σ_3	σ_2	I	R_{240}	R_{120}
σ_2	σ_2	σ_1	σ_3	R_{120}	I	R_{240}
σ_3	σ_3	σ_2	σ_1	R_{240}	R_{120}	I

Group C3 has subgroups D 3. Turn on $R_{120}$with reflection $\sigma_1$equals reflection σ_2. But if we first reflect $\sigma_2$and then rotate R_{120}, we get a reflection σ_3. That is: $R_{120} * \sigma_1 \neq \sigma_1 * R_{120}$. But the law of commutativity is not a property of the group, and it does not have to be observed. The group D 3 is not abelian, the subgroup C3 is abelian. But if we choose the specified triangle, its symmetry under reflections is already broken.

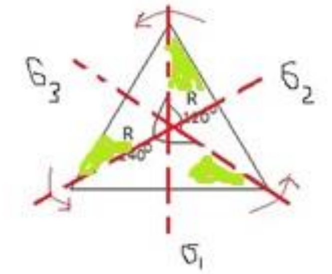


Figure 2.4

Such symmetry breaking is called spontaneous.

2.4 Representation of groups. As stated, the group operation can be any action, multiplication, rotation, inversion, whatever. Group elements can also be any abstract objects that can be replaced in isomorphic groups by prime numbers (1) and (-1) if the group is commutative. But there are also mathematical objects for which the commutativity of multiplication is not observed, for example, matrices . In other words, matrices can also be abstract elements of groups. In the considered D 3 matrix, the elements of the group can be represented by matrices, in the form:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad R_{120} = \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}. \ R_{240} = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}.$$

$$\sigma_1 = \frac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \ \sigma_2 = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}, \ \sigma_3 = \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}.$$

Now the operation of the group D 3 is matrix multiplication. In this case, the structure of the group is preserved:

$R_{120} * R_{120} = R_{240}$, or $\frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} * \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$, or

$R_{120} * \sigma_1 = \sigma_3$, as: $\frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} * \frac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$

$\sigma_1 * R_{120} = \sigma_2$, in the form: $\frac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} * \frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$.

Rotation matrices do not commute with reflection matrices along the specified axes. But the rotation matrices commute with each other. The product $R_{120} * R_{240} = I$, or $\frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} * \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$, gives

The single element of the group. All matrices are invertible. $R_{120}^{-1} = R_{240}$, $\frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}^{-1} = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$. The inverse element of the matrix group is represented by the inverse matrix: ${\sigma_1}^{-1} = \sigma_1$, or $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

The analysis of the abstract operations of a group can thus be replaced by the study of the properties of matrices. But matrices can also be considered as operators acting on vectors. For example, when rotating a vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}byR_{120}$, we get:$\frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} * \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\sqrt{3}/2 \\ -1/2 \end{pmatrix}$ rotated vector, or:

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$on R_{120}, we get $\frac{1}{2}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix} * \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 \\ -1/2 \end{pmatrix}$.

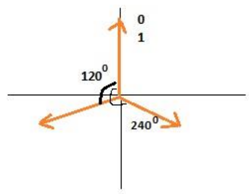


Figure 2.5

Multiplying the remaining matrices by any of these three vectors will translate the vector into one of these three. We get the same symmetrical triangle. That is, matrices are representations of operations.

2.5. In group theory there are many theorems: discrete groups, normal subgroups, classes, factor - groups ... . Let us consider the groups A and, in physical theories. In the previous group, for example D 3, we considered the symmetries of a triangle under rotations and reflections. Similarly, one can consider the symmetries of a square in the group D 4 for 4 turns, in a regular pentagon D 5 for 5 turns, a hexagon D 6 for 6 turns ... . A regular (N → ∞) square turns into a circle, with the radius rotated by the angle (α). The circle is invariant under rotations through any angle (α). But here there are no elements of the groups considered earlier $(R)$and (σ).

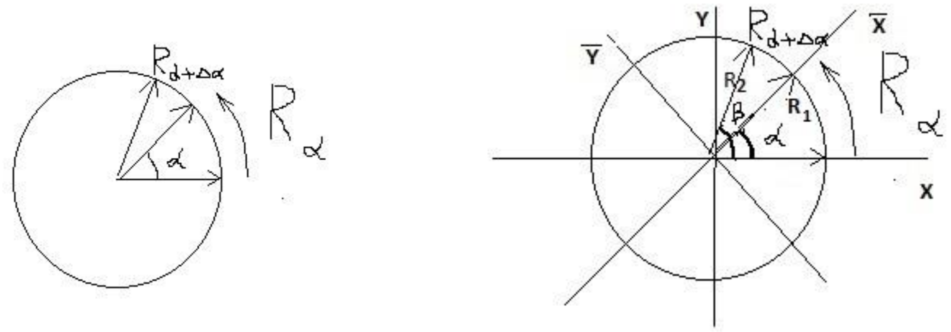


Figure 2.6

In such a symmetry group of a circle, a group parameter is introduced, by the angle of rotation R_α. In this group, we obtain a continuous transition from one element of the group $(R_\alpha)$to another $(R_{\alpha+\Delta\alpha})$. These are the groups L and. Here there is $(R_0 = I)$an identity element, the inverse element of the group $(R_\alpha^{-1} = R_{2\pi-\alpha})$. The elements of the group are also represented by matrices. If we consider rotations of the coordinate system $XY \to \bar{X}\bar{Y}$, we obtain for

