Delayed choice quantum erasure: the path information and Complementarity

Vijay Shankar A, Jyotirmayee Satapathy, Sitaram V, Rakhesh Vamadevan

Department of Physics, Amrita Vishwa Vidyapeetham, Amritapuri, India E-mail: jyotirmayees@am.amrita.edu

Photon wave functions collapse into particles only after one discerns their path, as was observed in certain experiments. However, scientists like Wheeler, and Scully contemplated that this causality and the uncertainty principle could be violated through quantum erasure. Complementarity and availability of path information are sufficient to explain quantum mechanics. Scientists widely debate this claim; in the process, they often try to reinterpret the tenets of quantum mechanics. Qureshi employs microscopic-macroscopic entanglement to save causality. Qureshi also posits that the experimenter's active choice of Hilbert space basis determines wave or particle nature. Qureshi insists that when the experimenter measures the photon in the x-basis, it entangles with his novel qubit which-way detector and the two detectors that constitute the screen and show interference. If the basis choice is z, then the interference is destroyed. In this paper, we peruse the shortcomings of Qureshi's analysis. The distinction between evolution and measurement is not acknowledged. The entanglement of a photon with experimental apparatus smears quantum-classical distinction. Qureshi forgets that the screen in quantum experiments can be a single entity. The quantum qubit which-way detector he contemplates will likely function classically. In assigning measurement basis to the photon, Qureshi forgets that the phase change due to path difference in the experimental setup does not influence quantum measurement. Thus, it does not contribute to a distinct quantum state. Scientists have studied wave-particle duality using entangled photons: entanglement alone cannot destroy the interference. The photon can choose its wave-particle option randomly; the experimenter's role is thus inactive. Wave nature formulation is not derived from Hilbert space basis, and the mathematical formulation in quantum mechanics is meant to predict probabilities in the particle nature; it does not say anything about the photon's physical realization. We observe that Complementarity ensures that causality violations are possible.

Quanta ; :

1 Introduction

In 1978, Wheeler proposed a radical thought experiment: photons from a distant quasar circumvent a gravitational lensing galaxy; when observed with an interferometer, that is, when the individual path information of the photon is lost, one shall see interference (49). However, if the two-photon paths are discerned, one does not see an interference pattern (49). One thus has the power to influence the source billions of years in the past to emit light along both paths or only one path: the past can be changed! (49).

Scully proposed a more laboratory-friendly version of Wheeler's thought experiment and called it the 'delayed-choice quantum eraser thought experiment' (39). Kim Et al. demonstrated this experimentally using Young's double slit and entangled pairs of photons, claiming to have fully realized Scully's thought experiment for the first time (24). One of the pairs controlled the which-way information availability, and the other was directed to the screen showing wave or particle nature (24). However, many similar experiments claim to espouse causality-violations (2; 3; 27; 28; 32; 33; 38).

Wigner posited that consciousness causes the collapse of the quantum wave to its complementary particle form (51). Modern scientists strongly espouse realism (8; 26): scientific results are 'mind-independent'; but some say quantum mechanics may imply otherwise (40) (13) (1) (21; 22) (29). The paradox of 'delayed choice' is also a part of this debate, key to understanding quantum phenomena.

If one can change the past, one can kill one's grandfather; the famous Grandfather paradox has intrigued physicists and science fiction enthusiasts alike (42). The Big Bang to create the universe and its subsequent expansion of the universe requires time to follow asymmetry: forward arrow (12). Scientists often have to look beyond the Copenhagen interpretation to salvage the principle of causality: the Relative States (31), consistent histories (19), proper and improper mixed states (9; 25; 43), and Bohmian mechanics (14). Tabish Qureshi attempts to denounce causality violation using macroscopic-microscopic entanglement and attributing physical realization ability to Hilbert space formalism: emphasizes the experimenter's role in choosing the photon nature, thereby avoiding causality violations (34). He argues that his analysis is per Bohr's Complementarity principle (34). This paper aims to elucidate the problems in his claim and points out the incompatibility with specific empirical observations and mathematical underpinnings.

We devise a thought experiment using Young's double-slit experiment with a beam splitter to set the tone: the way Scully and Kim see it (24; 39). Then, we explain the mathematical structure used in Qureshi's analysis (34): the spin analogy. We explain the key ideas behind Qureshi's analysis of the delayed choice quantum eraser experiment (34). Before we elucidate the problems in his analysis, we explain the working of the Mach-Zehnder interferometer and how it demonstrates causality violations. We then discuss the correct way of looking at the delayed choice quantum eraser experiment to contrast with Qureshi's analysis. In conclusion, we attempt to identify the severest issues in Qureshi's analysis. We must discuss many elementary aspects of quantum mechanics to assist our work.

2 Simple picture: Young's double slit

It is well known that Young's double slit is clear and historically significant in explaining quantum mechanics (15). We use the same concept with a beam splitter and two detector screen arrangement similar to a Mach Zehnder interferometer. This will help compare the standard Complementarity principle used by Scully to explain his delayed choice quantum eraser thought experiment (39) with Qureshi's analysis (34).

Alice has Young's double slit arrangement. She has a which-way detector placed in front of each slit. There is a screen behind the double slit; it has two sensors, a pair of mirrors, and a beam splitter. Alice places two which-way detectors in front of the slits; when the photon passes through either, the photon will light either bulb A or B. The source generates a single photon. The which-way detectors are turned off; the photon does not reveal its path, interfering with itself (as Paul Dirac would say (10)). This model only serves conceptual understanding and not experimental constancy.

$$\psi = \frac{1}{\sqrt{2}}\psi_A + \frac{1}{\sqrt{2}}\psi_B \qquad (1)$$

Equation 1 is a superposition of wave functions corresponding to position A and B with Eigenvalues x_A and x_B. The arrangement in figures 1 and 4, for the overall setup and the beam splitter admit phases of zero and π so that one can write equation 1 as in equation 2 to include a relative phase factor ϕ.

$$\psi = \frac{1}{\sqrt{2}}e^{i\phi}\psi_A + \frac{1}{\sqrt{2}}\psi_B \qquad (2)$$

If the which-way detector is switched off, there is no measurement. The superposition evolves to cause interference.

$$\langle\psi(t)|\psi(t)\rangle = \frac{1}{2}(|\psi_A|^2 + |\psi_B|^2) + \psi_A\psi_B cos(\omega t) \qquad (3)$$

Equation 3's last term represents wave-nature of the superposition in equation 1, where $\omega = \frac{x_B - x_A}{\hbar}$. Similarly, equation 4 gives the wave nature for equation 2. Alice places two reflectors behind the slits to direct the photons to the beam splitter. The photon reaches the beam splitter at phase zero. The beam splitter, according to figure 4, can induce a phase change of ϕ as in equation 4 and directs constructive interference to sensor D1, as shown in figure

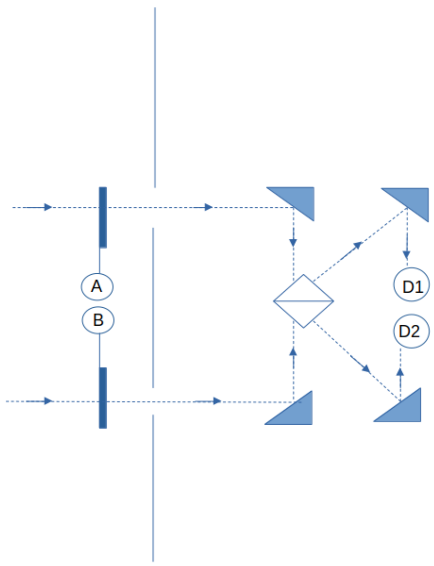

Figure 1: Young's double slit arrangement: Dashed lines indicate possible paths of the photon. The circles enclosing A and B indicate the which-way detectors. The triangles are the reflectors. The Beam splitter is the square between the two reflectors. D1 and D2 are the two sensors that constitute the screen.

