The path information is the measurement

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Photon wave functions collapse into particles only after they exhibit their location, as was observed in many experiments. However, scientists like Bohr, Wheeler, and Scully contemplated that this causality could be violated, after which many experimenters claim to have demonstrated the contemplated causality violation. Scientists widely debate this claim; this debate often tries to reinterpret many tenets of quantum mechanics. Qureshi posits that the experimenter's choice of Hilbert space basis determines wave or particle nature: the experimenter thus chooses to exhibit the which-way information or obscure it. Qureshi posits that when the experimenter measures the photon in the x-basis, it entangles with the which-way detector and the two detectors that constitute the screen, leaves a mathematical trail caused by phase change, and destroys interference. He contemplates a novel quantum qubit which-way detector. In this paper, we peruse the problems in Qureshi's analysis. The distinction between evolution and measurement is not acknowledged. The entanglement of a photon with experimental apparatus violates the quantum-classical distinction, which is key to wave function collapse. In an attempt to facilitate classical-quantum entanglement, Qureshi forgets that the screen in quantum experiments can be a single entity. The quantum qubit he contemplates will likely function like a classical which-way detector. Instantaneous phase change does not influence quantum measurement: it thus does not contribute to a distinct quantum state. Scientists have detected wave-particle duality using entangled photons: entanglement alone cannot destroy the interference; the photon can choose its wave-particle option randomly; the experimenter's role is thus inactive. Wave nature formulation is not derived from Hilbert space basis, and the mathematical formulation in quantum mechanics is meant to predict probabilities in the particle nature; it does not say anything about the photon's physical form choice. We thus reiterate that Complementarity is independent of causal constraints.

Quanta ; :

1 Introduction

In 1978, Wheeler proposed a radical thought experiment: photons from a distant quasar circumvent a gravitational lensing galaxy; when observed with an interferometer, the individual path information of the photon is lost one shall see interference; however, if the two photon paths are discerned one does not see an interference pattern (48). One thus has the power to influence the source billions of years away to emit light along both paths or only one path: the past can be changed! (48). Scully proposed a more laboratory-friendly version of Wheeler's thought experiment and called it the 'delayed-choice quantum eraser thought experiment' (38). Kim Et al. demonstrated experimentally using Young's double slit and entangled photons, claiming to have fully realized it for the first time (23); however, many such experiments espouse causality-violating (2; 3; 26; 27; 31; 32; 37).

Wigner posited that consciousness causes the collapse of the quantum wave to its complementary particle form (49). Modern scientists strongly espouse realism (8; 25): scientific results are 'mind-independent'; but some say quantum mechanics may imply otherwise (39) (13) (1) (21) (28). The paradox of 'delayed choice' is also a part of this debate, key to understanding quantum mechanics.

If one can change the past, one can kill one's grandfather; the famous Grandfather paradox has intrigued physicists and the general public alike (41). The Big-Bang to create the universe requires time to follow asymmetry: forward arrow (12). Scientists often have to look beyond the Copenhagen interpretation to expel causality-violating ideas about the delayed eraser: the Relative States (30), consistent histories (20), proper and improper mixed states (9; 24; 42), and Bohmian mechanics (14). Tabish Qureshi attempts to denounce causality violation using macroscopic-microscopic entanglement and attributing physical realization ability to Hilbert space formalism: emphasizes the experimenter's role in choosing the photon nature, thereby avoiding causality violations (33). He argues that his analysis is per Bohr's Complementarity principle (33); this paper aims to elucidate the problems in his claim and points out the incompatibility with specific empirical observations and mathematical underpinnings.

We illustrate the basic ideas of Young's double-slit experiment using a simple picture: we devise a thought experiment using Young's double-slit experiment with a beam splitter to set the tone: basic Copenhagen interpretation. Then we explain the mathematical structure used in Qureshi's analysis (33): the spin analogy. We explain the key ideas behind Qureshi's analysis of the delayed choice quantum eraser experiment (33). Before we elucidate the problems in his analysis, we explain the working of the Mach-Zehnder interferometer and how it can enable causality violations: we consider what Qureshi's thought experiment would say about an actual experiment. We then discuss the correct way of looking at the delayed choice quantum eraser experiment to contrast with Qurehsi's analysis. In conclusion, we attempt to identify the biggest problems in Qureshi's analysis.

