New Consideration on Electromagnetic Induction

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Faraday's Law of Electromagnetic Induction vs Lorentz's Magnetic Field Force Theorem

Abstract: Faraday's law of electromagnetic induction reveals that the induction electromotive force generated in a metal coil is proportional to the change rate of the magnetic fluxes passing through the coil. Lorentz's magnetic field force theorem reveals that an electric charge moving in a magnetic field is affected by the Lorentz magnetic field force. Lorentz's magnetic field force theorem is the microscopic physical nature of induction electromotive force. A metal wire moving in a magnetic field will generate an induction electromotive force between two ends of the wire. In this study, the calculation formulas of electromotive force of metal wires were derived based on Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem, respectively. When a metal wire moves at a uniform speed in a magnetic field, the calculation formulas derived from both of them are the same. When a metal wire moves back and forth sinusoidally in a magnetic field, the electromotive force of the wire, which derived from Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem, is different. Lorentz 's magnetic force theorem is a universal fundamental electromagnetic theorem. Therefore, Faraday's law of electromagnetic induction is an engineering approximation formula. This study proposes the electron motion resistance force theorem: An electron moves in the metal wire, it will be affected by the motion resistance force, and the electron motion resistance force is proportional to the speed of the electron. An electric charge moving in a uniform magnetic field is affected by the Lorentz magnetic field force, which is the microscopic physical nature of the motional electromotive force. An electric charge at rest in a magnetic field wave is also affected by the Lorentz magnetic field force, which is the microscopic physical nature of the induced electromotive force. The electromotive force in metal wires and coils is essentially the result of the counter-potential movement of electric charges under the action of the Lorentz magnetic field force. This study reveals that Faraday's law of electromagnetic induction is an engineering approximation formula, which is a great challenge for Maxwell's equations and the fundamental electromagnetic theorems.

Keywords: Faraday's Law of Electromagnetic Induction, Lorentz's Magnetic Field Force Theorem, Electron Motion Resistance Force Theorem, Induction Electromotive Force, Motional Electromotive Force, Induced Electromotive Force, Maxwell's Equations.

1. Introduction

In 1820, the Danish physicist Oster found that electric current could produce a magnetic field. Many scientists began to explore and study whether a magnetic field could also produce an electric current. In 1831, Faraday revealed for the first time through experiments that the change of magnetic flux in a metal coil could produce induced current and voltage in the metal coil.

As shown in Figure 1.1, a metal coil was connected in series with a voltmeter V. When the magnetic rod B was inserted or pulled out of the metal coil C, the coil C generated an induction electromotive force, and the pointer of voltmeter V deflected. The faster the insertion or withdrawal, the greater the induction electromotive force generated by coil C. When the magnetic rod was stopped in the coil, there was no induction electromotive force in the coil and the pointer of the voltmeter V did not deflect.

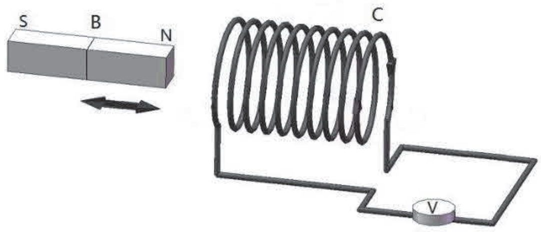

Figure 1.1 The electromagnetic induction of a metal coil

With a large number of experiments [1] [2] [3], Faraday revealed that the induction electromotive force generated in a metal coil was proportional to the change rate of the magnetic fluxes passing through the coil. This conclusion is called Faraday's law of electromagnetic induction. The induction electromotive force with Faraday's law is expressed as:

$$\varepsilon = - d\Phi/dt \qquad (1\text{-}1)$$

where "-" indicates the direction of the induction electromotive force, which is determined by Lenz's law.

Faraday's law of electromagnetic induction states that whenever the magnetic fluxes passing through the coil change, an induction electromotive force is generated in the coil. There are two ways in which magnetic fluxes change. The first is when the magnetic field intensity remains constant, and the whole or part of the metal loop moves in the magnetic field. The electromotive force generated in this way is called motional electromotive force. The second is when no part of the metal loop moves, and the space magnetic field changes. The electromotive force generated in this way is called induced electromotive force.