$$R_1(R_{X1}R_{Y1})\text{and } R_2(R_{X2}R_{Y2}): \quad R_1 * R_2 = |R_1||R_2|\cos(\beta-\alpha) = (R_{X1}R_{X2} + R_{Y1}R_{Y2}),$$

$$|R_1||R_2|\cos(\beta-\alpha) = |R_1|\cos(\alpha)\,|R_2|\cos(\beta) + |R_1|sin(\alpha)|R_2|sin(\beta),$$

$$\cos(\beta-\alpha) = \cos(\alpha)\cos(\beta) + sin(\alpha)sin(\beta),$$

$$\cos(\beta+\alpha) = \cos(\beta-(-\alpha)) = \cos(-\alpha)\cos(\beta) + sin(-\alpha)sin(\beta),$$

$$\cos(\beta+\alpha) = \cos(\alpha)\cos(\beta) - sin(\alpha)sin(\beta)$$

$$|R_1| * \ (|R_2|\cos(\beta+\alpha) = \bar{X}) = |R_1|\cos(\alpha) * (|R_2|\cos(\beta) = X) - |R_1|\,sin(\alpha) * (|R_2|si\,n(\beta) = \mathrm{Y})$$

$$\bar{X} = X\cos(\alpha) - Ysi\,n(\alpha).$$ Similarly, next:

$$|R_1||R_2|\ \sin(\beta+\alpha) = |R_1||R_2|\cos\big(90-(\beta+\alpha)\big) = \ |R_1||R_2|\cos((90-\alpha)-\beta)$$

$$|R_1||R_2|\sin(\beta+\alpha) = |R_1||R_2|\cos(90-\alpha)\cos(\beta) + \ |R_1||R_2|\,sin(90-\alpha)\,sin(\beta)$$

$$|R_1| * (|R_2|\sin(\beta+\alpha) = \bar{Y}) = |R_1|\sin(\alpha) * (|R_2|\cos(\beta) = \mathrm{X}) + \ |R_1|\cos(\alpha) * (|R_2|sin(\beta = Y)$$

$$\bar{Y} = X\sin(\alpha) + Ycos(\alpha).$$

Finally, we get the transformations:

$$\left|\begin{matrix}\bar{X} = X\cos(\alpha) - Ysi\,n(\alpha) \\ \bar{Y} = X\sin(\alpha) + Ycos(\alpha)\end{matrix}\right| \text{or } \begin{pmatrix}\bar{X}\\ \bar{Y}\end{pmatrix} = \begin{pmatrix}X\\ Y\end{pmatrix}\begin{pmatrix}\cos(\alpha) & -\sin(\alpha)\\ \sin(\alpha) & \cos(\alpha)\end{pmatrix}$$

$$\begin{pmatrix}\bar{X}\\ \bar{Y}\end{pmatrix} = \begin{pmatrix}X\\ Y\end{pmatrix} * R_\alpha \qquad \text{where is} \qquad R_\alpha = \begin{pmatrix}\cos(\alpha) & -\sin(\alpha)\\ \sin(\alpha) & \cos(\alpha)\end{pmatrix}\text{the matrix of the group A and.}$$

The previously considered cases of rotation by 120 ^{0} and 240 ^{0} are special cases of rotations R_α.

$$R_{120} = \begin{pmatrix}\cos(120) & -\sin(120)\\ \sin(120) & \cos(120)\end{pmatrix} = \frac{1}{2}\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}$$

This is a (R_α) SO (2) matrix , i.e. a Special (det (R_α)=1) Orthogonal ($R_\alpha(R_\alpha)^{\mathrm{T}} = I$) matrix where the transposed matrix $(R_\alpha)^{\mathrm{T}} = (R_\alpha)^{-1}$is equal to the inverse. This is (R_α) the rotation matrix, abelian.

The matrix of the scaling operation $\begin{pmatrix}M & 0\\ 0 & M\end{pmatrix}$with the parameter (M=2), performs $\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix} = \begin{pmatrix}2\\ 2\end{pmatrix}$an increase $(M > 1)$ or decrease ($0 < M < 1$) of the original vector. The parameter (M) can be taken out of brackets, then we will get M $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$a group generator in brackets, not attached to the elements of the group.