2 The double slit experiment

Young's double slit is historically significant and known for its conceptual clarity (15). We use the same concept but a beam splitter based two detector screen arrangement similar to a Mach Zehnder interferometer. This will help compare the standard Complementarity principle used by Scully to explain his delayed choice quantum eraser thought experiment (38) and help compare with Qureshi's analysis (33).

2.1 Simple picture

Alice has a Young's double slit arrangement. She has a which-way detector placed in front of each slit. There is a screen behind the double slit; it has two sensors, a pair of mirrors, and a beam splitter. Alice places two which-way detectors in front of the slits; when the photon passes through either, the photon will light either bulb A or B. The source generates a single photon. The which-way detectors are turned off; the photon does not reveal its path, interfering with itself, as Paul Dirac would say (10). We show the arrangement in figure 1; serves conceptual understanding and not experimental accuracy.

$$\psi = \frac{1}{\sqrt{2}}\psi_A + \frac{1}{\sqrt{2}}\psi_B \tag{1}$$

Equation 1 is a superposition of wave functions corresponding to position A:x_A and B:x_B, which are the eigenvalues. The arrangement in figures 1 and 3 admit phases of zero and π so that one can write equation 1 as in 2.

$$\psi = \frac{1}{\sqrt{2}}e^{i\phi}\psi_A + \frac{1}{\sqrt{2}}\psi_B \tag{2}$$

If the which-way detector is switched off, there is no measurement. The superposition evolves to cause interference.

$$\langle\psi(t)|\psi(t)\rangle = \frac{1}{2}(|\psi_A|^2 + |\psi_B|^2) + \psi_A\psi_B cos(\omega t) \tag{3}$$

Equation 3's last term represents wave-nature and $\omega = \frac{x_B - x_A}{\hbar}$. Alice places two reflectors behind the slits to direct the photons to the beam splitter. The photon reaches the beam splitter at phase zero. The beam splitter, according to figure 3, can induce a phase change of ϕ as in equation 4 and directs constructive interference to sensor D1, as shown in figure 1, the destructive counterpart to sensor D2; clicks are noted in detector D1 alone.

$$|e^{i\phi}\psi_A(t) + \psi_B(t)|^2 = \frac{1}{2}(|\psi_A|^2 + |\psi_B|^2) + \psi_A\psi_B cos(\omega t + \phi) \tag{4}$$

Measurement takes place when the which-way detector is turned on; the position projection operator can be given by $P_x = |\psi_A\rangle\langle\psi_A| + |\psi_B\rangle\langle\psi_B|$. Photons will pass through either slit A or slit B with a probability given by $|\langle\psi_A|P_x|\psi\rangle|^2$ or $|\langle\psi_B|P_x|\psi\rangle|^2 = 0.5$: Alice sees clicks on one of the detectors and the corresponding which-way detector will glow. The key to this picture is the *collapse* of the wave function from the superposition of two states to a single state particle.

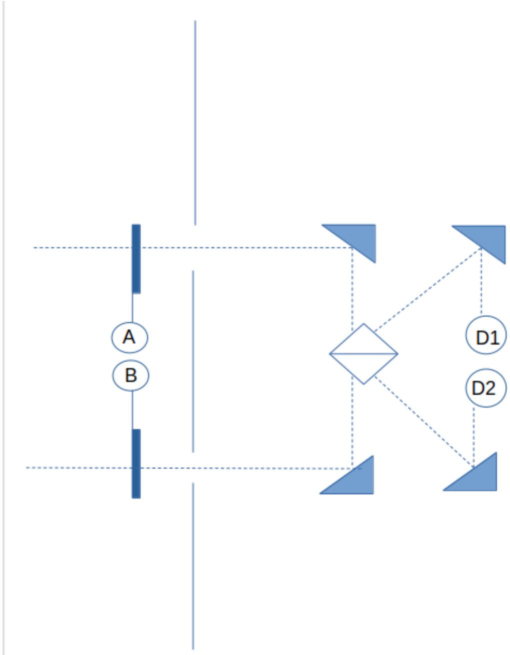

***Figure 1:** Young's double slit arrangement: Dashed lines indicate possible paths of the photon. The oval structure in yellow is the phase shift. The red and blue rectangles are the which-way detectors. D1 and D2 are the two sensors of the screen*

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3 Spin based picture

In this viewpoint, ψ_A and ψ_B are assigned the states of a one-half spin electron (33). Assign values for phase 0 and π in 4 to give $|+\rangle$ and $|-\rangle$ as in equations 5 and 6: $\frac{1}{2}$.