In Figure 1.1 above, the electromotive force generated by coil C is induced electromotive force. Below is an example to further analyze and illustrate the motional electromotive force.

As shown in Figure 1.2, it is cited from physics textbook [4]. In the uniform magnetic field B, place a metal wireframe ABCD. The wireframe has two parts. The fixed U-shaped part of the wireframe is composed of metal wires AD, AB and BC. The metal wire CD can slide left and right, and its length is L_{CD}.

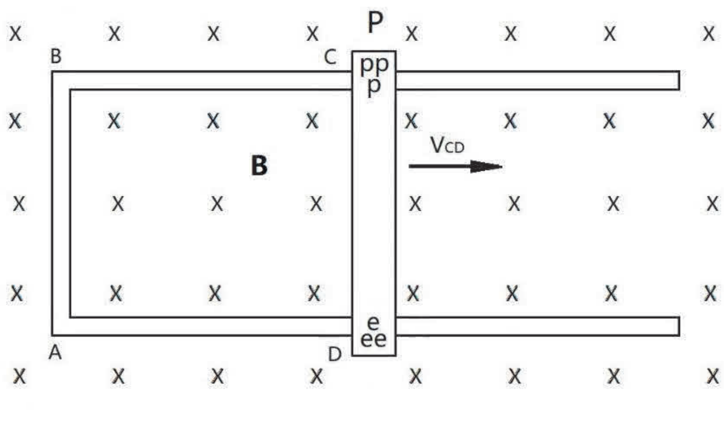

Figure1.2 Motional electromotive force

When the metal wire CD is not in contact with the fixed U-shaped part of the wireframe, that is, the circuit is open, the metal wire CD moves to the right at the speed of V_{CD}, and the magnetic fluxes increased in the wireframe ABCD within dt:

$$d\Phi = \mathbf{V}_{CD} L_{CD} B\, dt \qquad (1\text{-}2)$$

Then the change rate of the magnetic fluxes passing through the wireframe ABCD:

$$d\Phi/dt = \mathbf{V}_{CD} B L_{CD} \qquad (1\text{-}3)$$

According to equations (1-1) and (1-3), the electromotive force of metal wire CD:

$$\varepsilon_{CDO} = \mathbf{V}_{CD} B L_{CD} \qquad (1\text{-}4)$$

According to equation (1- 4), the electromotive force ε_{CDO} is proportional to the speed V_{CD} of the metal wire CD, that is, the change rate of the magnetic fluxes.

The metal wire CD is equivalent to a battery. The C end is positive and the D end is negative. A chemical battery is essentially the result of the counter-potential movement of electric charges under the action of the electrochemical forces; Similarly, the electromotive force of the metal wire CD is essentially the result of the counter-potential movement of electric charges under the action of the Lorentz magnetic field forces.

The microscopic physical essence of motional electromotive force is that the charges within the wire move relative to a magnetic field, and the charges are affected by Lorentz magnetic field force. Based on Lorentz magnetic field force, we will derive the motional electromotive force of the metal wire CD below.

When the metal wire CD is not in contact with the fixed U-shaped part of the wireframe and moves right at the speed of V_{CD}, the free electron e in the metal wire CD moves along the wire from C end to D end under the action of the Lorentz magnetic field force, so that the negative electrons e accumulate at the D end, and the positive charges p accumulate at the C end, thus generating an electromotive force ε_{CDO} from C end to D end. The charges on the metal wire CD are affected by both the Lorentz magnetic field force and the electric field force. When the Lorentz magnetic field force and the electric field force are balanced, the accumulation of charges stops, as shown in Figure 1.3.