Angle (α) the group parameter $R_\alpha = \begin{pmatrix}\cos(\alpha) & -\sin(\alpha)\\ \sin(\alpha) & \cos(\alpha)\end{pmatrix}$is also taken out $R_0 = I$, the rotation by 0 ^{0} does nothing, it gives the identity matrix. Turning an angle $(\Delta\,\alpha \to 0)$gives $(R_{\Delta\alpha} = I + \Delta\,\alpha L)$ transformation where(L) rotation generator. Then $(R_{\alpha+\Delta\alpha} = R_{\Delta\alpha}R_\alpha)$to rotate by an angle ($\alpha + \Delta\,\alpha$), you must first rotate by an angle (α), then by an angle ($\Delta\,\alpha$). Substituting the values, we get: $(R_{\alpha+\Delta\alpha} = (I + \Delta\,\alpha L)R_\alpha = R_\alpha + \Delta\,\alpha L R_\alpha)$. Further, in the usual order, we obtain: $(R_{\alpha+\Delta\alpha} - R_\alpha = \Delta\,\alpha L R_\alpha)$, $lim_{\Delta\alpha\to 0}\frac{R_{\alpha+\Delta\alpha}-R_\alpha}{} = LR_\alpha$, $\frac{dR_\alpha}{d\alpha} = LR_\alpha$, $\frac{dR_\alpha}{R_\alpha} = Ld\alpha$, $R_\alpha = e^{\alpha L}$, solution of the differential equation, with the group generator $(\frac{dR_\alpha}{d\alpha})_0 = L$. These equations are similar to the Schrödinger equation: $\frac{dU}{dt} = -iHU$, with solutions: $U = e^{-itH}$. Here the group generator is represented by the Hamilton operator, and instead of turning by an angle, time is considered. In our case of rotations, the group generator is equal to the derivative of the group elements at zero rotation angle. We take the derivatives, substitute the value of the angle and get the group generator.

$\frac{dR_\alpha}{d\alpha} = \begin{pmatrix}-sin(0) & -\cos(0)\\ \cos(0) & -\sin(0)\end{pmatrix} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} = L$. Or $\frac{dR_\alpha}{d\alpha} = LR_\alpha$, in the form: $\frac{dR_\alpha}{d\alpha} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\cos\ (\alpha) & -\sin(\alpha)\\ \sin(\alpha) & \cos(\alpha)\end{pmatrix}$.

Here: $\cos^2(\alpha) + \sin^2(\alpha) = 1$, as expected. Then the rotated and original vector is represented as: $\bar{V} = e^{\alpha\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}}V$, where: $e^{\alpha\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}}$ is the matrix of the group element. Now we rotate the vector by an angle (α) without using trigonometric functions. The generators themselves say a lot about the band itself. For example, the scale generator $e^{m\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}} = e^{\begin{pmatrix}m & 0\\0 & m\end{pmatrix}}$ is represented as a group element. The scale factor is: $M = e^m$ the exponent of the group parameter (m).

2.6. The elements of the groups L and are found by matrix exponentiation of the generators of the Lie groups. The elements themselves are considered as generators acting on the vector. These operators change the vector. But the invariant always remains unchanged in the group. Group generator L and: $L = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ changes the element of the Lie group $R_\alpha = e^{\pm\alpha\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}}$, as the scaling operator. Acting R_α on the column of point coordinates, we obtain radially diverging (converging) points with a constant angle (α). The group generator $L = \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$ gives the group element $R_\alpha = e^{\pm\alpha\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}}$, as a point moving in a circle. The length of the vector is constant.). The group generator $L = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ gives the group element $R_\alpha = e^{\pm\alpha\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}}$ as a point moving along a hyperbola. Exponentiating such a generator gives $\Lambda = \begin{pmatrix}\mathrm{ch}(\alpha) & \mathrm{sh}(\alpha)\\ \mathrm{sh}(\alpha) & \mathrm{ch}(\alpha)\end{pmatrix}$, the Lorentz group. This takes place: $\mathrm{ch}^2(\alpha) - \mathrm{sh}^2(\alpha) = 1$, as expected. Recall the graphs of these functions $Y = Y_0\mathrm{ch}\left(\alpha = \frac{X=Z}{Y_0}\right)$ in the form:

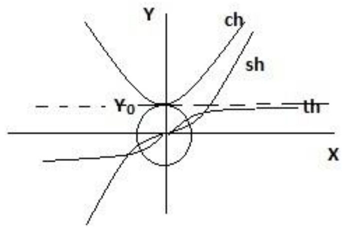


Figure 2.7

Here the Lorentz group $\Lambda = \begin{pmatrix}\mathrm{ch}(\alpha) & \mathrm{sh}(\alpha)\\ \mathrm{sh}(\alpha) & \mathrm{ch}(\alpha)\end{pmatrix}$, together with $\mathrm{ch}^2(\alpha) - \mathrm{sh}^2(\alpha) = 1$ and elements of the group in the form $R_\alpha = e^{\pm\alpha\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}}$ represented in hyperbolic functions $e^z = ch(z) + sh(z)$. At the same time, we derived transformations of the relativistic dynamics $R_\alpha = \begin{pmatrix}\cos(\alpha) & -\sin(\alpha)\\ \sin(\alpha) & \cos(\alpha)\end{pmatrix}$ of the Lie group matrix, with the generator already in the trigonometric $e^{iz} = cos(z) + isin(z)$ functions $R_\alpha = e^{i\alpha\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}}$ of the group elements. There is a problem in the relativistic solutions of the invariant Dirac equation. The action of a quantum cannot $\hbar = \Delta p\Delta\lambda = F\Delta t\Delta\lambda$ be fixed in space $\Delta\lambda$ or time Δt. This is due to the non-zero $(\varphi \neq const)$ angle of parallelism $(X-)$ or $(Y-)$ trajectory $(X\pm)$ or $(Y\pm)$ space-matter quantum. There is only a certain probability of action. Transformations of the relativistic dynamics of the wave ψ - function of the quantum field with the probability density ($|\psi|^2$) of interaction in $(X+)$ the field (Fig. 3) correspond to the Globally Invariant $\psi(X) = e^{-ia}\overline{\psi}(X)$, $a = const$ Lorentz group. These transformations correspond to rotations in the plane of the circle S, and relativistically - to the invariant Dirac equation.