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \quad (5)$$

$$|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} - \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \quad (6)$$

The $|+\rangle$ and $|-\rangle$ are equivalent to measuring the spin one-half state along the x-axis (horizontal or left-right). The x-axis eigenstates are given as a linear combination of z-axis states (vertical) $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$; which we can name as the $|\uparrow\rangle$ and $|\downarrow\rangle$ states. When the which-way detector is turned on, the path is available, and the wave function collapses into either the $|\uparrow\rangle$ or $|\downarrow\rangle$ states with a probability of 0.5.

Alice switches off the which-way detector and erases the path information. It is easy to see that up-and-down states can be written as a linear combination of $|+\rangle$ and $|-\rangle$ states. Consider that Alice turned the which-way detector on, and the A bulb glows. Alice then turns off the which-way detector and waits for some time. She then sends the photon again. This time she sees an interference pattern. One can assign phase values to equation 4 and obtain the wave nature in terms of $|+\rangle$ and $|-\rangle$ as in equation 7 and 8 (33). The key to this picture is that ϕ in equation 4 creates a distinct quantum state through the minus sign ($\phi = \pi$).

$$|+\rangle\langle+| = \frac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| + cos(\omega t)(|\downarrow\rangle\langle\uparrow|) \quad (7)$$

$$|-\rangle\langle-| = \frac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| - cos(\omega t)(|\downarrow\rangle\langle\uparrow|) \quad (8)$$

4 Qureshi picture

This is a modification of the spin picture marked by the entanglement of the photon with the detector. This methodology is used to analyze the delayed choice quantum eraser experiment by Tabish Qureshi (33). The which-way detector in figure 1 must be a qubit; takes $|\uparrow\rangle$ or $|\downarrow\rangle$ state when the photon takes path A and B, respectively (33): the which-way detector state maps to the photon state.

The interference occurs when the experimenter measures the which-way detector in the plus-minus states, so the photon entangles with the detectors D1 and D2, respectively.

$$\psi = \frac{1}{2}(|D1\rangle(|\uparrow\rangle + \downarrow\rangle) + |D2\rangle(|\uparrow\rangle - \downarrow\rangle)) \quad (9)$$

Equation 9 shows the $|+\rangle$ entangles with the detector D1 one sees clicks in detector D1 alone while D2 has no clicks since it entangles with $|-\rangle$: caused by constructive and destructive interference, respectively. When the which-way detector is turned on, either path A or path B is measured with a probability of 0.5.

$$\psi = \frac{1}{2}((|D1\rangle + |D2\rangle)|\uparrow\rangle + (|D1\rangle - |D2\rangle)|\downarrow\rangle) \quad (10)$$

Equation 10 shows the up-down states entangle with superposition states of the detectors D1 and D2, the particle nature of the photon. According to Qureshi, the photon's observation in the particle form is marked by the probability of finding clicks in either detector D1 or D2; when a large number of photons follow suit, one can see clicks in both the detectors (33). The key to this picture, contrary to the standard Copenhagen interpretation:

1. Experimenter measures the qubit in the z-basis equation 10 or in the x-basis equation 9, and hence selects between wave and particle.

2. Attributing quantum nature to a macroscopic object: which-way detector and screen.

3. Entanglement replaces the collapse of the wave function and suffices to remove interference.

4. Equation 1's plus and minus cases are taken as distinct quantum states; as in Section 3.

We will dedicate the rest of the paper to examining the problems in the Qureshi picture.

5 The Mach-Zehnder Interferometer

We will first illustrate the working of a typical Mach-Zehnder interferometer; this will help us understand Qureshi's thought experiment (33) and its fallacies. We explore how the which-way information discernibility influences the outcome.

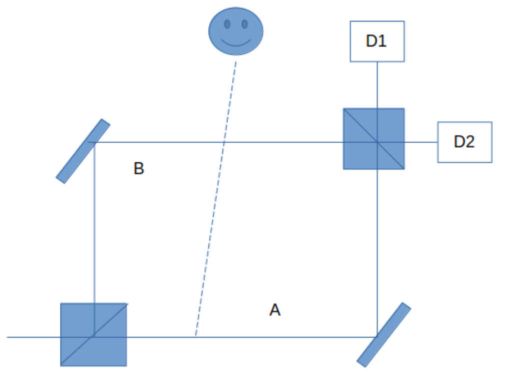

Figure 2: *The Mach-Zehnder interferometer arrangement consists of a pair of 50-50 beam splitter, mirrors, and sensors. The observer may observe the photon in one of the paths.*

