

On the equation $X + (X/X) = X$

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April 18, 2023

Abstract: In this note, we shall refer to the equation $X + (X/X) = X$ from our division by zero and division by zero calculus ideas against the Barukčić's idea.

Key Words: Division by zero, division by zero calculus, $1/0 = 0/0 = z/0 = \tan \frac{\pi}{2} = 0$, equation $X + (X/X) = X$.

2010 AMS Mathematics Subject Classification: 34A24, 41A30, 41A27, 51N20, 00A05, 00A09, 42B20, 30E20.

1 Introduction

The equation $X + (X/X) = X$ ([1]) may be considered simply with our division by zero and division by zero calculus ideas.

At first, our division by zero is given by the idea of the Yamada field axiom **Y** containing division by zero with generalized fractions. In its general fractions

$$\frac{a}{b},$$

we have

$$\frac{a}{0} = 0.$$

See [5] and [9] for the details.

Meanwhile, for the function case we need the concept of division by zero calculus.

The general definition of division by zero calculus is given by the following way:

For a function $y = f(x)$ which is n order differentiable at $x = a$, we will **define** the value of the function, for $n > 0$

$$\frac{f(x)}{(x - a)^n}$$

at the point $x = a$ by the value

$$\frac{f^{(n)}(a)}{n!}.$$

For the important case of $n = 1$,

$$\left. \frac{f(x)}{x - a} \right|_{x=a} = f'(a). \quad (1.1)$$

In particular, the values of the functions $y = 1/x$ and $y = 0/x$ at the origin $x = 0$ are zero. **We write them as $1/0 = 0$ and $0/0 = 0$, respectively.** Of course, the definitions of $1/0 = 0$ and $0/0 = 0$ are not usual ones in the sense: $0 \cdot x = b$ and $x = b/0$ (however, when we consider its solution in the sense of the Moore-Penrose general inverse, our result is the same as the solution of $0 \cdot x = b$). Our division by zero is given in this sense and is not given by the usual sense as in stated in [7, 9, 10, 11] with many applications.

In particular, note that for $a > 0$

$$\left[\frac{a^n}{n} \right]_{n=0} = \log a$$

and

$$\tan \frac{\pi}{2} = 0.$$

2 Conclusion

For the equation $X + (X/X) = X$, if $X \neq 0$, then we have

$$\frac{1}{X} = 0$$

and so $X(\neq 0)$ is not the solution.

Meanwhile, for the case $X = 0$, we have two interpretations for the expression (X/X) .

When we consider X/X as a function of X , by our division by zero calculus, we have

$$\frac{X}{X}(X = 0) = 1.$$

Therefore, with this interpretation, there is no solution for the equation.

Meanwhile, for $X = 0$, if we consider for X/X

$$\frac{0}{0},$$

then, by the division by zero, we have

$$\frac{0}{0} = 0.$$

Therefore, with this interpretation, the equation has the unique solution $X = 0$.

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