.

1, the destructive counterpart to sensor D2; clicks are noted in detector D1 alone.

$$|e^{i\phi}\psi_A(t) + \psi_B(t)|^2 = \frac{1}{2}(|\psi_A|^2 + |\psi_B|^2) + \psi_A\psi_B cos(\omega t + \phi) \quad (4)$$

Measurement takes place when the which-way detector is turned on; the position projection operator can be given by $P_x = |\psi_A\rangle\langle\psi_A| + |\psi_B\rangle\langle\psi_B|$. Photons will pass through either slit A or slit B with a probability given by $|\langle\psi_A|P_x|\psi\rangle|^2$ or $|\langle\psi_B|P_x|\psi\rangle|^2 = 0.5$: Alice sees clicks non-simultaneous detection on both the detectors. However, if the which-way detector is inactive only detector D1 will have clicks - because of interference. The key to this picture is the *correlation* between the photon's path information availability and wave-particle duality.

3 The Qureshi picture

Tabish Qureshi analyses the delayed choice quantum eraser using a spin analogy, where he uses the mathematical formulation of quantum spin using the well-known Pauli spin matrices (34). He later brings in the entanglement of the photon with the quantum detectors to formulate his argument (34).

3.1 Spin based picture

In this viewpoint, ψ_A and ψ_B are assigned the states of a one-half spin electron (34). Assign values for phase 0 and π in 4 to give $|+\rangle$ and $|-\rangle$ as in equations 5 and 6.

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \quad (5)$$

$$|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} - \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \quad (6)$$

The $|+\rangle$ and $|-\rangle$ are equivalent to measuring the one-half spin state along the x-axis (horizontal or left-right). The x-axis eigenstates are given as a linear combination of z-axis states (vertical) $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$; which we can name as the $|\uparrow\rangle$ and $|\downarrow\rangle$ states. When the which-way detector is turned on, the path information becomes available, and the wave function collapses into either the $|\uparrow\rangle$ or $|\downarrow\rangle$ states with a probability of 0.5.

Consider that Alice had turned the which-way detector on, and the bulb 'A' was on. Alice then turns off the which-way detector and waits for some time. She then sends the photon again. Alice switches off the which-way detector *erasing* the path information. This time she sees an interference pattern. It is easy to see that up-and-down states can be written as a linear combination of $|+\rangle$ and $|-\rangle$ states. One can assign phase values to equation 4 and obtain the wave nature in terms of $|+\rangle$ and $|-\rangle$ as in equation 7 and 8 (34). The key to this picture is that ϕ in equation 4 creates a distinct quantum state through the minus sign ($\phi = \pi$).

$$|+\rangle\langle+| = \frac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| + cos(\omega t)(|\downarrow\rangle\langle\uparrow|) \quad (7)$$

$$|-\rangle\langle-| = \frac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| - cos(\omega t)(|\downarrow\rangle\langle\uparrow|) \quad (8)$$

3.2 Classical-quantum entanglement picture

The spin picture modified by the entanglement of the photon with the detector is the methodology used to analyze the delayed choice quantum eraser experiment by Tabish Qureshi (34). The which-way detector in figure 1 takes the form of a qubit; takes $|\uparrow\rangle$ or $|\downarrow\rangle$ state when the photon takes path 'A' and 'B,' respectively (34). This is an analogous version to what Qureshi contemplated using a Mach-Zehnder interferometer and a qubit which-way detector in one of the paths, say 'A,' as in figure 3; the qubit is in a state 'up' when the photon takes path 'B'; if it takes path 'A,' it entangles with the qubit and proceeds further. The photon state is mapped onto the qubit (34).

The interference occurs when the experimenter measures the which-way detector in the plus-minus states, so the photon entangles with the detectors D1 and D2, respectively (34).

$$\psi = \frac{1}{2}(|D1\rangle(|\uparrow\rangle + \downarrow\rangle) + |D2\rangle(|\uparrow\rangle - \downarrow\rangle)) \tag{9}$$

Equation 9 shows the $|+\rangle$ entangles with the detector D1 one sees clicks in detector D1 alone while D2 has no clicks since it entangles with $|-\rangle$: caused by constructive and destructive interference, respectively. When the which-way detector is turned on, either path 'A' or path 'B' is measured with a probability of 0.5.

$$\psi = \frac{1}{2}((|D1\rangle + |D2\rangle)|\uparrow\rangle + (|D1\rangle - |D2\rangle)|\downarrow\rangle) \tag{10}$$

Equation 10 shows the up-down states entangle with superposition states of the detectors D1 and D2, the particle nature of the photon. According to Qureshi, the photon's observation in the particle form is marked by the probability of finding clicks in either detector D1 or D2. There are a few aspects here that are quite out of place :

1. Experimenter measures the qubit in the z-basis equation 10 or in the x-basis equation 9, and hence selects between wave and particle.
2. Entanglement as a reason for loss of interference.
3. Qubit as a which-way detector.
4. Equation 2's plus and minus cases ($\phi = 0$ or π) are taken as distinct quantum states, as in Section 3.1.

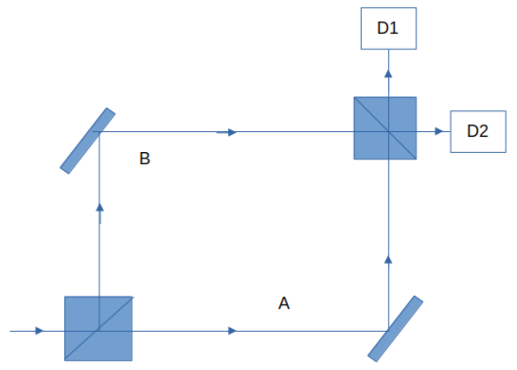

Figure 2: *The Mach-Zehnder interferometer arrangement consists of a pair of 50-50 beam splitter, mirrors, and sensors. A setup used to study photon wave-particle duality*

.