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When a single photon passes through the first 50-50 beam splitter, it splits the photon wave into two parts with equal intensity. The beam splitter is explained in figure 3; only the photon reflected from the glass surface undergoes a phase shift. The mirrors direct the two rays to another beam splitter, as shown in figure 2. The entire design comprising the beam splitters and mirrors is such that the photon reaching the second beam splitter from both paths arrives at the same phase. A pair of rays emerge from the last beam splitter; one pair interferes constructively and reaches detector D1, and another interferes destructively; the photon taking path B in the figure 2 after reflection undergoes a phase shift of π: all other cases have phase unchanged as in figure 3. The photon interference causes clicks on D1 (constructive) and no clicks on D2 (destructive).

Let us place a block that does not let the photon pass through in path B. The second beam splitter receives the photon only if the photon chooses path A: there is no

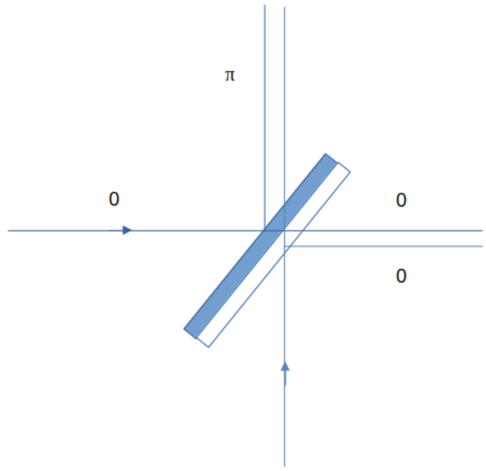

Figure 3: *Beam splitter with glass coating on one side(dark). Light is either reflected or transmitted; reflection from glass surface alone results in phase shift*

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interference, and detectors, either D1 or D2, will show clicks: now the photon behaves as a particle. One will not see clicks on both detectors simultaneously: proof of the photon's existence, a historic experiment by Alain Aspect and crew (17). We emphasize the role of path information: let us send a single photon directly through the second beam splitter alone, without the first beam splitter and the mirrors; the photon is either reflected to detector D1 or transmitted to detector D2 (18). The two detectors are placed appropriately so that the path information is evident. Now, the experimenters return to the mechanism as in figure 2; now, the path information is lost; one can observe interference. The correlation between wave-particle duality and path discernibility is evident.

6 Causality violations

Alice has set up a Mach-Zehnder interferometer in 2 to perform her moving beam splitter thought experiment (18). She attaches a remote-controlled motor to the second beam splitter; the beam splitter can be pushed away from the path of the single photon. The photon is sent through the first beam splitter. She suddenly removes the second beam splitter just before it reaches the second beam splitter. Alice notices clicks only in one of the detectors; the photon evinces particle nature. Alice was expecting the photon to show interference because while she sent the photon, the path was indiscernible because of the second beam splitter, so she must have seen interference but noticed the contrary. It is as if the photon has changed the past: when the second beam splitter was removed, at the last moment, the photon influenced the source to take only one path. Similarly, she starts with the second beam splitter out of the path and suddenly brings it into the path. Alice notes clicks only in detector D1 only. Bringing the beam splitter back has erased the path information.

However, moving the beam splitter is not practical. Hence, one must consider blocking a path: achieved using a Pockel cell operated by a switch placed in one of the paths: the cell is tuned to block a photon with a particular polarization. Hence the photon must travel through path B; this destroys interference (2; 18) or get stopped by the Pockel cell if it chooses path A: both detectors D1 and D2 don't show any click. If the Pockel cell is turned off, one can see interference through detector D1 alone, which shows clicks. Instead of moving the second beam splitter, Alice would use the switch with similar results.

However, Qureshi posits against causality violations, the experimenter decides wave or particle nature (33). Using equation 10 and 9, based on his qubit detector, Qureshi argues that path information availability depends on the experimenter's active choice of measurement basis (33).

Alice reanalyses her thought experiment. When the photon is sent without the second beam splitter, she chooses to measure the state of the photon in the z-basis; the photon entangles with the detectors D1 and D2 as per equation 10. If she suddenly inserts the second beam splitter in the path, she chooses to measure the photon in the x-basis, and the result is explained by 9. The experimenter's choice destroys interference; hence, there is no causality violation (33).

