Cosmological constant

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1 Strongly coupled hidden sector

Assume that our universe has a strongly coupled hidden sector on top of the visible (standard model) sector. They may be weakly coupled. The theory is defined in an evolving universe with a metric g_{ab},

$$S = \frac{1}{16\pi G_4}\int d^4x\sqrt{-g}\left[S_{vis} + S_{hid} + S_{int}\right]. \tag{1.1}$$

Furthermore, we assume that the cosmological constant of the boundary has a zero cosmological constant!

We will assume that the hidden sector has a holographic dual and that the four-dimensional hyper-surface is embedded into a five dimensional bulk space time $G_{\mu\nu}$ with a metric

$$ds^2 = -e^{2\lambda(r)}dt^2 + e^{2\nu(r)}dr^2 + \left(\frac{r}{L}\right)^\alpha d\vec{x}^2. \tag{1.2}$$

To find the embedding $t(r)$, we define the four un-normalised tangent vectors

$$R^\mu = \left(\frac{\partial t}{\partial r}, 0, 0, 0, 1\right), \qquad X^\mu = (0, 1, 0, 0, 0)\,, \tag{1.3}$$

$$Y^\mu = (0, 0, 1, 0, 0)\,, \qquad Z^\mu = (0, 0, 0, 1, 0)\,. \tag{1.4}$$

The normalised normal vector to the hyper-surface is

$$n_\mu = \pm\sqrt{\frac{e^{2\lambda}}{e^{2(\lambda-\nu)}t'^2 - 1}}\left(-1, 0, 0, 0, t'\right). \tag{1.5}$$

The induced metric can be written as

$$\gamma_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu, \tag{1.6}$$

and the extrinsic curvature as

$$K^{\mu\nu} = -\frac{1}{L}\gamma^{\mu\nu}. \tag{1.7}$$

The junction equation (1.7) allows us to solve for $\partial t/\partial r$,

$$\frac{\partial t}{\partial r} = \pm \frac{r e^{2\nu-\lambda}}{\sqrt{r^2 e^{2\nu} - \frac{1}{4}L^2\alpha^2}}. \tag{1.8}$$

The induced metric's line element is thus

$$ds^2 = -\left(\frac{L^2\alpha^2 e^{2\nu}}{4r^2 e^{2\nu} - L^2\alpha^2}\right) dr^2 + \left(\frac{r}{L}\right)^{\alpha} d\vec{x}^2. \tag{1.9}$$

The metric has no dependence on the function $\lambda(r)$.

We can now look for the time coordinate τ and the FRW scale factor $a(\tau)$ of the boundary space-time. The time coordinate is

$$\tau = \tau_0 \pm \int dr \frac{L\alpha e^{\nu}}{\sqrt{4r^2 e^{2\nu} - L^2\alpha^2}}. \tag{1.10}$$

In the late-time regime we have $r \gg 1$. Now, if $\lim_{r\to\infty} r^2 e^{2\nu} = 0$ then τ would become imaginary. If $\lim_{r\to\infty} r^2 e^{2\nu} = \mathcal{O}(1)$, as in the AdS-Schwarzschild, the integral depends on the details of the function ν. In that case

$$\nu = -\frac{1}{2}\log\left[\left(\frac{r}{L}\right)^2\left(1-\left(\frac{r_h}{r}\right)^4\right)\right] \tag{1.11}$$

If, however, $r^2 e^{2\nu} \gg \frac{1}{4}L^2\alpha^2$, we find that

$$\tau = \tau_0 + \frac{L\alpha}{2}\log r. \tag{1.12}$$

This result does not depend on the details of two functions specifying the metric.

Finally, we are able to write down the late-τ hyper-surface metric

$$ds^2 = -d\tau^2 + a(\tau)^2 d\vec{x}^2, \tag{1.13}$$

where the scale factor is

$$a(\tau) = e^{\frac{1}{2L}\tau}. \tag{1.14}$$

The Hubble time is

$$H = \frac{\dot{a}}{a} = \frac{1}{2L}, \tag{1.15}$$

which can be written in terms of the vacuum energy density, $H^2 = 8\pi G_4 \rho_V/3$. Furthermore, $\rho_V = 3/(32\pi L G_5)$ and $\Lambda = 8\pi G_4 \rho_V$, which gives us an *effective cosmological constant* on the brane, which is universal for all theories [**of some type**]. The cosmological constant is