$$i\gamma_\mu \frac{\partial\psi(X)}{\partial x_\mu} - m\psi(X) = 0, \quad \text{and} \quad \left[i\gamma_\mu \frac{\partial\overline{\psi}(X)}{\partial x_\mu} - m\overline{\psi}(X)\right] = 0.$$

Such invariance gives the conservation laws in the equations of motion. For transformations of relativistic dynamics in hyperbolic motion,

$$\psi(X) = e^{a(X)}\overline{\psi}(X). \qquad ch(aX) = \frac{1}{2}(e^{aX} + e^{-aX}) \cong e^{aX}. \qquad a(X) \neq const.$$

$$\left[i\gamma_\mu \frac{\partial\overline{\psi}(X)}{\partial x_\mu} - m\overline{\psi}(X)\right] + i\gamma_\mu \frac{\partial a(X)}{\partial x_\mu}\overline{\psi}(X) = 0.$$

The invariance of conservation laws is violated. To save them, calibration fields are introduced. They compensate for the extra term in the equation.

$$A_\mu(X) = \bar{A}_\mu(X) + i\frac{\partial a(X)}{\partial x_\mu}, \quad \text{and} \quad i\gamma_\mu\left[\frac{\partial}{\partial x_\mu} + iA_\mu(X)\right]\psi(X) - m\psi(X) = 0.$$

Now, substituting the value $\psi(X) = e^{a(X)}\bar{\psi}(X)$ of $a(X) \neq const$ the wave function into such an equation, we obtain an invariant equation of relativistic dynamics.

$$i\gamma_\mu\frac{\partial\psi}{\partial x_\mu} - \gamma_\mu A_\mu(X)\psi - m\psi = i\gamma_\mu\frac{\partial\bar{\psi}}{\partial x_\mu} + i\gamma_\mu\frac{\partial a(X)}{\partial x_\mu}\bar{\psi} - \gamma_\mu\bar{A}_\mu(X)\bar{\psi} - i\gamma_\mu\frac{\partial a(X)}{\partial x_\mu}\bar{\psi} - m\bar{\psi} = 0$$

$$i\gamma_\mu\frac{\partial\bar{\psi}}{\partial x_\mu} - \gamma_\mu\bar{A}_\mu(X)\bar{\psi} - m\bar{\psi} = 0, \quad \text{or} \quad i\gamma_\mu\left[\frac{\partial}{\partial x_\mu} + i\bar{A}_\mu(X)\right]\bar{\psi} - m\bar{\psi} = 0.$$

This equation is invariant to the original equation

$$i\gamma_\mu\left[\frac{\partial}{\partial x_\mu} + iA_\mu(X)\right]\psi(X) - m\psi(X) = 0$$

in conditions $A_\mu(X) = \bar{A}_\mu(X)$, and $A_\mu(X) = \bar{A}_\mu(X) + i\frac{\partial a(X)}{\partial x_\mu}$,

the presence of a scalar boson $(\sqrt{(+a)(-a)} = ia(\Delta X) \neq 0) = const$, within the gauge $(\Delta X) \neq 0)$ field (Fig. 3). These conditions ($\frac{\partial a(X)}{\partial x_\mu} \equiv f'(x) = 0$)give constant extremals (f_{max}) dynamic $a(X) = f(x) \neq const$ space-matter in global invariance. And there are no scalar bosons here. These are: $A_\mu(X) = \bar{A}_\mu(X) + i\frac{\partial a(X)}{\partial x_\mu}$, known gauge transformations. $a(X)$– 4-vector (A_0, A_1, A_2, A_3)electromagnetic scalar ($\varphi = A_0$)and vector ($\vec{A} = A_1, A_2, A_3$) potential in Maxwell electrodynamics: $\vec{E} = -\nabla\varphi - \frac{\partial\vec{A}}{\partial t}$, and $\vec{B} = -\nabla \mathrm{x}\vec{A}$, gradient and curl, or $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, with tensor ($F_{\mu\nu}$), $(E_X, E_Y, E_Z, E_X, E_Y, E_Z)$components and Lorentz transformations. The derivative of the scalar function is added to such a potential, which does not change the potential itself. This is the key point. In the Yang-Mills theory, it is represented by a symmetry group, $A_\mu = \Omega(x)A_\mu(\Omega)^{-1}(x) + i\Omega(x)\partial_\mu(\Omega)^{-1}(x)$, where $\Omega(x) = e^{i\omega}$, and $\omega$is an element of any (SU (N), SO (N), Sp (N), E_6, E_7, E_8, F_4, G_2) of the group L and, $A_\mu \to A_\mu + \partial_\mu\omega$. In reality, this is a fixed state of a dynamic function: $K_Y = \psi + Y_0$, in quantum relativistic dynamics. Relatively speaking, at each fixed point: $a\left(\frac{X \equiv Z}{Y_0}\right) = const$, there is its own (angle of inclination of the branches) hyperbolic cosine, $K_Y = Y_0 ch(\frac{X \equiv Z}{Y_0}) \equiv e^{a\left(\frac{X \equiv Z}{Y_0}\right)}$, already in the orthogonal ($YZ \perp X$)) plane, and, moreover, outside the dynamic in quantum relativistic dynamics (Y_0). Thus, scalar bosons in gauge fields are created artificially to eliminate the shortcomings of the Theory of Relativity in quantum fields.