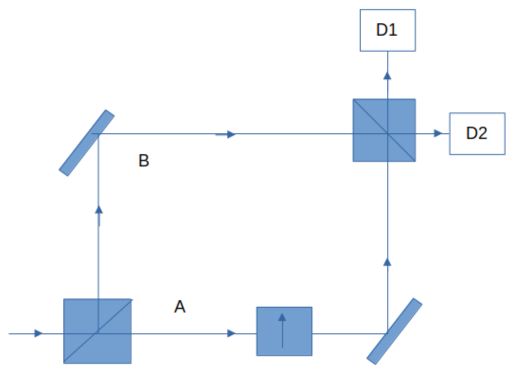

Figure 3: *Qureshi's Mach Zehnder interferometer with a qubit which-way detector in one of the paths (34)*

.

4 The Mach-Zehnder Interferometer

Let us discuss the Mach-Zehnder interferometer (figure 2) in detail to articulate the issues in Qureshi's analysis (34). The second beam splitter (near the detectors) in figure 2 is critical to our argument.

When a single photon passes through the first 50-50 beam splitter of the Mach-Zehnder interferometer, as shown in figure 2, it directs the photon wave into two parts. The beam splitter is explained in figure 4. Only the photon reflected from the glass surface undergoes a phase shift. The mirrors direct the two rays to another beam splitter, as shown in figure 2. The entire design comprising the beam splitters and mirrors is such that the photon reaching the second beam splitter from both paths arrives at the same phase. A pair of rays emerge from the last beam splitter; one pair interferes constructively and reaches detector D1, and another interferes destructively; the photon taking path 'B' in the figure 2 after reflection undergoes a phase shift of π: all other cases have phase unchanged as in figure 4. The photon interference causes clicks on D1 (constructive) and no clicks on D2 (destructive).

We emphasize the role of path information. Let us send a single photon directly through the second beam splitter alone, without the first beam splitter and the mirrors as in figure 5. The photon is either reflected to detector D1 or transmitted to detector D2 (17, chapter 2). Alternatively, one can block one of the paths as in figure 6. The two detectors are placed appropriately so that the path information is evident, and thus, the photon shows particle nature. Now, the experimenters return to the mechanism in figure 2; now, the path information is lost; one can observe interference. The correlation between wave-particle duality and path discernibility is evident.

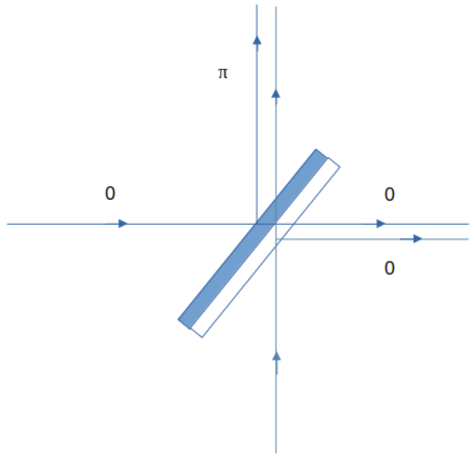

Figure 4: Beam splitter with glass coating on one side (dark). Light is either reflected or transmitted.

.

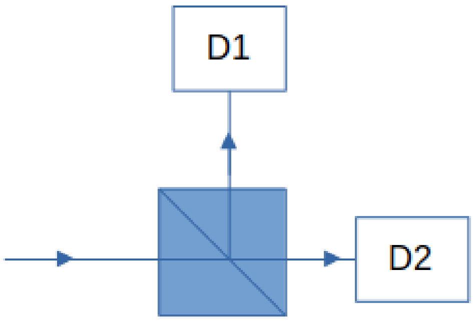

Figure 5: A sub-section of the Mach-Zehnder interferometer with a single 50-50 beam splitter. The photon path is clear in this setup.

.

5 Causality violations

Alice has set up a Mach-Zehnder interferometer in figure 2 to perform a moving beam splitter thought experiment (17, chapter 2)(50, page 182) (19). She attaches a remote-controlled motor to the second beam splitter; the beam splitter can be pushed away from the path of the single photon. The photon is sent through the first beam splitter. She suddenly removes the second beam splitter just before the photon reaches it; now the setup will be as in figure 7.

Alice notices clicks in both the detectors; the photons evince particle nature. Alice was expecting the photon to show interference because while she sent the photon, the path was indiscernible because of the second beam splitter. She should have seen clicks in only one detector but noticed the contrary. It is as if the last-minute change caused the photon nature to *change in the past*. The change in the experimental setup influenced the photon at the source to take only one of the paths (particle nature). Similarly, she starts with the second beam splitter out of the path and suddenly brings it into the path. She expects to see clicks only in both detectors. But to her shock, Alice notes clicks in detector D1 only (interference); the photon has traveled through both paths. Bringing the beam splitter back has erased the path information. This has once again caused the photon nature to change in the past.

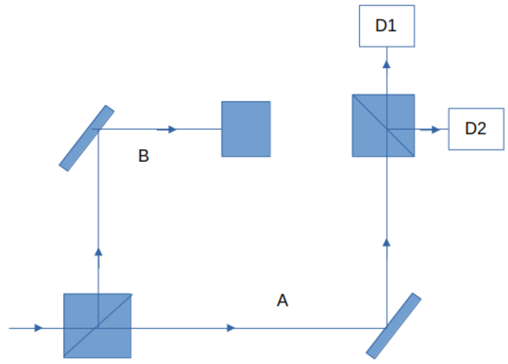

Figure 6: The Mach-Zehnder interferometer arrangement with one path blocked. The photon is forced to show particle nature since the path is explicit.

.

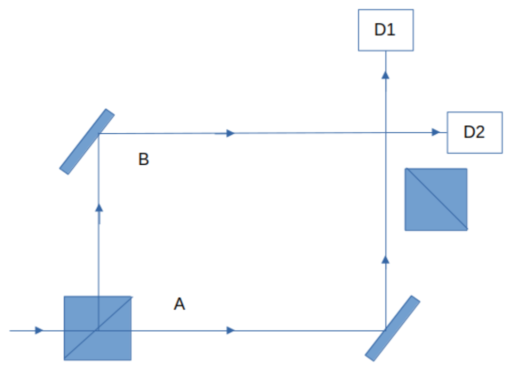

Figure 7: The Mach-Zehnder interferometer arrangement with the Beam splitter placed away from the path. The path is discernible, and the photon shows particle nature.

.

However, moving the beam splitter is not practical. Blocking a path can be achieved using a mechanism consisting of a Pockel's cell operated by a high-speed switch placed in one of the paths. The cell is tuned to block a photon with a particular polarization (17, chap. 2)(2; 20).

Assume that the Pockel's cell is on path 'A.' An analogous arrangement is shown in figure 6. If the Pockel's cell is on, the photon that travels through path 'B,' and both detectors show non-simultaneous clicks, otherwise, D1 and D2 don't show any clicks. If the Pockel's cell is turned off, one can see interference through clicks in detector D1 alone. Instead of moving the second beam splitter, Alice employs a switch that randomly chooses the on and off state of the Pockel's cell much later after the photon leaves the first beam splitter. The results are similar to the moving beam splitter thought experiment.

