Relativistic Interferometry Using Aqueous Waves

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Abstract

In this paper we investigate the geometry and sequence of events within a Michelson-Morley interferometer and generalise our findings into the aqueous domain. In doing so we uncover a conflict between the predictions of special relativity and the symmetry of nature.

Keywords— special relativity, paradox, symmetry of nature

1 Introduction

The Michelson-Morley (MM) experiment [1] and its resolution by the special theory of relativity (SR) [2] form a foundational truth in modern physics. Let us examine the validity of this truth by testing its compatibility against the symmetry of nature [3]. We investigate as follows:

1. We begin with the geometry of two flat triangles that are relevant to the discussions at hand.
2. Then we present a thought experiment involving ideal sinusoidal waves that travel, reflect and interfere with each other within the confines of a circular boundary.
3. Next we establish that our thought experiment is equivalent to the MM experiment and from this we generalise the MM result in order to predict the outcome of our thought experiment.
4. Finally we realise our thought experiment in a circular ripple tank and leverage on the equivalence of aqueous and optical interferometry to arrive at our conclusion.

2 Euclidean Geometry

On a flat surface, we draw any angle θ at origin Q bounded by two equal length line segments $QB = QB' = h$. We join points B and B' to points A and C such that the line segment AC is perpendicular to QB and centred at Q. We will restrict our arguments to the domain $x < h$. Fig. 1 illustrates.

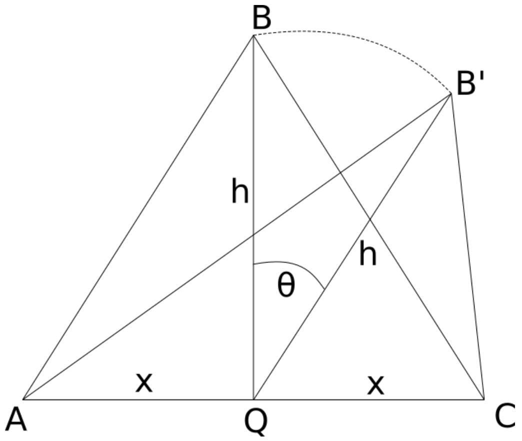

Figure 1: Triangles ABC and $AB'C$ rendered on a flat surface

From fig. 1, we establish the following geometric truths:

1. If $x > 0$, physical measurements of fig. 1 will verify the theoretical statement $AB + BC \neq AB' + B'C$ is true for all $\theta \neq 0, \pi, 2\pi...$
2. Since h is constant, curve BB' will take the form of a circle as $0 \leq \theta \leq 2\pi$.

3 A Thought Experiment

Imagine an ideal homogeneous flat surface S1 enclosed by an ideal rigid boundary of geometrically circular shape (radius = h) and capable of transporting a travelling wave of the form,

$$\frac{1}{c^2}\frac{\delta^2 y}{\delta t^2} = \frac{\delta^2 y}{\delta x^2} \tag{1}$$

where the terms are as follows:

1. x represents the displacement of the measurement point from the origin of the wave measured along surface S1,
2. c represents the velocity of the wave measured along surface S1,
3. y represents the instant displacement of the wave measured perpendicular to surface S1.
4. t represents the time elapsed since the instant that the wave was created.

From directly above, we may project fig. 1 onto S1 without distortion such that the boundary of S1 is defined by curve BB', a circle of radius h about point Q.

Now let us agree that surface S1 supports the geometry of fig. 1 over all $0 \leq \theta \leq 2\pi$ and $0 \leq x < h$. We choose any point A on S1 and disturb the equilibrium causing an isotropic sinusoidal wave (wavelength = λ) to emanate from that point. As this primary wave expands, its wavefront will interact with S1's boundary generating innumerable secondary waves as it does so. Each reflection event along curve BB' generates its own isotropic wave and from physical measurements of fig. 1, we find that if $x \neq 0$ the statement $AB + BC \neq AB'_1 + B'_1C... \neq AB'_i + B'_iC$ is true (See fig. 2 which is a generalisation of fig. 1 over all $0 \leq \theta \leq 2\pi$). Let us invoke the following assumptions to debate the nature of the interference pattern at point C:

1. The wave we generate originates from a single point and comprises exactly one complete cycle of a sinusoidal travelling wave

2. λ remains constant in accordance with the law of conservation of energy [4]
3. Reflections are instantaneous and lossless

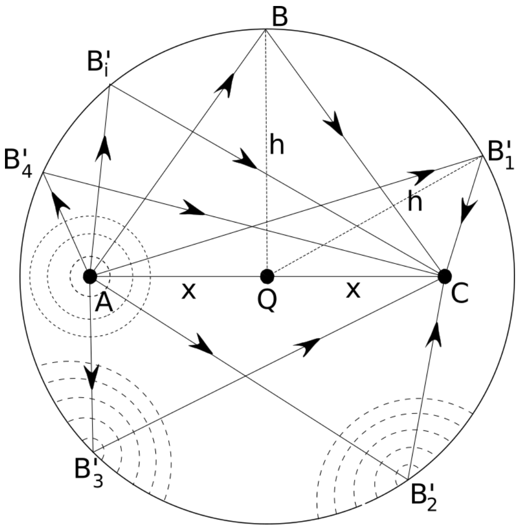

Figure 2: A single isotropic sinusoidal wave is emitted from point A and reflects from the circular boundary generating innumerable secondary wavefronts. With reference to sec. 6.1.1, we readily observe in both MM and WW experiments that if $x > 0$ then reflection events from any two points B'_i and B'_j occur simultaneously only if $\sin\theta_i = \sin\theta_j$ i.e. only if the line segment $B'_iB'_j$ is perpendicular to AC.

4 The Michelson-Morley Experiment

Now we turn to theoretical aspects of the MM experiment in order to establish it's equivalence with our thought experiment.

4.1 Frames of Reference

For the purpose of further discussion, we refer to fig. 1 and establish the following euclidean frames of reference:

1. A stationary reference frame I_0 centered at point Q.
2. A moving reference frame I_1 that translates from point A to point C with some constant velocity v relative to arbitrarily selected origin Q.

4.2 Geometry and Sequence of Events

Consider an MM interferometer [1] moving through space under inertial rules (see fig. 3). By fixing $\angle B'_1QB'_2 = \pi/2$, line segments QB'_1 and QB'_2 form the arms of the interferometer. The arms are free to rotate about point Q and consequently each arm subtends its own angle θ measured from a perpendicular to line segment AC. Reference frame I_1 is fixed to the interferometric source and moves with constant velocity v from point A to point C.

The event cycle begins with the source at point A marking the simultaneous emission of a pair of photons (wavelength=λ). As the entire apparatus moves with some constant ($AQ = QC$) velocity v relative to origin Q along line segment AC, the photons are emitted at point A, reflect from mirrors B_1 and B_2 to finally arrive simultaneously (in phase with each other) at point C.

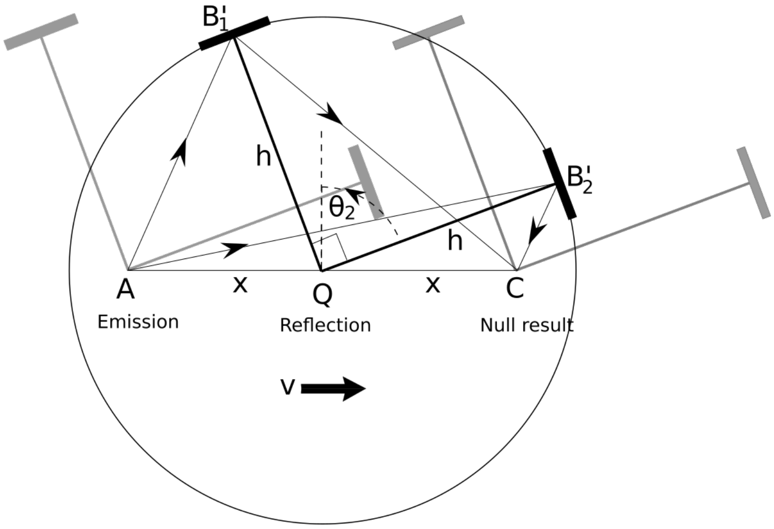

Figure 3: Geometry of the Michelson-Morley experiment depicting the general case $x \neq 0$. Equivalent to our thought experiment and identical to fig. 1, we find $AB_1' + B_1'C \neq AB_2' + B_2'C$ but yet we agree that the outcome is a null result at point C.