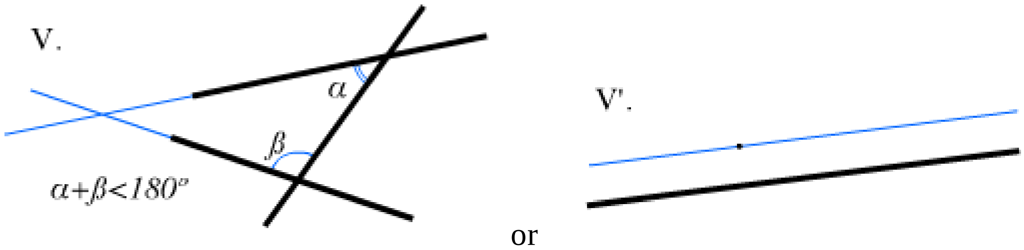
3. symmetries in classical and quantum relativistic dynamics

3.1. Lorentz transformations in physics are considered as: $\bar{x} = \frac{x+wt}{\sqrt{1-w^2}}$, $\bar{t} = \frac{t+wx}{\sqrt{1-w^2}}$, $c = 1$. These two formulas are represented as a single matrix expression.

$$\frac{1}{\sqrt{1-w^2}}\begin{pmatrix}1 & w\\ w & 1\end{pmatrix}\begin{pmatrix}t\\ x\end{pmatrix} = \frac{1}{\sqrt{1-w^2}}\begin{pmatrix}t+wx\\ wt+x\end{pmatrix} = \begin{pmatrix}\bar{t}\\ \bar{x}\end{pmatrix}. \qquad \Lambda = \begin{pmatrix}\mathrm{ch}(\alpha) & \mathrm{sh}(\alpha)\\ \mathrm{sh}(\alpha) & \mathrm{ch}(\alpha)\end{pmatrix}$$

The matrix action converts non-primed vector coordinates to primed ones. The angle of rotation in hyperbolic transformations is related to the hyperbolic arc tangent .$\alpha = arc\ \ th(w)$ and find this angle from the speed, which approaches unity: $w \to (\mathrm{c} = 1)$. The mathematical apparatus of group theory is thus quite universal **in the Euclidean axiomatics** of space-time. They are known:

1. "A point is that, part of which is nothing") ("Beginnings" of Euclid) . and whether the Point is that which has no parts,
2. Line - length without width.
3. And the 5th postulate about parallel straight lines that do not intersect. If a line intersecting two lines forms interior one-sided angles less than two lines, then, extended indefinitely, these two lines will meet on the side where the angles are less than two lines.



drawing. 3.1 Euclidean axiomatics

That is, through a point outside the line, you can draw only one straight line, parallel to the line.

3.2. in fact, in the "Unified Theory 2", undecidable in the Euclidean axiomatics are noted contradictions. That is, many lines in one line (length without width), again a line. Is it a line or many lines? Similarly, the set of points in one point is again a point. Is it a point or a set of them? The Euclidean Elements do not provide answers to such questions. well-known problems
5th postulate, the solution of which opened Lobachevsky geometry and Riemannian space.

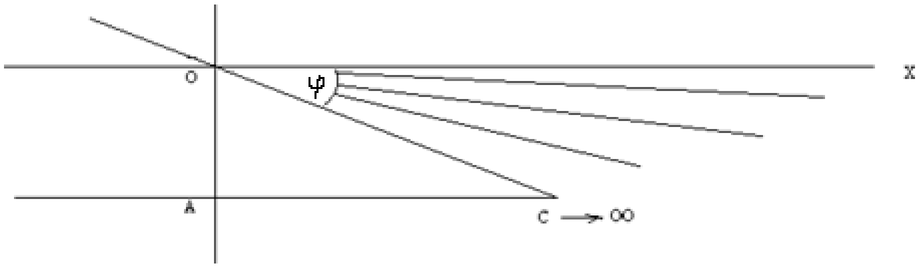


Figure 3.2 dynamic space of a pencil of parallel lines

There are real facts of the dynamic space of a bunch of straight lines that do not intersect, that is, parallel to the original line AC at infinity, presented in the "Unified Theory 2". And moving along the line (AC), there will be a dynamic space nearby, **which we will not be able to get into in principle.**
Infinity cannot be stopped, so this already dynamic space always exists. And already the properties of this dynamic $(\varphi \neq const)$ spaces are presented as properties of matter, the main property of which is movement. There is no matter outside such space, and there is no space without matter. Space-matter is one and the same.

In such a dynamic space-matter, the Euclidean axiomatics is presented as a special case zero $(\varphi = 0)$angle of parallelism. At the same time solving the set problem exactly straight lines in one straight parallel lines as " length without breadth " .

The main property of a dynamic space-matter is a dynamic $(\varphi \neq const)$ angle of parallelism. In this case, the Euclidean space in the XYZ axes loses its meaning.