$$\Lambda = \frac{3}{4L^2}, \tag{1.16}$$

and is provided in the dual description solely by the dynamics of the strongly coupled hidden sector. In reality $\Lambda \approx 10^{-52}m^{-2}$ or $\Lambda \sim 10^{-121}M_P^2 = 10^{-121}10^{38}GeV^2 = 10^{-83}GeV^2$ in $c = h = 1$ units. Thus $L^2 \sim 10^{83}GeV^{-2}$ and $L^4 \sim 10^{166}GeV^{-4}$. Assume $1/\ell_s = M_p$, so

$$\frac{L^4}{\ell_s^4} \sim 10^{166} \cdot 10^{76} = 10^{242} \tag{1.17}$$

or

$$L \sim 10^{26}m \tag{1.18}$$

The diameter of the sphere which is the observable universe is $8.8 \times 10^{26}m$.

In the standard AdS/CFT example with D3 branes the supergravity description is applicable in the regime

$$\frac{L^4}{\ell_s^4} \sim g_s N \sim g_{YM}^2 N = \lambda \gg 1. \tag{1.19}$$

Hence, $\Lambda \sim \frac{1}{\ell_s^2\sqrt{\lambda}}$

$$\Lambda \sim \frac{1}{L^2} \ll \frac{1}{\ell_s^2}. \tag{1.20}$$

This is not a very strong bound, but at least we know that the cosmological constant needs to be much smaller that the string scale.

Gubser states that the cut-off of the theory can be measured as the energy of a fundamental string stretched form the Planck brane to the horizon of AdS_5 when measure in time τ. This is equivalent to separating one D3 brane from the rest so that the string tension give the mass of the heaviest W boson. Now, since at large τ requires large r, by UV/IR the dual theory should be able to access the extreme UV degrees of freedom. Hence, we require

$$\Lambda_{\text{cut-off}} \sim \frac{L}{\alpha'} \sim \frac{L}{\ell_s^2}, \tag{1.21}$$

so for supergravity in the bulk to be a good approximation

$$\Lambda \sim \frac{1}{L^2} \sim \frac{1}{\ell_s^4 \Lambda_{\text{cut-off}}^2} \ll \frac{1}{\ell_s^2} \Longrightarrow 1 \ll \ell_s^2 \Lambda_{\text{cut-off}}^2 \tag{1.22}$$

Rough estimate of the string scale near the Planck scale gives

$$\Lambda \ll M_p^2, \tag{1.23}$$

which is much better than the QFT calculation giving

$$\Lambda \sim G_4 M_p^4 = \hbar c M_p^2. \tag{1.24}$$

The Ricci tensor is given by

$$R_{ab} = \frac{3}{2L^2}\,\text{diag}\left[-1, e^{\sqrt{2}t/L}, e^{\sqrt{2}t/L}, e^{\sqrt{2}t/L}\right], \tag{1.25}$$

and the Ricci scalar is given by

$$R = \frac{6}{L^2}. \tag{1.26}$$

The Einstein's equation without higher-curvature terms should be satisfied in the late universe with small curvatures. If we use the holographic stress-energy tensor expression, from which the embedding equation was derived,

$$R^{\mu\nu} - \frac{1}{2}R\gamma^{\mu\nu} + \mathcal{O}\left(R^2\right) = \frac{2}{L}8\pi G_5 T^{\mu\nu}_{vis} + \frac{2}{L}8\pi G_5 T^{\mu\nu}_{hid}, \tag{1.27}$$

should contain higher-curvature terms. However, in order for each term to be sub-leading, we require

$$|R| \gg \left|R^2\right| \Longrightarrow 1 \gg \frac{1}{L^2} \Longrightarrow L^2 \gg 1. \tag{1.28}$$

The validity of the Einstein-Hilbert action on the boundary thus imposes a stronger bound on the effective cosmological constant

$$\Lambda \sim \frac{1}{L^2} \ll 1 \tag{1.29}$$

2 Cosmological laboratory

Given the bulk metric

$$ds^2 = -e^{2\lambda(r)}dt^2 + e^{2\nu(r)}dr^2 + \left(\frac{r}{L}\right)^{2\beta} d\vec{x}^2, \tag{2.1}$$

the induced metric is

$$ds^2 = -\left(\frac{\beta^2 L^2 e^{2\nu}}{r^2 e^{2\nu} - \beta^2 L^2}\right) dr^2 + \left(\frac{r}{L}\right)^{2\beta} d\vec{x}^2. \tag{2.2}$$