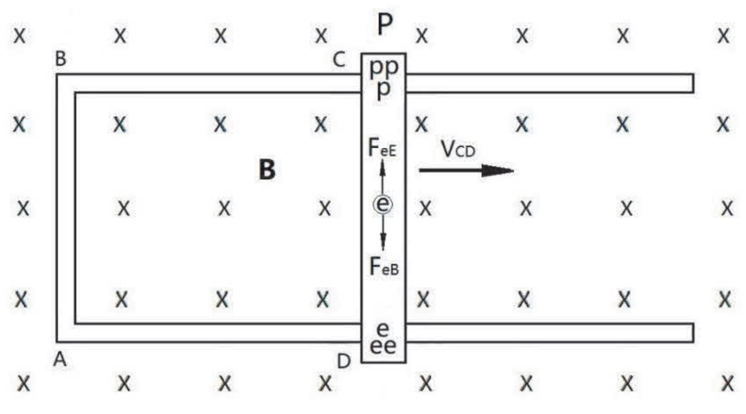

Figure 1.3 Force analysis of electrons in metal wire CD

Let the motional electromotive force between the two ends of wire CD be ε_{CDO}, and the electric field intensity：

$$E_{CD} = \varepsilon_{CDO}/L_{CD}$$

The electric field force on a free electron e:

$$F_{eE} = e\ E_{CD}$$

$$F_{eE} = e\ \varepsilon_{CDO}/L_{CD} \qquad (1\text{-}5)$$

The electron e within wire CD moves together at V_{CD} relative to magnetic field B. According to Lorentz’s magnetic field force theorem, the Lorentz magnetic field force on the free electron e:

$$F_{eB} = e\ V_{CD}\ B \qquad (1\text{-}6)$$

The electric field force F_{eE} and the Lorentz magnetic field force F_{eB} are equal in magnitude and opposite in direction. From Formulars (1-5) and (1-6), it can be obtained:

$$e\ \varepsilon_{CDO}/L_{CD} = e\ V_{CD}\ B$$

Then the motional electromotive force of metal wire CD:

$$\varepsilon_{CDO} = B\ L_{CD}\ V_{CD} \qquad (1\text{-}7)$$

Formulars (1-7) and (1-4) are the same. Formular (1-7) is derived based on Lorentz’s magnetic field force theorem, and Formular (1-4) is derived based on Faraday's law of electromagnetic induction. Lorentz’s magnetic field force theorem is the microscopic physical nature of electromotive force, while Faraday's law of electromagnetic induction is the macroscopic physical manifestation of electromotive force.

2. Further study on Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem

The above equations (1-7) and (1-4) are the same, and the wire CD moves at a constant speed V_{CD} relative to the magnetic field B. If the speed V_{CD} is variable, is the motional electromotive force of the wire CD derived from Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem still the same.

Before derivation, the electron motion resistance force theorem is first introduced. Under the action of the electric field force F_{eE}, the electrons moving in the metal wire will be affected by the electron motion resistance force F_{eR}. As shown in Figure 2.1, the circuit has a constant voltage power supply, the electrons move at a uniform speed V_e in the wire, and the electric field force F_{eE} and the motion resistance force F_{eR} are balanced, and they are equal in magnitude and opposite in direction.

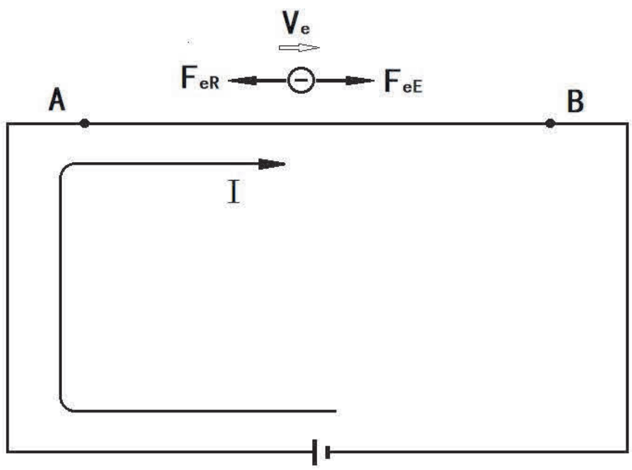

Figure 2.1 Force analysis of an electron flowing through a wire

Let the voltage between two ends of wire AB be U_{AB}, then the electric field force on an electron:

$F_{eE} = e\ U_{AB}/L_{AB}$

$F_{eE} = e\ E_{AB}$ (2-1)

Let the length of metal wire AB be L_{AB}, the cross-sectional area be S, and the resistivity be ρ, the resistance R_{AB} of wire AB:

$R_{AB} = L_{AB}\ \rho/S$

According to Ohm's law, the current passing through wire AB:

$I_{AB} = U_{AB}/R_{AB}$

$I_{AB} = U_{AB}\ S\ /\ (L_{AB}\ \rho)$ (2-2)

Let the number of free electrons per unit volume of the above metal wire be n_e, and the speed of electrons in the wire be V_e, the amount of electricity flowing through the section of the metal wire within 1 second, that is, the current passing through wire AB:

$I_{AB} = e\ n_e\ S\ V_e$ (2-3)

From Formulars (2-2) and (2-3), it can be obtained:

$U_{AB}\ S\ /\ (L_{AB}\ \rho) = e\ n_e\ S\ V_e$

$U_{AB}\ /\ L_{AB} = V_e\ (e\ n_e\ \rho)$

Then the electric field intensity of wire AB:

$E_{AB} = (e\ n_e\ \rho)\ V_e$

The electrons move at a constant speed V_e in the wire, and the electric field force F_{eE} and the motion resistance force F_{eR} are balanced, and the two are equal in magnitude and opposite in direction. Then the electron motion resistance force:

$F_{eR} = F_{eE}$

$= e\ E_{AB}$

$= (e^2\ n_e\ \rho)\ V_e$

Let $k_{ev} = e^2 n_e \rho$ be the coefficient of resistance of electron motion, then the electron motion resistance force:

$$F_{eR} = - k_{ev} V_e \quad (2\text{-}4)$$

The above equation indicates that electrons moving in the metal wire, under the action of the motion resistance force F_{eR}, and the magnitude of the electron motion resistance force is proportional to the speed of the electrons, and the direction is opposite. Equation (2-4) is called electron motion resistance force theorem.

The coefficient of resistance of electron motion k_{ev} is related to wire materials. For copper wires, the number of free electrons per unit volume n_e is $8.5 \times 10^{28} m^{-3}$, the charge amount of an electron e is $1.6 \times 10^{-19} C$, and the resistivity ρ is $1.75 \times 10^{-8} \Omega m$, then the coefficient of resistance of electron motion of the copper wire: $k_{ev} = e^2 n_e \rho = 3.81 \times 10^{-17}\ \Omega C^2 m^{-2}$。

The electron motion resistance force theorem reveals that an electron moving in the metal wire will be affected by the motion resistance force, and the electron motion resistance force is proportional to the speed of the electron. The electrons moving in the metal wire will be affected by the motion resistance force, which heats up the metal wire. In Figure 2.1, the driving force of electrons is the electric field force F_{eE}, the electric field force F_{eE} and the motion resistance force F_{eR} balanced, both of them are equal in magnitude and opposite in direction. In addition to the electric field force, the driving force of electrons can also be Lorentz magnetic field force, electrochemical driving force, etc.

Below, based on Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem, the electromotive force of the metal wire CD at variable speed will be derived, respectively.

Without losing the generality, the wire CD moves back and forth sinusoidally with O as the central zero point along the x-axis, and X_M is the maximum distance of the left and right movement of the wire CD, as shown in Figure 2.2.