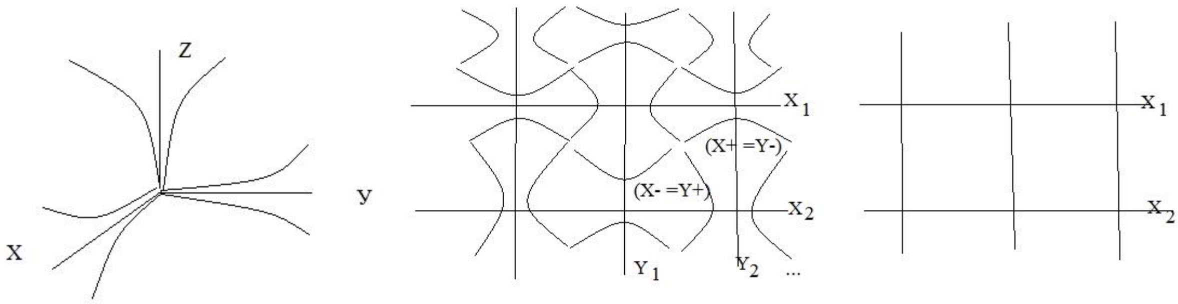


figure 3.3 dynamic space-matter

In "Unified Theory 2 " (), Transformations of relativistic dynamics in the Special Theory of Relativity and quantum relativistic dynamics (it is fashionable to say Quantum Relativity Theory), are presented in one mathematical truth, form. We are talking about the relativistic dynamics of the radius-vector of a dynamic sphere with a non-stationary Euclidean space-time , on the trajectory (X-) or (Y -) of the quantum (X ±) (Y ±), respectively, of the dynamic space-matter. Consider, for example, a quantum (X ±) of a dynamic space-matter.

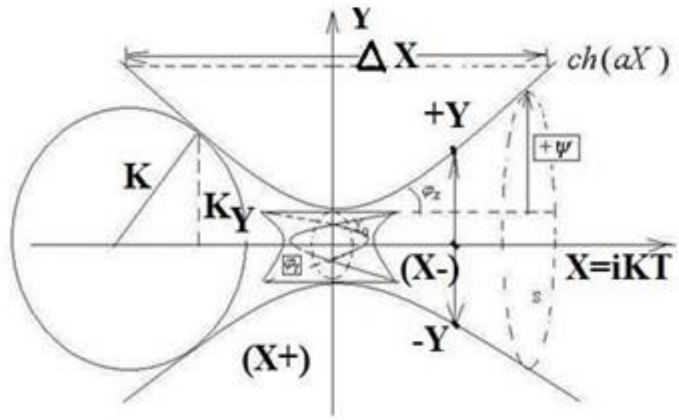


Figure 3.4 quantum of dynamic space-matter

We see that the dynamic radius-vector (К) in the sphere with non-stationary Euclidean space-time has projections ($К_Y$) in the plane of the circle of the dynamic sphere, and projections ($К_X$) on the (X-) trajectories. On (n) convergence, as we already know and see, there are two trajectories of (Y ±) quantum closed on (Y -) . As already noted, at each point fixed in the experiment with ($i\psi = \sqrt{(+\psi)(-\psi)}$) wave function, there is a hyperbolic cosine with different slope angles of the graph branches. In the section of the circle, the point fixed in the experiment with ($i\psi = \sqrt{(+\psi)(-\psi)}$) wave function, we have trigonometric functions, with different radii of the circle in different fixed points (X-) of the trajectory of the space-matter quantum. As we see at fixed points, fixed experimental facts, both representations of the Lorentz group are valid and correspond to the truth. Terms of $A_\mu(X) = \bar{A}_\mu(X) + i\frac{\partial a(X)}{\partial x_\mu}$ the Dirac equation and conditions $A_\mu = \Omega(x)A_\mu(\Omega)^{-1}(x) + i\Omega(x)\partial_\mu(\Omega)^{-1}(x)$, where: $\Omega(x) = e^{i\omega}$, in the Yang-Mills theory are not violated. Here:

$e^{i\omega} = \cos\omega + isin\,\omega)$, and ($isin\,\omega \equiv K_Y = \sqrt{(+\sin\,\omega)(-\sin\,\omega)} = i\psi = \sqrt{(+\psi)(-\psi)}$.

The identity matrices of (R_α)group elements, $\cos^2(\alpha) + \sin^2(\alpha) = 1$for any rotations and (Λ) groups: $\mathrm{ch}^2(\alpha) - \mathrm{sh}^2(\alpha) = 1$, and their derivatives in the form of group generators (reducible to zero initial conditions) are unchanged. But the very dynamics of such conditions, that is, the quantum relativistic dynamics of the dynamic sphere radius vector with non-stationary Euclidean space-time, we have lost it $a(X) \neq const$. represented by a matrix with a dynamic wave function: $i\psi = isin\,\omega \equiv \pm K_Y$in an experiment, as an argument, as a fixed fact of reality. But there is no theory, or models, equations of such "hidden processes", as we see. It must be said that in the dynamic space-matter, there is a space-matter, which we cannot get into in principle. We can't get in, by definition.