However, Qureshi posits against causality violations, the experimenter decides wave or particle nature (34). Using equation 10 and 9, Qureshi argues that path information availability depends on the experimenter's active choice of measurement basis (34). Bob reanalyses Alice's arguments with Qureshi's insight. When the photon is sent and the second beam splitter is suddenly removed, she chooses to measure the state of the photon on the z-basis. The photon entangles with the detectors D1 and D2 as per equation 10 and shows particle nature. Next, she sends the photon without the second beam splitter. When she suddenly inserts the second beam splitter into the path, she chooses to measure the photon in the x-basis, and the result is explained by 9 and shows interference. The experimenter's choice actively determines wave or particle nature; hence, there is no causality violation (34).

6 Qureshi picture's problems

The problems with Qureshi's picture include a combination of incompatibility with Bohr's complementarity, a mistaken understanding of mathematical formalism, and excessive dependence on thought experiments. We will analyze the problems in this section.

6.1 The qubit which-way detector

According to Qureshi, the qubit which-way detector lies in the up state when the photon chooses path 'B.' If it chooses path 'A,' it interacts with the which-way detector, they both entangle and the photon proceeds further (34).

However, It is well known that a quantum system will lie in a superposition of up and down before measurement (41). Moreover, Gambetta Et al. study qubit photon interaction in a quantum electrodynamic cavity: observe a blurring of the qubit phase, known as dephasing, during the inter-state transition and photons undergo Stark splitting (16); one does not find the scope for an entanglement model of the photon and the qubit as Qureshi contemplates in section 3.

Experimenters have conducted the delayed choice quantum eraser experiment using a macroscopic *classical* device, Pockel's cell operated by a high-speed switch to block one of the photon paths of a Mach Zehnder interferometer (2; 17), as discussed previously. Even if the qubit detector is implemented, the result will be similar to that of the Pockel's cell-based experiment (2; 16; 20) rather than the entanglement of the photon with the which-way detector.

Bohr's complementarity emphasizes the apparatus themselves follow a different type of physics than the quantum entities they describe (7; 41). When Qureshi proposes his analysis using a qubit which-way detector, one notes the inability to make such an apparatus reiterates the quantum-classical divide. The analysis of quantum mechanics must be based on practical considerations, not concepts that can not be designed. Even thought experiments must map to realizable experiments.

6.2 The detectors' role

Qureshi says that the role played by detectors D1 and D2 in figure 2 is evident in equations 10 and 9: photon Entanglement with the *two* detectors is critical to the particle or wave nature of the photon.

But we observe that the role played by the detectors is that of a single 'screen' where information is presented. The question of interest in quantum mechanics is which path the photon has taken: this event occurs before the second beam splitter. The photon interaction with the screen has no impact on the result; the discernability of the photon path determines the result. Townsend Et al. conducted a single-photon interference experiment where the detector was a single Avalanche Photo Diode used as a photon detector, where both interfering fringe and non-interfering patterns were discerned (46). If we were to apply Qureshi's analysis (34) to Townsend's experiment (46), we must replace D1 and D2 as a single physical entity, say D0, this would make equations 10 and 9 pointless.

Consider our Young's double slit set up in figure 1; here again, the beam splitter or the detectors beyond it do not decide on the availability or unavailability of path information because we could have contemplated the experiment with a single screen. So the detectors are merely elements of screen design. So Equations 9 and 10 are not required in the quantum analysis of a single photon in a Mach-Zehnder interferometer. The quantum mechanical problem in the delayed choice quantum eraser experiment is the availability or unavailability of the path information, not how the screen interacts with the photon. The screen thus gives binary information: interference or particle; detectors D1 and D2 conceptually must be taken as a single entity.

6.3 The role of phase

According to Qureshi, photons that reach detectors D1 and D2 carry phase information as indicated by the minus sign in equation 6, which makes it a distinct state in addition to equation 5. But, Kim Et al. clearly reiterate the well known fact that equations 6 and 5 are the same (24): the minus sign does not affect the which-path probability.

As indicated in figure 4, the phase is due to the beam splitter: the experiment can be designed to vary ϕ in equation 4 in multiple ways. Equation 4 shows how ω and ϕ are distinct: ω is a probability-wave component associated with the quantum observable: position, and thus inherently quantum mechanical, and ϕ is an aspect of experimental design that can be modified without affecting the probabilities and the quantum observables. In equation 4, ω multiplies time, and ϕ has nothing to do with time. The instantaneous or relative phase change ϕ attributed to the wave function is classical. It is quite obvious that one can assign any value to the phase ϕ from 0 to π in equation 2, which will still not affect the observables or their probabilities.

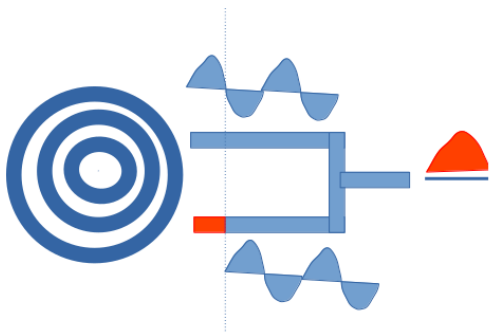

Figure 8: *A trivial thought experiment explaining the impact of path length on phase. The concentric circles indicate a single photon wave. Red indicates in phase (peak), and blue indicates destructive interference (flat line)*

.

A simple thought experiment given in figure 8: a single photon wave is sent through two different paths initially and finally into a single path. If both the wave branches are in phase, they interfere constructively. One can see the resultant wave has high peaks. Now one of the path lengths is altered slowly till one observes a decline in the peak height and, eventually, a flat line: destructive interference. Because the relative distance changes, one cannot argue that the two are different states. Townsend Et al. varied the relative optical path length using a Piezo-electric transducer to obtain single photon interference (46). Rueckner and Peidle demonstrate that changing the Young's double slit's distance from the photon source improves the quality of the interference fringes (37).

6.4 The role of entanglement

According to Qureshi, entanglement with the which-way detector (qubit) alone can destroy interference (34). We observe that this is an attempt to undermine the role of which-way information. Entanglement with the qubit detector may destroy the interference: not because of entanglement alone but because the path information will be available through entanglement.

Kim Et al. used coincidence detection using entangled photons in their delayed choice quantum eraser experiment; they directed one entangled photon to a screen D0 and another to the which-way detectors (24). The which-way photon may randomly choose a path where the which-way information is available or unavailable; correspondingly, one can see interference or otherwise. Thus we can see that there is interference in the presence of entanglement. Kaur and Singh have also demonstrated that two-photon entangled photons show interference when the which-slit information is unavailable (23). Qureshi's argument that entanglement alone can destroy interference (34), we observe, is untenable.