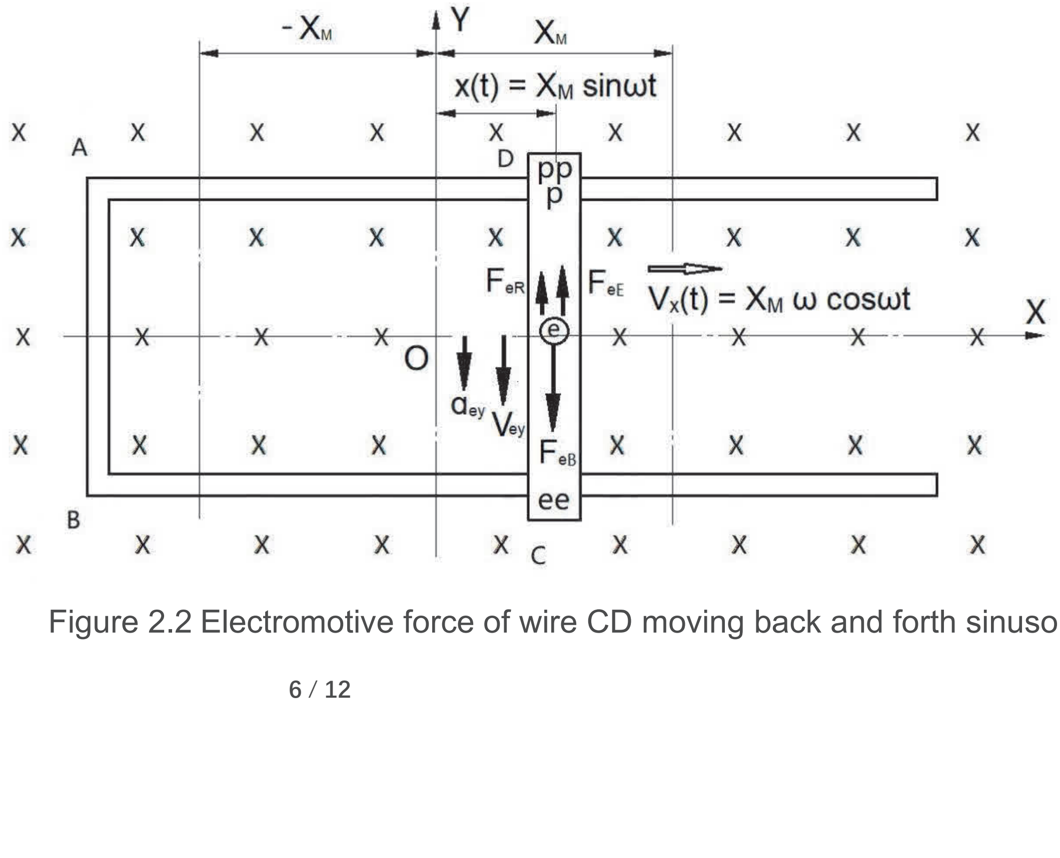

Figure 2.2 Electromotive force of wire CD moving back and forth sinusoidally

At time t, the position of the wire CD on the x-axis:

$$x(t) = X_M \sin\omega t \qquad (2\text{-}5)$$

where ω=2πf is the angular frequency, f is the frequency, and the unit is Hz. Then the speed of the wire CD in the x direction at time t:

$$V_x(t) = X_M\ \omega \cos\omega t \qquad (2\text{-}6)$$

Below, at first the electromotive force of the wire CD will be derived according to Faraday's law of electromagnetic induction. At time t, the wire CD is at the position of x(t), and its speed is $V_x(t)$, then the magnetic fluxes increased by the wire frame ABCD within dt time

$$d\Phi = V_x(t)\ L_{CD}\ B\ dt$$
$$= L_{CD}\ B\ dt\ X_M\ \omega \cos\omega t$$

Then the change rate of the magnetic fluxes:

$$d\Phi/dt= L_{CD}\ B\ X_M\ \omega \cos\omega t \qquad (2\text{-}7)$$

According to Faraday's law of electromagnetic induction, at time t and the position of x(t), the electromotive force of the wire CD is obtained from equations (2-7):

$$\varepsilon_{CDO}(t)= L_{CD}\ B\ X_M\ \omega \cos\omega t \qquad (2\text{-}8)$$

Equation (2-8) is derived from Faraday's laws of electromagnetic induction.

Below, the electromotive force of the wire CD will be derived based on Lorentz's magnetic field force theorem.

