Vacuum energy near the earth's surface. 5.8·10^10 J/m^3

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Abstract. Gravitational energy is localized. The problem, from which Einstein, Eddington, and other classics have distanced themselves, has been solved. It is shown that the gravitational energy that, for example, a falling apple receives, is released by space during its additional curvature caused by the fall of this apple. The local density of gravitational energy is determined by the characteristics of the space-time curvature.

Key words: curvature of space; mass-energy conservation; collapse

1. Introduction

The apple, which Newton watched, acquired kinetic energy $mv^2/2 = mgh$ when falling from a height h and, accordingly, acquired the mass $mgh/c^2 = \Delta m$, where m is the initial mass of the apple. Thus, the law of mass conservation of the closed Earth-apple system was violated. The place where the apple fell absorbed this additional mass Δm in the form of the mass-energy of heating this place after the fall of the apple.

Similarly, during a gravitational compression of a star, the star heats up with a corresponding increase in the mass and, accordingly, with a violation of the law of mass conservation of a closed system.

The law of conservation of mass-energy is usually violated during gravitational interaction. Einstein geometrized the Newtonian gravitational field. "Gravitational field" does not exist within the framework of general relativity. Weyl writes about “leading” (Führung) [1]. All gravitational phenomena are explained by the curvature of space-time. At the same time, no energy or mass is attributed to space itself. That's why the mutual gravitational attraction of masses is fundamentally different from the mutual electrical attraction of electric charges of different signs. To compare electrical and gravitational attraction, it is convenient to consider an isolated centrally symmetric system.

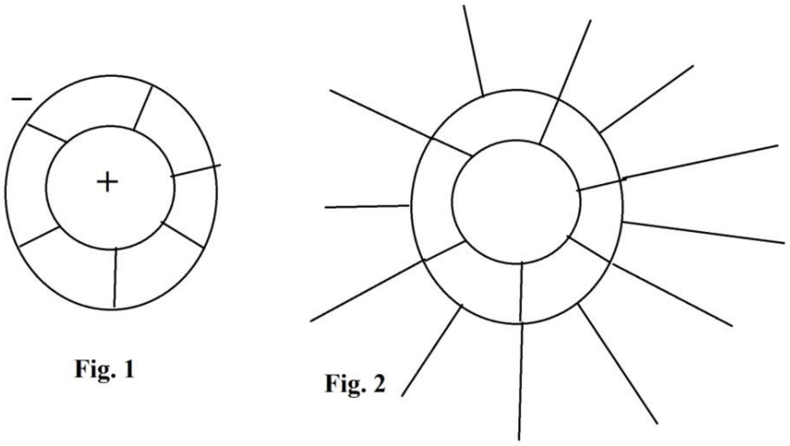

Fig. 1 Fig. 2

In the case of electricity, this is an oppositely charged ball and a shell located at some distance from the ball (Fig. 1). Between them, and only between them, there is an electric field with its mass-energy. When the shell falls on the ball, the field is eliminated, and the field energy goes

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first into kinetic energy, and then into heat. So the mass-energy of the system remains unchanged during the fall.

In the gravitational case (Fig. 2), this is a massive ball and a massive shell (we will assume that the masses are the same). Unlike the electrical case, the so-called gravitational field exists not only between the ball and the shell. The field of double strength exists outside the shell. But the entire “gravitational field” exists in empty space, to which no mass is attributed. When the shell falls on the ball, the field is not eliminated. On the contrary, the field doubles in this interval. However, the kinetic mass-energy, and then the thermal mass, arise in the same way as in the electrical case. So in the gravitational case, the mass of the system increases!

In the process of attraction of an electron to a proton, the total mass-energy of the system does not change. When an electron and a proton approach each other, the area occupied by the electric field decreases. As a result, the energy of the electric field decreases. The electric field is eliminated so that a neutral atom is obtained. In this case, the field energy is first converted into the kinetic energy of the electron and the proton flying towards each other. The negative mass defect occurs at the end of the process, when part of the energy in the amount of 13.6 eV is radiated. As a result, the mass of the neutral hydrogen atom is less than the sum of the masses of the electron and the proton by 13.6 eV. This can be seen as an electromagnetic mass defect, analogous to the nuclear mass defect that, for example, an alpha particle has with respect to the mass of four nucleons.