As is true in our thought experiment, it is straightforward to recognise that in one emission-reflection-result cycle of an MM interferometer and for all $0 \leq v < c$, the locus of all points in space where a reflection event can occur is a physical circle of radius h about origin Q. In terms of scope, our thought experiment is equivalent to one cycle of an MM interferometer having infinite arms (See fig. 2). It is also a well established fact of modern science [5] that the MM experiment presents a null result for all $0 \leq v < c$, where c represents the velocity of light.

4.3 Conflict Resolution

The geometry of the MM experiment and its sequence of events present a paradox of unequal path lengths but only from the perspective of a stationary observer (reference frame I_0) i.e. in all experimental cases where $v \neq 0$. This conflict is traditionally resolved by the application of SR. In order to reconcile the paradox of unequal path lengths, SR predicts the existence of measurable distortions in the structure of space and time known as lorentz contraction and time dilation. The magnitude of these effects is proportional to the lorentz factor [2] given by,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2}$$

Equation 2 predicts that in cases where $v \approx c$, lorentz contraction and time dilation grow to infinite magnitudes. For the purpose of further discussion, let us stipulate that the predictions of SR are true [6] [7].

5 Predicted Outcome in our Thought Experiment

We now generalise the results of the MM experiment to predict the outcome of our thought experiment (sec. 3). We reason this outcome as follows:

1. Both experiments occur within equivalent spatial geometries i.e. Points A and C are always diametrically opposite each other and always contained within a circle of radius h about point Q.
2. In both experiments, eq. 1 equivalently governs the properties of the waves under investigation.
3. Therefore if the emission event is identical in both experiments, the sequence and character of all other events within both experiments must be identical as well.

From this,

1. We expect identical results i.e. null results in both experiments.
2. Null results would create equivalent paradoxes in both experiments.
3. If SR can be applied in the optical domain, it may be also be equivalently applied to reconcile the paradox of unequal path lengths presented by our thought experiment. Noting that the terms v and c in the MM experiment are equivalent to x and h respectively in our thought experiment, we must predict imaginary equivalents of lorentz contraction and time dilation to manifest in our thought experiment according to the rule,

$$\gamma = \frac{1}{\sqrt{1 - \frac{x^2}{h^2}}} \tag{3}$$

6 Practical Implications

Let us now realise our thought experiment onto the surface of a circular container (arbitrary radius $= h$) of fluid such as water. The experiment may be performed by gently allowing a single droplet of water to disturb the surface equillibrium, the location of the drop (point A) being randomly chosen (see fig. 4). The reader will soon see that practical concerns such as non-ideal waveform, circularity errors of the boundary, bottom interactions, meniscus, dispersion (non-constant wavelength), measurement errors etc. are irrelevant to the argument being presented. We refer to this experiment as the Water Wave (WW) experiment.

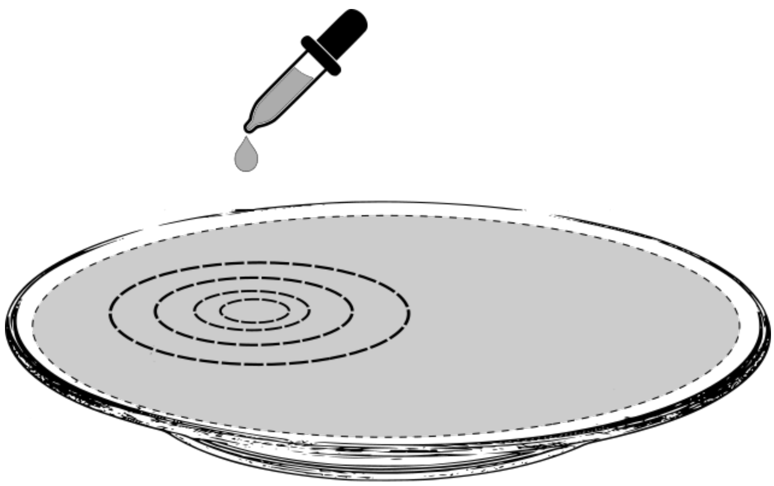

Figure 4: The Water Wave experiment may be performed using a circular platter and any suitable means to initiate an isotropic wave on the surface of the water.