6.5 Experimenter's role

According to Qureshi, the interference appears and disappears when the experimenter chooses to measure the photon in the x or z basis respectively (34). However, Kim Et al. show that the photon's choice to display or destroy interference can be random, and the experimenter does not have an active role (24).

Qureshi argues that during time evolution, the photon is *measured* in the x-basis as in equation 7 (34). But, we calculate the system *evolution* in the x-basis if we represent the measurement in the z-basis. One can choose the system's initial state in the x-basis and construct the evolution in the z-basis. One can choose any arbitrary basis; one can try many different combinations for measurement and evolution if one uses $\begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} \end{pmatrix}$ and $\begin{pmatrix} e^{-i\phi}\sin\frac{\theta}{2} \\ -\cos\frac{\theta}{2} \end{pmatrix}$ corresponds to aligning the spin along various directions (18). The unitary transformation matrices are a perfect tool to assist in changing into another compatible basis; for example, the Hadamard matrix $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$, can be applied to transform from z-basis to x-basis (52). A more radical change will be a transformation from position to momentum coordinate system, highlighting the argument that a Hilbert basis is only a mathematical tool (5; 41). One does not find any reason to map the choice of basis to the physical nature of the photon.

The basis can not determine the physical nature of the photon. The equation 4 that represents wave nature is not derived from Hilbert space mathematics. When the which-way information is unavailable, one observes interference; it represents that the system lies in a superposition of two states: we don't know which state it is in. One does not obtain any information about the quantum observables from the interference pattern. One obtains the observable probabilities when the wave becomes a particle upon measurement; this is the basic idea of the mathematical formulation of quantum mechanics.

7 Discussion

From a glimpse of the setups described in figures 7 and 5, one can easily see that the path information availability correlates with the photon nature is a critical aspect. However, Qureshi's qubit which-way detector model (34) is not an inevitable aspect of any of these configurations. Significantly, the path information availability in the experimental setup decides wave or particle nature, not the active role of the experimenter.

Let us explain the delayed choice quantum erasure with a qubit path detector without Qureshi's help; the qubit detector, due to dephasing effects (16), will act as a block, similar to the Pockel's cell arrangement.

$$\psi = \frac{1}{\sqrt{2}}\psi_A + \frac{1}{\sqrt{2}}\psi_B \tag{11}$$

Alice sends the photon through the first beam splitter. Equation 11 shows the quantum state after the photon passes the first beam splitter, where states A and B correspond to paths A and B of the Mach-Zhender arrangement. The experiment is designed so that when the photon reaches the second beam splitter, the photons have the same phase if they travel through both paths. The $P_x = |\psi_A\rangle\langle\psi_A| + |\psi_B\rangle\langle\psi_B|$ is the projection operator. Alice switches on the qubit detector. There is a probability of 0.5 for both the detectors showing no clicks $|\langle\psi_A|P_x|\psi\rangle|^2$ since the qubit impedes the photon's travel. There is a 0.5 chance for the photon to take path 'B'. It will continue that way and reach the beam splitter: the state of the photon is now given by equation 12.

The photon has a quantum state, not the detector, contrary to equations 10 and 9, where the quantum apparatus is conferred a quantum state. The projection operator now is $P_x = |\psi_{D1}\rangle\langle\psi_{D1}| + |\psi_{D2}\rangle\langle\psi_{D2}|$ after the photon reaches the beam splitter; the the wave function is now per equation 12.

$$\psi = \frac{1}{\sqrt{2}}\psi_{D1} + \frac{1}{\sqrt{2}}\psi_{D2} \tag{12}$$

The detectors' placement ensures a way to get the photon path, The probability of clicks in D1 is $|\langle\psi_{D1}|P_x|\psi\rangle|^2 =$ 0.5 and D2 $|\langle\psi_{D2}|P_x|\psi\rangle|^2 = 0.5$. Let us look at another way to make path information unavailable: replace D1 and D2 with two Mach-Zehnder interferometers. In that case, the path is now ambiguous, and one will notice interference results on both Mach-Zehnder interferometers' detectors: further reiterating the role of path information.

Alice turns off the which-way detector, and the photon evolves in time as a wave function, as in equation 4. The photon wave travels through both paths in the form of 3. According to figure 4, the wave function has the same phase for all the components but for the one reflected from path 'B'. So at detector D1, the two interfering wave functions are out of phase by π. So equation 4 becomes two *components*: equations 13 and 14, for the zero and pi cases for ϕ in figure 4, respectively.

$$\psi = \frac{1}{2}(|\psi_A|^2 + |\psi_B|^2) + \psi_A\psi_B \cos\omega t \tag{13}$$

$$\psi = \frac{1}{2}(|\psi_A|^2 + |\psi_B|^2) - \psi_A\psi_B \cos\omega t \tag{14}$$

The single photon which has traveled both paths now has different components because of the phase change induced by the apparatus, three according to equation 13 and one according to equation 14 have a value of 0 and π respectively for ϕ in equation 4 as evident in figure 4. The distinct components are due to different paths and phases due to path lengths, but they all represent the same wave function or state. This is because, as it is not difficult to see from equation 2 that the relative phase component will cancel out during measurement without affecting the probabilities.

Consider Young's double slit with the slits denoting the position to help with a spin analogy. Let us consider the origin at the middle of the two slits and the position of the slits (or the photon), $-x_1$ and x_1: this is analogous to path 'A' and path 'B' in the Mach-Zehnder interferometer. Let us consider the initial $|\uparrow\rangle$ state for the x_1 position: which-way detector on. One can write the $|\uparrow\rangle$ in the linear combination of $|+\rangle$ and $|-\rangle$ and consider the time evolution with the positions as eigenvalues in equation 16.

$$|\chi(0)\rangle = \begin{pmatrix}1\\0\end{pmatrix} \tag{15}$$

$$|\chi(t)\rangle = \frac{1}{2}(e^{\frac{-ix_1 t}{\hbar}}\begin{pmatrix}1\\1\end{pmatrix} + e^{\frac{ix_1 t}{\hbar}}\begin{pmatrix}1\\-1\end{pmatrix}) \tag{16}$$

Equation 13 and 14's equivalent representation of interference in the spin basis: using the simplified form of equation 16 as in equation 17 can be given as $|\chi(t)\rangle\langle\chi(t)|$.

$$|\chi(t)\rangle = \begin{pmatrix}cos(\frac{x_1 t}{\hbar})\\isin(\frac{x_1 t}{\hbar})\end{pmatrix} \tag{17}$$

Equation 17 says that the superposition evolves, causing interference when the which-way detector is turned off. One can add an instantaneous phase factor ϕ to equation 17 as in equation 4 and obtain plus and minus waves as in Equations 13 and 14. We observe that the phase shift is an instantaneous change induced by the apparatus and does not contribute to a different quantum state: it does not say anything about the position probability. The wave function *evolves* in the x-basis and is *measured* in the z-basis.