7 Qureshi picture's problems

The problems with Qureshi's picture include a combination of incompatibility with Bohr's complementarity, a mistaken understanding of the mathematical formalism, and excessive dependency on thought experiments. We will analyze the problems in this section.

7.1 The qubit which-way detector

Qureshi imagines a quantum qubit as a which-way detector in the path A of the Mach-Zehnder interferometer 2; according to him, the which-way detector is in the 'up' state when the photon does not pass through the path A, and if it does travel through the path, the which-way detector acquires the 'down' state.(33). But it is well known that a quantum system will lie in a superposition of up and down before measurement (40). Gambetta et al. study qubit photon interaction in a quantum electrodynamic cavity: observe a blurring of the qubit phase, known as dephasing, during the inter-state transition and photons undergo Stark splitting (16); one does not find the scope for a simple entanglement model of the photon and the qubit as Qureshi contemplates 4.

Experimenters have conducted the delayed choice quantum eraser experiment using a macroscopic electro optic device, Pockels cell placed in one of the paths of a Mach Zehnder interferometer; a photon switch with a random choice decides to block or allow the photon with a particular polarization to pass through (2; 18); a much more realistic and cost-effective solution: If one path is blocked, either the photon has to go through the other path and destroy interference or misdirected by the block and show now clicks in the detector. Even if the qubit detector is implemented, the result will be similar to that of the Pockels cell based experiment (2) rather than the entanglement of the photon with the which-way detector.

The standard interpretation of quantum mechanics, with its origins in Bohr's ideas, emphasizes the inability of experimental apparatus results in the uncertainty principle; the apparatus themselves follow a different type of physics than the quantum entities they describe (7). When Qureshi proposes his analysis using a qubit which-way detector, one notes the inability to make such an apparatus possible is similar to what Bohr had indicated. The analysis of quantum mechanics must be based on practical considerations, not concepts that can not be designed: even thought experiments must map to realizable experiments.

7.2 The detectors' role

Qureshi says that role played by detectors D1 and D2 in figure 2 is evident in equations 10 and 9:

1. Only one detector glows when the photon path is unknown: wave nature.
2. Both detectors glow when the photon path is known: particle nature.
3. Entanglement with the *two* detectors is critical to the particle or wave nature of the photon.

But we observe that the role played by the detectors is that of a single 'screen' where information is presented. The question of interest in quantum mechanics is which path the photon has taken: this event occurs before the second beam splitter. The photon interaction with the screen has no impact on the result; the discernability of the photon path determines the result. Townsend et al. conducted a single-photon interference experiment where the detector was a single Avalanche Photo Diode used as a photon detector, where both interfering fringe and non-interfering patterns were discerned (45). If we were to apply Qureshi's analysis (33) to Townsend's experiment (45), we must replace D1 and D2 as a single physical entity, say D0, this would make equations 10 and 9 pointless.

Consider figure 1; here again, the beam splitter or the detectors beyond it do not decide on the availability or unavailability of path information because we could have contemplated the experiment with a single screen. So the detectors are merely elements of screen design. So Equations 9 and 10 are not required in the quantum analysis of a single photon in a Mach-Zehnder interferometer. The quantum mechanical problem in the delayed choice quantum eraser experiment is the availability or unavailability of the path information, not how the screen is designed. The screen thus gives binary information: interference or particle; detectors D1 and D2 conceptually must be taken as a single entity.

7.3 The role of phase

According to Qureshi, photons that reach detectors D1 and D2 carry phase information as indicated by the minus sign in equation 8, which makes it a distinct state in addition to the 7. But, Kim et al. say that equations 7 and 8 are the same (23): the minus sign does not affect the which-path probability.

As indicated in figure 3, the phase is due to the beam splitter: the experiment can be designed to vary ϕ in equation 4 in multiple ways. Equation 4 shows how ω and ϕ are distinct: ω is a probability-wave component associated with the quantum observable: position, and thus inherently quantum mechanical, and ϕ is an aspect of experimental design that can be modified without affecting the probabilities and the quantum observables. In equation 4, ω multiplies time, and ϕ has nothing to do with time. The instantaneous phase change ϕ attributed to the wave function is classical or electromagnetic. One can add any value to the phase ϕ from 0 to π in equation 4, which will still not affect the observables or the probabilities.