We emphasize that electromagnetic and nuclear negative mass defects are a decrease in mass during the formation of a compact neutral or colorless (in the case of nucleons) object only after an emission of a field quanta. A gravitational mass defect is principally positive. It is an increase in mass. An example is constructed for the creation of matter in the form of a compact body in an gravitational field [2]. But since the gravitational field does not exist within the framework of general relativity, this mass increase comes from nothing from the modern point of view. Or you need to assign a negative sign to the energy density of the gravitational field, since the field increases simultaneously with the increase in mass. However Weyl writes: "Einstein left to the mercy of fate the principle of unique localization for gravitational energy. This negative sign was the main stumbling block" [1].

We indicate the source of the emerging mass-energy. Consider a gravitational compression of a large dusty cloud, initially distributed in a practically flat space. It leaves an empty curved Schwarzschild space behind its surface upon the compression. Therefore, in order to fulfill the law of energy conservation, one has to accept that the energy that the dust acquires during gravitational compression comes from the space during its curvature. It seems that the curvature is energetically favorable to space. When curving, the energy of space decreases. In the process of gravitational compression, matter allows space to curve and receives energy from it. If we assume that the energy of flat space is zero, then the energy of curved space is negative. So, during the gravitational compression of matter, the sum of the increasing positive mass-energy of matter and the negative energy of space is preserved. This sum can be called the *total* energy of the star. Tolman represents the negative energy of curved space by the use of the usual Newtonian potential $\psi < 0$. This sum is clearly expressed by the formula [3 (97.10)],

$$m = \int \rho dV_0 + \int \frac{1}{2} \rho \psi dV . \tag{1}$$

Tolman wrotes: “the *total* energy of a fluid sphere would reduce to the sum of the *total proper* energy and the usual Newtonian expression for potential gravitational energy”. The constant m equals to the mass of the cloud before compression.

2. Space-time curvature

Einstein's great discovery is that matter turns out to bend space (space-time). The curvature of space depends both on the mass-energy density of the substance and on the pressure in the substance. This curvature of space occurs both inside the substance and in adjacent empty regions of space. In particular, the three-dimensional space around the Sun is curved. This curvature in the literal sense is observed as an additional curvature of the flight trajectory of photons attracted to the Sun with respect to the trajectory in Euclidean space [4-6].

Mathematically, Einstein's discovery is expressed by the fact that the Einstein tensor G^{α}_{β}, which describes the local geometric properties of space-time, is proportional to the matter energy-momentum tensor T^{α}_{β}, which locally describes the mass-energy and momentum density of matter:

$$G^{\alpha}_{\beta} \equiv R^{\alpha}_{\beta} - R^{\mu}_{\mu}\delta^{\alpha}_{\beta} / 2 = 8\pi\gamma T^{\alpha}_{\beta}, \quad \gamma = 7.4 \cdot 10^{-28} \text{ m/kg.} \tag{2}$$

The nature of the curvature of the space surrounding a star was studied, for example, in [7 § 100, problem 3]. In any case, the curvature of the centrally symmetric space is described by the linear element

$$dl^2 = g_{rr}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) . \tag{3}$$

The value of the metric coefficient g_{rr} was found by Schwarzschild in two cases [3 § 96]. For the empty space surrounding a star,

$$g_{rr} = \frac{1}{1 - r_g / r} ; \tag{4}$$

for the interior of a star of constant density,

$$g_{rr} = \frac{1}{1 - r^2 / R^2} , \tag{5}$$

moreover, if the radial coordinate of the star's surface is r_1, then

$$r_g = r_1^3 / R^2 . \tag{6}$$

This is the condition that the coefficients g_{rr} (4) and (5) coincide on the surface of the star.