Alice sends Bob an encryption key in quantum cryptography (35; 46) after sending the photons through one of the polarization choices. The photon will be in a superposition of different polarization choices and transmitted as a wave. Bob can not directly obtain the polarization (information) from the interference without a measurement: he needs to communicate with Alice to identify the correct filter. If Eve eavesdropped their communication, the wave would collapse and destroy the information. The cutting-edge area of quantum cryptography is an example of the distinction between evolution and measurement.

Consider the photon's initial state in the x-basis as in equation 18 and evolves in the z-basis 19. Equation 19 shows a result of the same form as 17. Representing the photon on one basis for measurement and expanding it on another is only a mathematical exercise. One cannot choose the physical nature of the photon through the Hilbert space basis choice. The path information availability causes the photon's choice of physical nature, but the effect can precede the cause: this is an empirical observation.

$$|\chi(0)\rangle = \begin{pmatrix}1\\1\end{pmatrix} \tag{18}$$

$$|\chi(t)\rangle = \frac{1}{2}(e^{\frac{-ix_1 t}{\hbar}}\begin{pmatrix}1\\0\end{pmatrix} + e^{\frac{ix_1 t}{\hbar}}\begin{pmatrix}0\\1\end{pmatrix}) \tag{19}$$

Further, one can transform the equation 11 can be transformed into the momentum-coordinate system: a direct consequence of the uncertainty principle. The action of the position operator (x) on a position wave function can be written according to equation 20. As a result, one can write equation 11 as in equation 21; it is not difficult to see that this momentum space representation is a sufficient replacement for the spin-based representation.

$$x|\psi_A\rangle = i\hbar\frac{\partial}{\partial p}|\psi_A\rangle \tag{20}$$

$$\psi = \frac{1}{\sqrt{2}}e^{\frac{-ipx_1}{\hbar}} + \frac{1}{\sqrt{2}}e^{\frac{-ipx_2}{\hbar}} \tag{21}$$

This is yet another mathematical exercise that does nothing more than predict probabilities.

Qureshi's analysis (34) is motivated primarily by the need to explain the delayed choice quantum eraser experiment without causality violations. Carlo Rovelli has observed that entropy is the only concept in physics that follows causality; many areas, including special relativity, can allow causality violations (36). Feynman's path integral formulation, essential to quantum field theory, can allow causality violations (11). Avoiding causality violations cannot be the only motivation behind an analysis; Qureshi does not specify what problems other than causality violations exist in the state-of-the-art analysis of the delayed choice quantum eraser experiments.

The aspects of reality are space and time. There are many studies on the study of the constituent aspects of space; quantum mechanics corresponds to a set of experiments of such constituent aspects. Theoretically and experimentally, quantum mechanics is well known to allow causality violations. One can say that just like space, time can also follow a separate set of rules for quantum mechanics in contrast with classical mechanics as posited by Bohr (6); our paper reiterates this fundamental aspect of the Complementarity principle. Studying the constituent aspects of time just as scientists have done for space would be necessary.

Qureshi claims that the detectors D1 and D2 must entangle with the photon for the photon's particle nature to reappear post interference or vice-versa: as in equations 9 and 10. Bohr's Complementarity principle contradicts Qureshi's claim of the quantum nature of the macroscopic detector (34): the detectors are classical, and the statistical results of quantum mechanics and collapse result from the classical-quantum distinction between the quantum apparatus and the photon (7; 41). The detectors are merely a tool to observe or measure the duality of the quantum entity. Von-Neuman attempted to confer quantum status to the classical apparatus in his original formulation itself : the composite systems formalism (microscopic-macroscopic entanglement) (48). Stapp shows the composite systems formulation does not comply with the Copenhagen interpretation (41):

1. One cannot conceptualize collapse for macroscopic systems: the world's objects do not exist in a superposition state and collapse when we perceive them. If quantum superposition would describe all the realities of the universe, one will have to argue that the moon exists only when we look at it (30).

2. Quantum states do not distinguish clearly from classical states: no explicit mechanism is defined for the quantum-to-classical transition in the Composite systems formulation. Wave-particle dual realities and collapse are a clear result of quantum-to-classical distinction.

Wave-particle duality dictates that the wave function must collapse (41); quantum systems entangling with macroscopic objects can not happen with collapse: Becker's take on Von Neuman's composite systems formulation (4; 47). In the Many-Worlds interpretation of quantum mechanics, there is no distinction between classical and quantum systems, no collapse or wave-particle duality. Entanglement of quantum systems with macroscopic detectors is allowed there (44; 45; 47). But using it to help explain the delayed choice quantum eraser is beyond the scope of this work.

Quantum erasure has indicated that erasure of which way information can bring back the wave nature of the photon which collapsed when the path was discernible. Here the idea of collapse needs alternatives. So Qureshi's bold attempt to explain the delayed choice quantum eraser independent of the Copenhagen Interpretation's collapse is necessary despite its limitations. Although we criticize his work, we are in no position to conclusively state that Complementarity alone is sufficient to explain quantum mechanics, since the collapse postulate has been a key aspect of the Complementarity narrative.

8 Conclusion

The biggest problem with Qureshi's analysis (34): the screen in quantum mechanical experiments can be designed as a single entity. This is because even if the analysis were considered in the many-worlds interpretation, where the classical-quantum distinction is nonexistent, the arguments of the analysis would not survive. Next, attributing quantum characteristics to phase change due to path lengths aggravates this further. Classical phase change does not affect the probability of path choice: this is a poignant result. Because Qureshi envisions that the classical phase can contribute to a distinct state, he misguides Hilbert space's role. Hilbert space predicts probabilities of the photon observables in the particle nature of quantum mechanics; the basis is merely a skeletal structure to enable that. One cannot obtain this 'framework' from the wave evolution: which is equivalent to an undisturbed quantum system, not a measured one. There is a significant mathematical gap between wave and particle nature: wave nature can not be derived from particle nature.

Qureshi's central concept: experimenter chooses to show the which-way information: all of his arguments are designed to support this claim. It is convenient for Qureshi when he argues that entanglement alone is enough to destroy the interference: a strategy to downgrade the which-way information's role and upgrade the experimenter's active role. We emphasize the significance of the photon's random choice of wave-particle duality. Since wave and particle detection can be random, the experimenter can not choose to show the wave nature in one instance and the particle nature in another and thus can not actively decide whether the photon will show wave or particle nature. The experimenter's only explicit role is to design the experiment to enable wave and particle detection. Qureshi's argument that entanglement alone can destroy interference is also not true.

We observe that the qubit which-way detector is unnecessary for the analysis: when Qureshi considers the detectors that constitute screen quantum mechanically. He can easily consider the same classical-quantum entanglement methodology for any macroscopic which-way detector already available in practice. The Qubit detector we have shown will only act as a block to the photon's path.