The metal wire CD moves back and forth sinusoidally, and the electrons in the wire CD are no longer stationary, so that the electrons within the wire CD move along the y axis at a speed of $V_{ey}(t)$ and an acceleration of $\alpha_{ey}(t)$, as shown in Figure 2.2. Below we will derive the electromotive force of the metal wire CD based on the force analysis of electrons within the wire CD.

At time t, the metal wire CD is at the x (t) position, let the motional electromotive force of the metal wire CD be ε_{CDO}, then the electric field force on an electron:

$$F_{eE}(t)=e\ \varepsilon_{CDO}(t)\ /L_{CD} \qquad (2\text{-}9)$$

The Lorentz magnetic field force on an electron:

$$F_{eB}(t) = e\ B\ X_M\ \omega \cos\omega t \qquad (2\text{-}10)$$

Driven by the Lorentz magnetic field force, let the speed of electrons along y axis within the wire CD be $V_{ey}(t)$, then the electron motion resistance force:

$$F_{eR}(t) = k_{ev}\ V_{ey}(t) \qquad (2\text{-}11)$$

Driven by the Lorentz magnetic field force, let the acceleration of electrons along y axis within the wire CD be $\alpha_{ey}(t)$, and the mass of an electron be m_e, then the electron acceleration driving force:

$$F_{e\alpha}(t) = m_e\, \alpha_{ey}(t) \qquad (2\text{-}12)$$

According to the force analysis of the electron in Figure 2.2, it can be obtained:

$$F_{e\alpha}(t) = F_{eB}(t) - F_{eE}(t) - F_{eR}(t)$$

$$m_e\, \alpha_{ey}(t) = e\, B\, X_M\, \omega\, \cos\omega t - e\, \varepsilon_{CDO}(t) / L_{CD} - k_{ev}\, V_e(t)$$

At time t and the position of x (t), the electromotive force of the wire CD:

$$\varepsilon_{CDO}(t) = L_{CD}\, B\, X_M\, \omega\, \cos\omega t - (k_{ev}\, L_{CD}/e)\, V_{ey}(t) - (m_e\, L_{CD}/e)\, \alpha_{ey}(t) \qquad (2\text{-}13)$$

Equations (2-5) and (2-13) are derived based on Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem, respectively.

There is only one item of $L_{CD}BX_M\omega\cos\omega t$ on the right side of equation (2-5); The first item on the right side of equation (2-13) is the same as the $L_{CD}BX_M\omega\cos\omega t$ in equation (2-5), and equation (2-13) has two additional items $(k_{ev}\, L_{CD}/e)\, V_{ey}(t)$ and $(m_e\, L_{CD}/e)\, \alpha_{ey}(t)$, which are related to the speed $V_{ey}(t)$ and the acceleration $\alpha_{ey}(t)$ of electrons. Therefore, when the speed of wire CD changes relative to the magnetic field, the electromotive force of the wire CD is different derived from Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem. Lorentz's magnetic field force theorem, in the high-energy electron cyclotrons, its theoretical and experimental observations are consistent within a fairly high degree of accuracy, even when the electron speed is close to the speed of light, reaching 0.99C, after considering the relativistic effect, its theoretical and experimental observations are still consistent. Lorentz's magnetic force theorem is a universal fundamental electromagnetic theorem. Therefore, Faraday's law of electromagnetic induction is an engineering approximation formula.

In equations (2-5) and (2-13), the item of $L_{CD}BX_M\omega\cos\omega t$ can be calculated directly by mathematical formula. However, in equation (2-13), the two additional items $(k_{ev}\, L_{CD}/e)V_{ey}(t)$ and $(m_e\, L_{CD}/e)\alpha_{ey}(t)$ are difficult to calculate directly by mathematical formulas, and it is necessary to perform the force analysis on each electron with Lorentz's magnetic field force and electric field force to obtain the position, speed $V_{ey}(t)$ and acceleration $\alpha_{ey}(t)$ of each free electron within the wire CD. For the determined wire material and structure, with the help of computer numerical processing, the calculation results that meet the engineering application can be obtained.