In both cases $g_{rr} > 1$. This means that the curvature of space by a star is as follows. The distance between two spherical surfaces, the equators of which have lengths $2\pi r$ and $2\pi(r + dr)$, is not equal to dr, as it would be in Euclidean space, but more, namely $\sqrt{g_{rr}}dr$. Therefore, the volume of space enclosed between these spherical surfaces is equal to

$$dV = 4\pi r^2 \sqrt{g_{rr}} dr . \tag{7}$$

This means that the volume of space inside a sphere with an equator of length $2\pi r$ is greater than the Euclidean value $4\pi r^3 / 3$. This volume is obtained by integrating

$$V = \int_0^r 4\pi r^2 \sqrt{g_{rr}} dr . \tag{8}$$

And if this volume is filled with a substance with density ρ kg/m3, then the mass of this substance in the volume is obtained by integrating

$$M = \int_0^r \rho 4\pi r^2 \sqrt{g_{rr}} dr . \tag{9}$$

Such an integral was considered by Tolman. He called it "the *total proper* energy (of the liquid sphere)" [3 (97.4), (97.10)]:

$$\int \rho dV_0 , \quad dV_0 = \sqrt{-g_{xx}g_{yy}g_{zz}} dxdydz . \tag{10}$$

The calculation of the integral (9) for the metric coefficient (5) is presented in [8-10]. We repeat it here for the convenience of readers.

3. Mass of a ball of constant density ρ

According to Einstein's equation (2), $G^t_t = 8\pi\gamma\rho$. The component G^t_t is presented in [3 (96.7)] $G^t_t = 3 / R^2$. So

$$\rho = \frac{3}{8\pi\gamma R^2} = \frac{3r_g}{8\pi\gamma r_1^3} . \tag{11}$$

Calculation of the proper mass of the ball using formula (9) or (10) gives the expression

$$M=\int_0^{r_1}\rho\sqrt{g_{rr}}\,4\pi r^2dr=\frac{3}{2\gamma R}\int_0^{r_1}\frac{r^2dr}{\sqrt{R^2-r^2}}=\frac{3r_1}{4\gamma}\left(\frac{\arcsin\xi}{\xi}-\sqrt{1-\xi^2}\right),\quad \xi^2=\frac{r_g}{r_1}\tag{12}$$

If the star is slightly compressed, then $\xi^2=\frac{r_g}{r_1}\ll 1$. Then

$$M\approx\frac{3r_1}{4\gamma}\left(\frac{2}{3}\xi^2+\frac{1}{5}\xi^4\right)=\frac{r_g}{2\gamma}+\frac{3r_g^2}{20\gamma r_1}=m+\frac{3r_g}{10r_1}m\,.\tag{13}$$

It can be seen that as the compression increases, the mass of the star increases due to the second term. We denoted

$$m=\frac{r_g}{2\gamma}\tag{14}$$

the initial mass of the initially rarefied cloud when $r_1\to\infty$ and the curvature of space is absent. So m is obtained by integration in flat space, without $\sqrt{g_{rr}}$ [7 (100.24)],

$$m=\int_0^{r_1}\rho 4\pi r^2dr=\int_0^{r_1}\frac{3r_g}{8\pi\gamma r_1^3}4\pi r^2dr=\frac{r_g}{2\gamma}\tag{15}$$

The increase in mass when compressing is easy to explain. According to (11), as the radius of the star r_1 decreases, the matter density increases $\rho\sim 1/r_1^3$. At the same time, the volume of the star decreases more slowly than $4\pi r_1^3/3$, due to the curvature of space. This is taken into account by the metric coefficient $g_{rr}>1$.

The increase in the mass-energy of a star during compression is beyond doubt, since the star heats up during gravitational collapse, which corresponds to an increase in the mass of iron during heating. Thus, both the calculation and the observation show that the *gravitational mass defect is positive*. This result was published [10].

It should be noted that the authorities of physics, in a strange way, object to the obvious fact of an increase in mass in the process of gravitational compression. Thus, Misner, Thorne and Wheeler write [11]: "The mass-energy of a neutron star is less than the mass-energy of the same number of baryons at infinite separation".

4. Energy density of curved space

Now our task is to find an expression for the volume energy density of curved space such that its integration over the entire space gives this additional mass-energy $\frac{3r_g^2}{20\gamma r_1}$. We denote this volume energy density of curved space as u kg/m^3. So it should be:

$$\int u 4\pi r^2dr=\frac{3r_g^2}{20\gamma r_1}\,.\tag{16}$$

Since this quantity is small compared to m, the factor $\sqrt{g_{rr}}$ can be neglected in integrand (16). This factor was necessary in formulas (8) – (10), (12) when calculating the main energy.