Qureshi is dogmatic to remove the notion of causality violations. The ideological basis for Qureshi's argument is classical-quantum entanglement. The classical-quantum distinction and collapse have been critical to Complementarity. However, quantum erasure cannot be explained through collapse and requires complementarity without collapse. While Qureshi tries to explain erasure through microscopic-macroscopic entanglement, the focus should be to strengthen the limitations of his arguments rather than focus mainly on protecting causality. He fails to prove that entanglement has any obligation to save causality.

Acknowledgements

The authors thank the Amrita Vishwa Vidyapeeth University and its Chancellor for their support.

References

[1] P Achuthan and Karuppath Narayanankutty. On mathematical modeling of quantum systems. In *AIP Conference Proceedings*, volume 1146, pages 105–112. American Institute of Physics, 2009. URL: `https://doi.org/10.1063/1.3183535`.

[2] CO Alley, O Jakubowicz, CA Steggerda, and WC Wickes. A delayed random choice quantum mechanics experiment with light quanta. In *Proceedings of the international symposium foundations of quantum mechanics in the light of new technology*, 1984. URL: `https://inis.iaea.org/search/search.aspx?orig_q=RN:16072640`.

[3] Ulrik L Andersen, Oliver Glöckl, Stefan Lorenz, Gerd Leuchs, and Radim Filip. Experimental

demonstration of continuous variable quantum erasing. *Physical review letters*, 93(10):100403, 2004. URL: https://link.aps.org/doi/10.1103/PhysRevLett.93.100403, doi:10.1103/PhysRevLett.93.100403.

[4] Lon Becker. That von neumann did not believe in a physical collapse. *British Journal for the Philosophy of Science*, 55(1), 2004. URL: https://doi.org/10.1093/bjps/55.1.121.

[5] Mani Bhaumik. Is the quantum state real in the hilbert space formulation? *Quanta*, 9(1):37–46, 2020. URL: http://quanta.ws/ojs/index.php/quanta/article/view/142, doi:10.12743/quanta.v9i1.142.

[6] Niels Bohr. Causality and complementarity. *Philosophy of Science*, 4(3):289–298, 1937.

[7] Niels Bohr et al. *The quantum postulate and the recent development of atomic theory*, volume 3. Printed in Great Britain by R. & R. Clarke, Limited, 1928. URL: https://www.informationphilosopher.com/solutions/scientists/bohr/Como_Nature.pdf.

[8] Richard N Boyd. On the current status of the issue of scientific realism. *Methodology, Epistemology, and Philosophy of Science: Essays in Honour of Wolfgang Stegmüller on the Occasion of His 60th Birthday, June 3rd, 1983*, pages 45–90, 1983. URL: https://doi.org/10.1007/978-94-015-7676-5_3.

[9] Bernard d'Espagnat. Reply to ka kirkpatrick. *arXiv preprint quant-ph/0111081*, 2001. URL: https://arxiv.org/abs/quant-ph/0111081.

[10] Paul Adrien Maurice Dirac. *The principles of quantum mechanics*. Number 27. Oxford university press, 1981. URL: https://global.oup.com/academic/product/the-principles-of-quantum-mechanics-9780198520115?cc=cn&lang=en&.

[11] John F Donoghue and Gabriel Menezes. Arrow of causality and quantum gravity. *Physical Review Letters*, 123(17):171601, 2019. URL: https://journals.aps.org/prl/pdf/10.1103/PhysRevLett.123.171601.

[12] Jonathan Simon Eakins. Classical and quantum causality in quantum field theory. or," the quantum universe". *arXiv preprint gr-qc/0408037*, 2004. URL: https://eprints.nottingham.ac.uk/10069/1/sophie.pdf.

[13] Hugh Everett III. " relative state" formulation of quantum mechanics. *Reviews of modern physics*, 29(3):454, 1957. URL: https://link.aps.org/doi/10.1103/RevModPhys.29.454, doi:10.1103/RevModPhys.29.454.

[14] Johannes Fankhauser. Taming the delayed choice quantum eraser. *arXiv preprint arXiv:1707.07884*, 2017. URL: https://arxiv.org/abs/1707.07884.

[15] RP Feynman. Feynman lectures on physics. vol. 3. quantum mechanics. le cours de physique de feynman. vol. 3. mecanique quantique. 1979. URL: https://www.osti.gov/etdeweb/biblio/5667847.

[16] Jay Gambetta, Alexandre Blais, David I Schuster, Andreas Wallraff, L Frunzio, J Majer, Michel H Devoret, Steven M Girvin, and Robert J Schoelkopf. Qubit-photon interactions in a cavity: Measurement-induced dephasing and number splitting. *Physical Review A*, 74(4):042318, 2006. URL: https://rsl.yale.edu/sites/default/files/physreva.74.pdf.qubit-photon_interactions_0.pdf.

[17] George Greenstein and Arthur Zajonc. *The quantum challenge: modern research on the foundations of quantum mechanics*. Jones & Bartlett Learning, 2006.

[18] David J Griffiths and Darrell F Schroeter. *Introduction to quantum mechanics*. Cambridge university press, 2018.

[19] Robert B Griffiths. Consistent histories and quantum delayed choice. *Fortschritte der Physik: Progress of Physics*, 46(6-8):741–748, 1998. URL: https://plato.stanford.edu/archives/sum2019/entries/qm-consistent-histories/.

[20] Thomas Hellmuth, Herbert Walther, Arthur Zajonc, and Wolfgang Schleich. Delayed-choice experiments in quantum interference. *Physical Review A*, 35(6):2532, 1987. URL: https://journals.aps.org/pra/abstract/10.1103/PhysRevA.35.2532.

[21] Narayanan Kutty Karuppath and Achuthan Panajikunnath. Quantum nonlocality, einstein-podolsky-rosen argument, and consciousness. *NeuroQuantology*, 8(2), 2010. URL: https://www.jneuroquantology.com/index.php/journal/article/view/289/287.

[22] Narayanankutty Karuppath. Critical studies in aspects of quantum reality, time and some applications. 2010. URL: https://api.semanticscholar.org/CorpusID:124696359.

[23] Manpreet Kaur and Mandip Singh. Quantum double-double-slit experiment with momentum entangled photons. *Scientific Reports*, 10(1):11427, 2020. URL: https://www.nature.com/articles/s41598-020-68181-1.

[24] Yoon-Ho Kim, Rong Yu, Sergei P. Kulik, Yanhua Shih, and Marlan O. Scully. Delayed "choice" quantum eraser. *Phys. Rev. Lett.*, 84:1–5, Jan 2000. URL: https://link.aps.org/doi/10.1103/PhysRevLett.84.1, doi:10.1103/PhysRevLett.84.1.

[25] KA Kirkpatrick. Indistinguishability and improper mixtures. *arXiv preprint quant-ph/0109146*, 2001. URL: https://arxiv.org/abs/quant-ph/0109146.

[26] Jarrett Leplin. *Scientific realism*, volume 323. Univ of California Press, 1984.