Let us present a tabular (comparative) analysis of the representations of the Lorentz groups of the relativistic dynamics of the Special Theory of Relativity and quantum relativistic dynamics, in full, without the condition (c=1) of the speed of light.

$R_\alpha = \begin{pmatrix}\cos(\alpha) & -\sin(\alpha)\\ \sin(\alpha) & \cos(\alpha)\end{pmatrix}$,$\cos^2(\alpha) + \sin^2(\alpha) = 1$ $R_{\alpha=0} = I$,$(R_{\Delta\alpha} = I + \Delta\,\alpha L)$ $(R_{\alpha+\Delta\alpha} = (I + \Delta\,\alpha L)R_\alpha = R_\alpha + \Delta\,\alpha L R_\alpha)$. $(R_{\alpha+\Delta\alpha} - R_\alpha = \Delta\,\alpha L R_\alpha)$, $lim\frac{R_{\alpha+\Delta\alpha}-R_\alpha}{\Delta\alpha\to 0} = LR_\alpha$, $\frac{dR_\alpha}{d\alpha} = LR_\alpha$, $R_\alpha = e^{\alpha L}$, solution of the differential equation, with group generator $(\frac{dR_\alpha}{d\alpha})_0 = L$. $(\frac{dR_\alpha}{d\alpha})_{\alpha=0} = \begin{pmatrix}-sin(0) & -\cos(0)\\ \cos(0) & -\sin(0)\end{pmatrix} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} = L$ $\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} = L$, group generator $R_\alpha = e^{\alpha L} = e^{\alpha * \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}}$, α- group parameter	$\Lambda = \begin{pmatrix}\mathrm{ch}\left(\frac{X\equiv Z}{Y_0}\right) & \mathrm{sh}\left(\frac{X\equiv Z}{Y_0}\right)\\ \mathrm{sh}\left(\frac{X\equiv Z}{Y_0}\right) & \mathrm{ch}\left(\frac{X\equiv Z}{Y_0}\right)\end{pmatrix}$,$\mathrm{ch}^2\left(\frac{X\equiv Z}{Y_0}\right) - \mathrm{sh}^2\left(\frac{X\equiv Z}{Y_0}\right) = 1$ $\Lambda_0\left(\frac{X=0}{Y_0}\right) = I$, $(\Lambda_{\Delta X} = I + \Delta\,(\frac{X}{Y_0}) * L)$ $\Lambda_{\frac{X+\Delta X}{Y_0}} = (I + \Delta * L)\,\Lambda_{\frac{X+\Delta X}{Y_0}}$,$(x/y_0)$ $\Lambda_{x+\Delta x/y_0} - \Lambda_{(x/y_0)} = \Delta\,(x/y_0) * L\,\Lambda_{(x/y_0)}$, $\frac{d\,\Lambda_{(x/y_0)}}{d(x/y_0)} = L\,\Lambda_{(x/y_0)}$, $x \neq const$, $y_0 \neq const$, dynamic sphere, $\Lambda_{(x/y_0)} = e^{(x/y_0)L}$, $(\frac{d\,\Lambda_{(x/y_0)}}{d(x/y_0)})_{(x/y_0)=0} = L$. $(\frac{d\,\Lambda_{(x/y_0)}}{d(x/y_0)})_{(x/y_0)=0} = \begin{pmatrix}sh(0) & \mathrm{ch}(0)\\ \mathrm{ch}(0) & sh(0)\end{pmatrix} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = L$ $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = L$, group generator $\Lambda_{(x/y_0)} = e^{(x/y_0) * \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}}$, (x/y_0)- group parameter
$R_\alpha * \Lambda_{(x/y_0)} = e^{\alpha * \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}} * e^{(x/y_0) * \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}}$, simultaneous dynamics of circular and hyperbolic motion Radius vector (its vertices) of dynamic($y_0 \neq const$) spheres.	