In order to find the energy transferred from space to matter when an additional region of space is bent, consider a thin massive sphere of radius *r*, mass *m*. The free fall acceleration in the surrounding Schwarzschild space is defined in the framework of general relativity as the curvature of the world line of a stationary observer, for which $r=Const$, [4 (12)], [5 (13)], [6 (16)]:

$$g=\frac{\sqrt{g_{rr}}}{g_{tt}}\frac{D^2r}{dt^2}=\frac{\sqrt{g_{rr}}}{g_{tt}}\Gamma^r_{tt}=\frac{r_g}{2r^2}\sqrt{\frac{r}{r-r_g}}\approx\frac{r_g}{2r^2}\,,\tag{17}$$

where Γ^r_{tt} is the Christoffel symbol. This acceleration varies across the thickness of the sphere from zero inside to $r_g/2r^2$ outside. So the average acceleration is $\bar{g}=r_g/4r^2$. Due to this acceleration, the particles of the sphere experience a net radial gravitational force $F = m\bar{g} = r_g^2 / 8\gamma r^2$, so that when displaced by dr they gain energy $dU = r_g^2 dr / 8\gamma r^2$. In this case, the space is curved in the volume $4\pi r^2 dr$, so that the density of the supplied energy is equal to

$$u = \frac{dU}{4\pi r^2 dr} = \frac{r_g^2}{32\gamma\pi r^4} = \frac{g^2}{8\pi\gamma}. \tag{18}$$

This is the desired quantity to be integrated in (16).

The magnitude of this density of gravitational energy near the Earth's surface is of interest. For Earth

$$g = r_g / 2r_1^2 = 8.87\cdot 10^{-3}\text{m} / 2\cdot(6{,}37\cdot 10^6\text{m})^2 = 1.1\cdot 10^{-16}/\text{m} \quad \text{or}$$

$$g = 9.8\text{m/s}^2 / (3\cdot 10^8\text{m/s})^2 = 1.1\cdot 10^{-16}/\text{m}.$$

So we have $u = g^2 / 8\pi\gamma = (1.1\cdot 10^{-16}/\text{m})^2 / (8\pi\cdot 7.4\cdot 10^{-28}\text{m/kg}) = 6.5\cdot 10^{-7}\text{kg/m}^3 = 5.85\cdot 10^{10}\text{J/m}^3$

It is not surprising that with such a high energy density, a small additional curvature of space due to the fall of an apple will cause discomfort in the case of the apple falling on a head.

5. Counting the energy of curved space

For the external Schwarzschild solution with respect to the ball under consideration, the acceleration depends on the coordinate *r* according to (17). Inside the ball, the acceleration decreases linearly from the maximum value on the surface of the ball $r_g / 2r_1^2$ to zero in the center:

$$g_{in} = \frac{r_g r}{2r_1^3}. \tag{19}$$

Therefore, integration (16) must be divided into two parts:

$$\int_0^\infty u 4\pi r^2 dr = \int_0^{r_1} \frac{g_{in}^2}{8\pi\gamma} 4\pi r^2 dr + \int_{r_1}^\infty \frac{g^2}{8\pi\gamma} 4\pi r^2 dr = \int_0^{r_1} \frac{r_g^2 r^4 dr}{r_1^6 8\gamma} + \int_{r_1}^\infty \frac{r_g^2 dr}{r^2 8\gamma} = (\frac{1}{5}+1)\frac{r_g^2}{8\gamma r_1} = \frac{3r_g^2}{20\gamma r_1}, \tag{20}$$

which was to be shown, see (16).

6. Conclusion

The square of gravitational acceleration characterizes the density of negative mass-energy contained in curved space (18), just as the square of the electric or magnetic field characterizes the density of positive mass-energy contained in an electromagnetic field. This means the localization of gravitational energy in space.

We considered the gravitational compression of a star, during which the matter of the star received the so-called gravitational energy and, because of this, was heated; its mass grew. Consider now the removal of the baryons of a neutron star to infinity. This requires mass-energy. But there is nowhere to take this mass from, except from the star itself. So when you remove baryons, you have to reduce the mass of the star. The mass has to be taken away from the baryons and introduced into the curved Schwarzschild space, spending this mass on straightening the space, on increasing the space energy from a negative value to zero. Therefore, the total mass of baryons after their removal turns out to be less than the mass of the neutron star.

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Notes

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