In equations (2-5) and (2-13), the radiation of electric and magnetic fields is not considered, and for general transformers and motors, their operating frequencies are below several hundred hertz, and the radiated energy of electric and magnetic fields can be negligible. When the angular frequency ω is high enough, a considerable part of the energy in the circuit becomes the radiated energy of the electric and magnetic fields. So, Faraday's law of electromagnetic induction and the formula (2-15) derived from Lorentz's magnetic field force theorem both are engineering approximation formulas for calculating electromotive force.

3. An extension of Lorentz's magnetic field force theorem

Lorentz's magnetic field force theorem reveals that an electric charge moving in a uniform magnetic field is affected by the Lorentz magnetic field force, which is the microscopic physical essence of the electromotive force generated in metal wires and coils. For a charge q moving at speed **v** in a magnetic field with magnetic induction intensity **B**, the charge q is affected by the Lorentz magnetic field force:

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} \qquad (3\text{-}1)$$

The magnetic force $\mathbf{F}_B$, charge velocity **v**, and magnetic induction intensity **B** obey the right-handed spiral rule.

As shown in Figure 3.1, a ball with a charge of q and a mass of m, is suspended with a quartz filament from a beam.

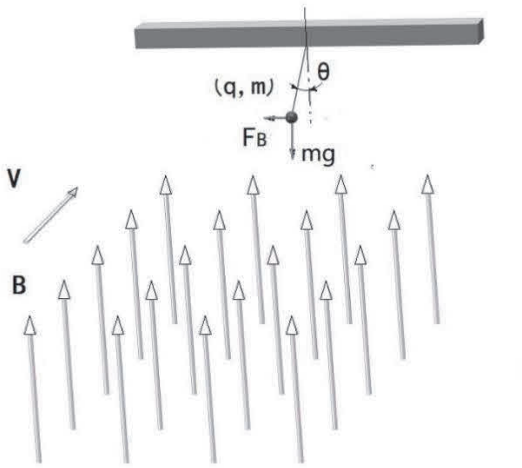

Figure 3.1 Lorentz magnetic field force of a charged moving ball in a uniform magnetic field

Let a uniform magnetic field with a magnetic induction intensity B move at V relative to a charged ball, the Lorentz magnetic field force affected on the charged ball:

$$F_B = q\, v\, B$$

The charged ball is simultaneously affected by the gravity mg and tensile force of quartz filament, and the Lorentz magnetic field force is obtained from the force analysis of the charged ball in Figure 3.1:

$$F_B = mg\, \mathrm{tg}\theta$$

A charge at rest in a magnetic field wave is also affected by the Lorentz magnetic field force, which is the microscopic physical nature of the induced electromotive force in metal wires and coils. For a sinusoidal magnetic field wave with an angular frequency ω and a maximum magnetic induction intensity B_{max}. Based on preliminary research, it can be concluded that: the Lorentz magnetic field force on the charge:

$$F_B(t) = k_B\, q\, \omega\, B_{max} \sin(\omega t) \qquad (3\text{-}2)$$

In the above equation, $F_B(t)$ is also a sinusoidal function, and k_B is the proportional coefficient of magnetic field force, which requires further theoretical and experimental studies. When sin(ωt) = 1, the sinusoidal alternating Lorentz magnetic field force $F_B(t)$ of equation (3-2) is the maximum:

$$F_{Bmax} = k_B\, q\, \omega\, B_{max} \qquad (3\text{-}3)$$

As shown in Figure 3.2, a ball with a charge of q and a mass of m, is suspended with a quartz filament from a beam.