Special theory of relativity $\bar{x} = \frac{x-wt}{\sqrt{1-(w/c)^2}}, \bar{t} = \frac{t-wx/c^2}{\sqrt{1-(w/c)^2}}$ $\bar{w} = \frac{x-wt}{t-wx/c^2},$	Lorentz group $\Lambda = \frac{1}{\sqrt{1-(w/c)^2}}\begin{pmatrix}1 & w/c^2\\ w & 1\end{pmatrix}, \Lambda * \begin{pmatrix}t\\ x\end{pmatrix} = \begin{pmatrix}\bar{t}\\ \bar{x}\end{pmatrix}$ $\frac{1}{\sqrt{1-(w/c)^2}}\begin{pmatrix}1 & w/c^2\\ w & 1\end{pmatrix}\begin{pmatrix}t\\ x\end{pmatrix} = \frac{1}{\sqrt{1-(w/c)^2}}\begin{pmatrix}t - wx/c^2\\ -wt + x\end{pmatrix} = \begin{pmatrix}\bar{t}\\ \bar{x}\end{pmatrix},$ $\bar{t} = \frac{t-wx/c^2}{\sqrt{1-(w/c)^2}}. \bar{x} = \frac{-wt+x}{\sqrt{1-(w/c)^2}}.$ exactly the same dynamics
quantum relativistic dynamics $\overline{K_Y} = \frac{a_{11}K_Y - cT}{\sqrt{1-(a_{22})^2}}. \bar{T} = \frac{a_{22}T - K_Y/c}{\sqrt{1-(a_{22})^2}}.$ $a_{11} = \cos(\varphi_Y) \neq const,$ $a_{22} = \cos(\varphi_X) \neq const,$ $\bar{w} = \frac{a_{11}K_Y - cT}{a_{22}T - K_Y/c} = \frac{a_{11}W_Y - c}{a_{22} - W_Y/c},$	(Quantum Theory of Relativity) $Q = \frac{1}{\sqrt{1-(a_{22})^2}}\begin{pmatrix}a_{22} & 1/c\\ c & a_{11}\end{pmatrix}, Q * \begin{pmatrix}T\\ K_Y\end{pmatrix} = \begin{pmatrix}\bar{T}\\ \bar{K}\end{pmatrix}$ $\frac{1}{\sqrt{1-(a_{22})^2}}\begin{pmatrix}a_{22} & 1/c\\ c & a_{11}\end{pmatrix}\begin{pmatrix}T\\ K_Y\end{pmatrix} = \frac{1}{\sqrt{1-(a_{22})^2}}\begin{pmatrix}a_{22}T - K_Y/c\\ a_{11}K_Y - cT\end{pmatrix} = \begin{pmatrix}\bar{T}\\ \bar{K}\end{pmatrix}$ $(a_{11} \neq a_{22}) \neq const,$ $\begin{pmatrix}a_{22} & 1/c\\ c & a_{11}\end{pmatrix} = a_{11} * a_{22} - c * \frac{1}{c} = 0. a_{11} * a_{22} = c * \frac{1}{c} = 1 .$ from where follows: $a_{11} * a_{22} = \cos(\varphi_Y) * \cos(\varphi_X) = 1 ,$

In the case of quantum relativistic dynamics, as we see, the symmetry condition follows: under the conditions $(a_{11} \neq a_{22}) \neq const$, we get: $a_{11} * a_{22} = \cos(\varphi_Y) * \cos(\varphi_X) = 1$, in nonzero values angles of parallelism $(\varphi_Y \neq 0)$, $(\varphi_X \neq 0)$, for the conditions of the denominator $\sqrt{1-(a_{22})^2} \neq 0$, $(X\pm)$quantum. Exactly the same transformations $(Y\pm)$ quantum, with the terms of the denominator $\sqrt{1-(a_{11})^2} \neq 0$. But the angles of parallelism cannot be 90^0. This means $(\varphi \neq 90^0)$that there are limit angles of parallelism, which correspond to the constants of interactions, in the form: $\cos(\varphi_Y)_{max} = \alpha(Y\pm) = 1/137.036$, and: $\cos^2(\varphi_X)_{max} = G(X\pm) = 6.67 * 10^{-8}$.

In the "Unified Theory 2 ", we considered the unified Criteria for the Evolution of dynamic space-matter in multidimensional space-time. In particular, charge: q = ΠK (Y+ =X -) in electro (Y+ =X -) magnetic fields, and mass: m = ΠK (X+=Y-) in gravity (X+=Y-) mass fields . We have also considered models of proton quantum fields:

$(X\pm = p^+) = (\boldsymbol{Y}-= \gamma_0^+)(\boldsymbol{X}+= \nu_e^-)(\boldsymbol{Y}-= \gamma_0^+)$, and: $(Y\pm = e^-) = (\boldsymbol{X}+= \nu_e^-)(\boldsymbol{Y}-= \gamma^+)(\boldsymbol{X}+= \nu_e^-)$ electron.

Then the conditions: ${a_{22}}^2 * a_{11} = \cos^2(\varphi_X)\cos(\varphi_Y) = 1$, quantum relativistic dynamics $(X\pm)$ quantum take the form:

$$(X \pm) = (X+= Y-)^2 * (Y+ = X -), \text{or: } \Pi K * \cos^2(\varphi_X)\cos(\varphi_Y) = 1 * \Pi K.$$

$$(\Pi K(X+= Y -) = m_0 = 1) * cos^2(\varphi_X)_{max} cos(\varphi_Y)_{max} = 1 * (\Pi K(Y+ = X -) = q_0 = 1),$$

Scale (a_{22}), in quantum state:${a_{22}}^2 * a_{11} = \cos^2(\varphi_X)\cos(\varphi_Y) = 1$ matrix $\begin{pmatrix}1 & \alpha\\ 1 & 1\end{pmatrix}^2 = (1-\alpha)^2$. Then:

$$(m_0 = 1) * (1-\alpha)^2 (cos^2(\varphi_X)_{max} = G)(cos(\varphi_Y)_{max} = \alpha) = 1 * q,$$

$$q(Y+ = X -) = (1-\alpha)^2 * G * \alpha = (1 - 1/137.036)^2 * 6.67 * 10^8 * (1/137.036) = 4.8 * 10^{-10}.$$

We have obtained an electric charge in the symmetry group of its quantum relativistic dynamics.

In the same way, by scaling the symmetry group $Q = e^{(X,Y)*L}$ of quantum relativistic dynamics (it is fashionable to say in the Quantum Relativity Theory), but already of mass fields, one can search for the mass spectrum of elementary particles. This is different from the symmetries of Lorentz groups in gauge fields.