Hence the boundary time is

$$\tau = \tau_0 \pm \int dr \frac{\beta L e^{\nu}}{\sqrt{r^2 e^{2\nu} - \beta^2 L^2}}, \tag{2.3}$$

and the scale factor

$$a(\tau) = \left(\frac{r(\tau)}{L}\right)^{\beta}. \tag{2.4}$$

Our goal is to reconstruct the bulk metric from our choice of $a(\tau)$. We assume that the foliation function $r(\tau)$ is invertible, so that

$$\frac{d\tau(r)}{dr} = \frac{L\beta e^{\nu(r)}}{\sqrt{r^2 e^{2\nu(r)} - \beta^2 L^2}} \Longrightarrow \frac{dr(\tau)}{d\tau} = \frac{\sqrt{r(\tau)^2 e^{2\nu(\tau)} - \beta^2 L^2}}{L\beta e^{\nu(\tau)}}, \tag{2.5}$$

and hence

$$\frac{d\log r(\tau)}{d\tau} = \frac{\sqrt{r(\tau)^2 e^{2\nu(\tau)} - \beta^2 L^2}}{\beta L r(\tau) e^{\nu(\tau)}}. \tag{2.6}$$

We know that

$$r(\tau) = La(\tau)^{1/\beta}, \tag{2.7}$$

which means that we can solve for

$$e^{2\nu(\tau)} = \frac{\beta^2}{a(\tau)^{2/\beta}\left(1 - L^2\left(\frac{d\log r(\tau)}{d\tau}\right)^2\right)} = \frac{\beta^2}{a(\tau)^{2/\beta}\left[1 - \left(\frac{L}{\beta}\frac{d\log a(\tau)}{d\tau}\right)^2\right]}. \tag{2.8}$$

Hence, we know both $r(\tau)$ and $\nu(\tau)$, which are required for the metric, assuming we are able to invert $r(\tau)$ to find $\tau(r)$. The function $\lambda(r)$ is left undetermined. The bulk metric can be written in terms of new coordinates $(t, \tau, \vec{x})$,

$$ds^2 = -e^{2\lambda(\tau)}dt^2 + \frac{\beta^2 L^2 \left(\frac{d\log a(\tau)}{d\tau}\right)^2}{\beta^2 - L^2\left(\frac{d\log a(\tau)}{d\tau}\right)^2} d\tau^2 + a(\tau)^2 d\vec{x}^2. \tag{2.9}$$

or

$$ds^2 = -e^{2\lambda(\tau)}dt^2 + \frac{\beta^2 L^2 \dot{a}(\tau)^2}{\beta^2 a(\tau)^2 - L^2 \dot{a}(\tau)^2} d\tau^2 + a(\tau)^2 d\vec{x}^2. \tag{2.10}$$

In terms of the Hubble parameter,

$$H \equiv \frac{\dot{a}}{a}, \tag{2.11}$$

the metric is

$$ds^2 = -e^{2\lambda(\tau)}dt^2 + \frac{\beta^2 L^2 H(\tau)^2}{\beta^2 - L^2 H(\tau)^2} d\tau^2 + a(\tau)^2 d\vec{x}^2. \tag{2.12}$$

We can think of λ as parametrising a family of different metrics, which all give FRW on the boundary. We should set $\beta = 1$, most likely without the loss of generality, so that the metric is

$$ds^2 = -e^{2\lambda(\tau)}dt^2 + \frac{L^2 H(\tau)^2}{1 - L^2 H(\tau)^2} d\tau^2 + a(\tau)^2 d\vec{x}^2. \tag{2.13}$$

To simplify the metric, we could make the standard choice of $\lambda = -\nu$,

$$ds^2 = -\frac{a(\tau)^{2/\beta}\left(\beta^2 - L^2 H(\tau)^2\right)}{\beta^4} dt^2 + \frac{\beta^2 L^2 H(\tau)^2}{\beta^2 - L^2 H(\tau)^2} d\tau^2 + a(\tau)^2 d\vec{x}^2. \tag{2.14}$$

The simplest choice we can make is to take $\beta = 1$, and $e^{2\lambda} = (r/L)^2$. The bulk metric is then

$$ds^2 = a(\tau)^2\left(-dt^2 + d\vec{x}^2\right) + \frac{L^2 H(\tau)^2}{1 - L^2 H(\tau)^2} d\tau^2. \tag{2.15}$$