Conceding that the physical surface of the water and the boundary of the container are far from ideal, we invoke the well established theoretical [8] and practical [9] [10] equivalence of optical and aqueous interferometry to assert that this physical arrangement at least to some small degree *approximates* the ideal properties of our thought experiment and equivalently, the MM experiment. Therefore if we were to physically conduct this experiment in a circular ripple tank, it is reasonable to assume that the outcome should at the very least *approximate* the ideal outcomes we have obtained from our thought experiment and the MM experiment.

Put another way, we expect the ideal theoretical predictions of the MM experiment and results of an equivalent aqueous experiment to agree with each other within some acceptable bounds due to practical limitations. Accordingly we predict for the WW experiment,

1. An *approximately* null result. This result is easily verified by **experiment**. Rather than chaos on the water surface, it is easy observe a definite convergence of waves around point C, supporting the assumption that ideal theoretical predictions of the MM experiment are indeed manifested *approximately* in the WW experiment.
2. The symmetry of nature [3] implies that *every* outcome of practical optical interferometry/ thought experiment be manifested *approximately* in the conduct of an equivalent aqueous experiment. Indeed, in general practice, we observe the travel, reflective, refractive, diffractive and interference properties of water waves are *approximately* equivalent to that of optical waves.

6.1 Relativistic Effects in Aqueous Interferometry

Let us now investigate if the relativistic effects observed in optical interferometry are also manifested equivalently in the conduct of aqueous interferometry.

6.1.1 Relativity of Simultaneity

Consider the spatial and temporal perspectives of two observers separated in the velocity domain. Recall that in the MM experiment, the observational perspective of the moving reference frame (I_1) is revealed by setting $v = 0$ (equivalently $x = 0$ in the WW experiment) and that of the stationary reference frame (I_0) by setting $0 < v < c$ (equivalently $0 < x < h$ in the WW experiment). Recall also from fig. 2 or fig. 3 that if $\theta \neq 0$ then in both WW and MM experiments, points B and B'_i are separated in space from the perspective of both observers.

In **conducting** the WW experiment we readily observe that (i) from the perspective of the moving observer (revealed by setting $x = 0$), the reflection events from B and B'_i occur at *approximately* the same instant in time and (ii) from the stationary observer's perspective (revealed by setting $x > 0$), the reflection events from B and B'_i are separated in time *approximately* as a function of x and $\sin\theta$.

In relativistic optical interferometry, this difference in observational perspectives is recognised as that of distant simultaneity [11]. Therefore we conclude that relativistic effects are also manifested *approximately* equivalently in the WW experiment.

6.1.2 Lorentz Contraction and Time Dilation

We have already stipulated that the predictions of SR namely lorentz contraction and time dilation are true when we conduct relativistic interferometry using optical waves. We now invoke the impartiality of nature [3] to predict that *approximate* effects equivalent to lorentz contraction and time dilation must also be physically manifested in accordance with eq. 3 when we conduct relativistic interferometry using aqueous waves. Let us test this prediction.

7 Physical Experiment

As this **demonstration video** of the WW experiment clearly shows, independent of x^2/h^2, the time interval T taken from emission to result remains *approximately* a constant showing that an aqueous equivalent of time dilation is absent. Further, independent of x^2/h^2, the boundary of the surface remains *approximately* a circle showing that an aqueous equivalent of lorentz contraction is also demonstrably absent. By setting $x \approx h$, eq. 3 predicts infinitely large magnitudes of lorentz contraction and time dilation, but instead we readily observe that not an iota of these effects are physically manifested.

8 Conclusion

At this stage it is reasonable to recall the perceived equivalence of optical and aqueous interferometry and ask: if the predictions of SR are true, has nature abandoned her impartiality and *preferred* not to equivalently implement **even a trace** of lorentz contraction and time dilation when we conduct relativistic interferometry using aqueous waves?

9 Statements and Declarations

The author has no competing interests to declare that are relevant to the content of this article. There are no data associated with this article.

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