[27] Xiao-Song Ma, Johannes Kofler, Angie Qarry, Nuray Tetik, Thomas Scheidl, Rupert Ursin, Sven Ramelow, Thomas Herbst, Lothar Ratschbacher, Alessandro Fedrizzi, et al. Quantum erasure with causally disconnected choice. *Proceedings of the National Academy of Sciences*, 110(4):1221–1226, 2013. URL: https://doi.org/10.1073/pnas.1213201110.

[28] Xiao-song Ma, Johannes Kofler, and Anton Zeilinger. Delayed-choice gedanken experiments and their realizations. *Reviews of Modern Physics*, 88(1):015005, 2016. URL: https://link.aps.org/doi/10.1103/RevModPhys.88.015005, doi:10.1103/RevModPhys.88.015005.

[29] Efstratios Manousakis. Founding quantum theory on the basis of consciousness. *Foundations of Physics*, 36(6):795–838, 2006. URL: https://doi.org/10.1007/s10701-006-9049-9.

[30] N David Mermin. Is the moon there when nobody looks? reality and the quantum theory. *Physics today*, 38(4):38–47, 1985. URL: https://doi.org/10.1063/1.880968.

[31] Ulrich Mohrhoff. Restoration of interference and the fallacy of delayed choice: Concerning an experiment proposed by englert, scully, and walther. *American Journal of Physics*, 64(12):1468–1475, 1996. URL: https://doi.org/10.1119/1.18411.

[32] Mehwish Nawaz, Hinna Tariq, Manzoor Ikram, et al. Delayed-choice tunable quantum eraser under cavity qed mach-zehnder setup. *Optics Communications*, 399:13–15, 2017. URL: https://doi.org/10.1016/j.optcom.2017.04.046.

[33] Tao Peng, Hui Chen, Yanhua Shih, and Marlan O Scully. Delayed-choice quantum eraser with thermal light. *Physical review letters*, 112(18):180401, 2014. URL: https://link.aps.org/doi/10.1103/PhysRevLett.112.180401, doi:10.1103/PhysRevLett.112.180401.

[34] Tabish Qureshi. Demystifying the delayed-choice quantum eraser. *European Journal of Physics*, 41(5):055403, 2020. URL: https://iopscience.iop.org/article/10.1088/1361-6404/ab923e, doi:10.1088/1361-6404/ab923e.

[35] George L Roberts, Marco Lucamarini, James F Dynes, Seb J Savory, ZL Yuan, and Andrew J Shields. Modulator-free coherent-one-way quantum key distribution. *Laser & Photonics Reviews*, 11(4):1700067, 2017. URL: https://onlinelibrary.wiley.com/doi/full/10.1002/lpor.201700067.

[36] Carlo Rovelli. *The order of time*. Penguin, 2019. URL: https://www.penguinrandomhouse.com/books/551483/the-order-of-time-by-carlo-rovelli/.

[37] Wolfgang Rueckner and Joseph Peidle. Young's double-slit experiment with single photons and quantum eraser. *American Journal of Physics*, 81(12):951–958, 2013. URL: https://pubs.aip.org/aapt/ajp/article-abstract/81/12/951/1044320/Young-s-double-slit-experiment-with-single-photons?redirectedFrom=fulltext.

[38] Giuliano Scarcelli, Yu Zhou, and Yanhua Shih. Random delayed-choice quantum eraser via two-photon imaging. *The European Physical Journal D*, 44:167–173, 2007. URL: https://doi.org/10.1140/epjd/e2007-00164-y.

[39] Marlan O. Scully and Kai Drühl. Quantum eraser: A proposed photon correlation experiment concerning observation and "delayed choice" in quantum mechanics. *Phys. Rev. A*, 25:2208–2213, Apr 1982. URL: https://link.aps.org/doi/10.1103/PhysRevA.25.2208, doi:10.1103/PhysRevA.25.2208.

[40] Abner Shimony. Role of the observer in quantum theory. *American Journal of Physics*, 31(10):755–773, 1963. doi:10.1119/1.1969073.

[41] Henry Pierce Stapp. The copenhagen interpretation. *American journal of physics*, 40(8):1098–1116, 1972. URL: https://doi.org/10.1119/1.1986768.

[42] Ian Stewart. Grandfather paradox. *Nature*, 464(7293):1398–1398, 2010. URL: https://www.nature.com/articles/4641398a.

[43] Christopher G Timpson and Harvey R Brown. Proper and improper separability. *International Journal of Quantum Information*, 3(04):679–690, 2005. URL: https://doi.org/10.1142/S0219749905001493.

[44] Frank J Tipler. Nonlocality as evidence for a multiverse cosmology. *Modern Physics Letters A*, 27(04):1250019, 2012. URL: https://doi.org/10.1142/S0217732312500198.

[45] Frank J Tipler. Quantum nonlocality does not exist. *Proceedings of the National Academy of Sciences*, 111(31):11281–11286, 2014. URL: https://doi.org/10.1073/pnas.1324238111.

[46] Paul D Townsend, JG Rarity, and PR Tapster. Single photon interference in 10 km long optical fibre interferometer. *Electronics Letters*, 7(29):634–635, 1993. URL: https://www.infona.pl/resource/bwmeta1.element.ieee-art-000000211832.

[47] Lev Vaidman. Many-Worlds Interpretation of Quantum Mechanics. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Fall 2021 edition, 2021. URL: https://plato.stanford.edu/archives/fall2021/entries/qm-manyworlds.

[48] John Von Neumann. *Mathematical foundations of quantum mechanics: New edition*, volume 53. Princeton university press, 2018. URL: https://press.princeton.edu/books/hardcover/9780691178561/mathematical-foundations-of-quantum-mechanics.

[49] John Archibald Wheeler. The “past” and the “delayed-choice” double-slit experiment. In *Mathematical foundations of quantum theory*, pages 9–48. Elsevier, 1978. URL: https://doi.org/10.1016/B978-0-12-473250-6.50006-6.

[50] John Archibald Wheeler and Wojciech Hubert Zurek. volume 40. Princeton University Press, 2014. URL: https://books.google.co.in/books?hl=en&lr=&id=L7r_AwAAQBAJ&oi=fnd&pg=PP1&dq=Quantum+Theory+and+Measurement&ots=7kSwo8zlom&sig=L7hCyO16cnr72EliNcFEu-OYlBg&redir_esc=y#v=onepage&q=Quantum%20Theory%20and%20Measurement&f=false.

[51] Eugene P. Wigner. The Problem of Measurement. *American Journal of Physics*, 31(1):6–15, 01 1963. doi:10.1119/1.1969254.

[52] Nouredine Zettili. Quantum mechanics: concepts and applications. 2009. URL: https://books.google.co.in/books?hl=en&lr=&id=TAjVyzm57YwC&oi=fnd&pg=PR5&dq=quantum+mechanics+zettili&ots=-nbLm3Jt6Y&sig=h19fPlgayULdi4RsMVhJtNB84Fs#v=onepage&q=quantum%20mechanics%20zettili&f=false.