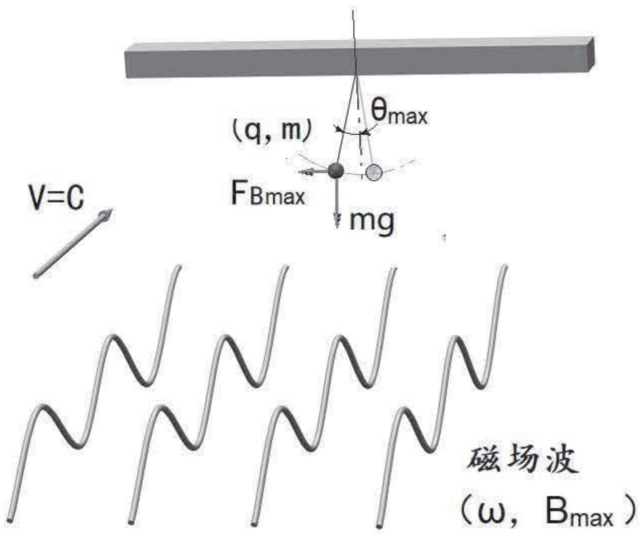

Figure 3.2 Lorentz magnetic field force of a charged stationary ball in a sinusoidal magnetic field wave

The sinusoidal magnetic field wave with an angular frequency of ω=2πf, and the maximum magnetic induction intensity of B_{max} passes through the charged ball at the speed of light, and the charged ball oscillates left and right at a frequency of f under the action of the alternating Lorentz magnetic field force, Let the maximum swing angle be θ_{max} and the corresponding maximum Lorentz magnetic field force be F_{Bmax}. The charged ball is simultaneously affected by the gravity mg and tensile force of quartz filament. When the swing angle is θ_{max}, the maximum Lorentz magnetic field force is obtained from the force analysis of the charged ball in Figure 3.2:

$$F_{Bmax} = mg\, tg\theta_{max} \qquad (3\text{-}4)$$

From equations (3-3) and (3-4) , it can be concluded:

$$k_B\, q\, \omega\, B_{max} = mg\, tg\theta_{max}$$

Then the proportional coefficient of magnetic field force:

$$k_B = mg\, tg\theta_{max} /(q\, \omega\, B_{max}) \qquad (3\text{-}5)$$

Equation (3-5) provides an experimental method for calibrating the proportional coefficient of magnetic field force.

4. Conclusion

Faraday's law of electromagnetic induction reveals that the induction electromotive force generated in a metal coil is proportional to the change rate of the magnetic fluxes passing through the coil. Lorentz's magnetic field force theorem reveals that an electric charge moving in a magnetic field is affected by the Lorentz magnetic field force. Lorentz's magnetic field force theorem is the microscopic physical nature of induction electromotive force, and Faraday's law of electromagnetic induction is the macroscopic physical manifestation of induction electromotive force.

A metal wire moving in a magnetic field will generate an induction electromotive force at its two ends of the wire. In this study, the calculation formulas of electromotive force of metal wires were derived based on Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem, respectively. When a metal wire moves at a uniform speed in a magnetic field, the results derived from both of them are the same. When a metal wire moves back and forth sinusoidally in a magnetic field, the electromotive force of the wire, which derived from Faraday's law of electromagnetic induction and Lorentz's magnetic field force theorem, is different. Lorentz 's magnetic force theorem is a universal fundamental electromagnetic theorem, so Faraday's law of electromagnetic induction is an engineering approximation formula.

This study proposes the electron motion resistance force theorem: An electron moves in the metal conductor, it will be affected by the motion resistance force, and the electron motion resistance force is proportional to the speed of the electron. Electrons moving in a metal wire are affected by the motion resistance force, which heats up the metal wire.

An electric charge moving in a uniform magnetic field is affected by the Lorentz magnetic field force, which is the microscopic physical nature of the motional electromotive force generated in metal wires and loops. An electric charge at rest in a magnetic field wave is also affected by the Lorentz magnetic field force, which is the microscopic physical nature of the induced electromotive force in metal wires and loops. The electromotive force in metal wires and loops is essentially the result of the counter-potential movement of electric charges under the action of the Lorentz magnetic field force. Faraday's law of electromagnetic induction and the formula for calculating electromotive force derived from Lorentz's magnetic field force theorem are both engineering approximation formulas.

Faraday's law of electromagnetic induction is one of the 4 equations of Maxwell's equations. This paper reveals that Faraday's law of electromagnetic induction is an engineering approximation formula, which is a great challenge for Maxwell's equations and the fundamental electromagnetic theorems. It will have a profound impact on scientific discovery and technological progress.

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