A simple thought experiment: consider two similar waves in phase and interfere constructively; one can see the resultant wave has high peaks. Now the distance between one of the sources and the screen is altered till one observes a decline in the peak height and, eventually, a flat line: destructive interference. The photon sources' relative distance affects the phase, not the quantum observables. Because the relative distance changes, one cannot argue that the two are different states. Townsend et al. varied the relative optical path length using a Piezo-electric transducer to obtain single photon interference (45). Rueckner and Peidle demonstrate that changing the Young's double slit's distance from the photon source improves the quality of the interference fringes (36).

7.4 The role of entanglement

According to Qureshi, entanglement with the which-way detector (qubit) alone can destroy interference (33). We observe that this is an attempt to undermine the role of which-way information. Entanglement with the qubit detector may destroy the interference: not because of entanglement alone but because the path information will be available through entanglement.

Kim et al. used coincidence detection using entangled photons in their delayed choice quantum eraser experiment (23); they direct one entangled photon to a screen D0 and another to the which-way detectors. The which-way photon may randomly choose a path where the which-way information is available or unavailable; correspondingly, one can see interference or otherwise. Thus we can see that there is interference in the presence of entanglement. Kaur and Singh have also demonstrated that two-photon entangled photons show interference when the which-slit information is unavailable (22). Qureshi's argument that entanglement alone can destroy interference (33), we observe, is untenable.

7.5 Classical-quantum distinction

Qureshi claims that the detectors D1 and D2 must entangle with the photon for the photon's particle nature to reappear post interference or vice-versa: as in equations 9 and 10. Bohr's Complementarity principle contradicts Qureshi's claim of the quantum nature of the macroscopic detector (33): the detectors are classical, and the statistical results of quantum mechanics and collapse result from the classical-quantum distinction between the detectors and the photon (7; 40). The detectors are merely a tool to observe or measure the duality of the quantum entity. Von-Neuman attempted to confer quantum status to the classical apparatus in his original formulation itself (47): the composite systems formalism (microscopic-macroscopic entanglement). Stapp shows the composite systems formulation does not comply with the Copenhagen interpretation (40):

1. One can not conceptualize collapse for macroscopic systems: the world does not exist in a superposition state and collapse when we perceive it; If quantum superposition would describe all the realities of the universe, one will have to argue that the moon exists only when we look at it (29).

2. Quantum states do not distinguish clearly from classical states: no explicit ontology is defined for the

quantum-to-classical transition in the Composite systems formulation. Wave-particle duality and collapse are a clear result of quantum to classical transition.

Wave-particle duality dictates that the wave function must collapse (40); quantum systems entangling with macroscopic objects can not happen with collapse: Becker's take on Von Neuman's composite systems formulation (4; 46). In the Many-Worlds interpretation of quantum mechanics, there is no distinction between classical and quantum systems; no collapse and no wave-particle duality: entanglement of quantum systems with macroscopic detectors is allowed there (43; 44; 46). So if Qureshi claims that photons can entangle with experimental apparatus (33): there is no wave-particle duality.

The wave-nature collapses to particle form; this is the idea of collapse in Bohr's complementarity. Let us see Qureshi's analysis in this purview. Consider the arguments in section 6: Alice sends the photon initially without the second beam splitter when she suddenly brings in the beam splitter; Qureshi's causality preserving argument says that the experimenter has thus actively chosen to erase the path information. If his argument is correct, we observe that the photon must suddenly uncollapse to a wave nature and interfere. We thus conclude that his argument is incompatible with the idea of collapse, which is critical to complementarity.

7.6 Role of basis choice

According to Qureshi, the interference appears when the experimenter chooses to measure the photon in the x-basis (33) and thus chooses to lose the which-way information. Kim et al. show that the photon's choice to display or destroy interference can be random, and the experimenter does not have an active role (23).

Qureshi argues that during time evolution, the photon is *measured* in the x-basis as in equation 7 (33); but, we calculate the system *evolution* in the x-basis if we represent the measurement in the z-basis. One can choose any arbitrary basis; one can try many different combinations for measurement and evolution if one uses $\begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} \end{pmatrix}$ and $\begin{pmatrix} e^{-i\phi}\sin\frac{\theta}{2} \\ -\cos\frac{\theta}{2} \end{pmatrix}$ corresponds to aligning the spin along various directions (19). The unitary transformation matrices are a perfect tool to assist in such a change of basis; for example, the Hadamard matrix $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$, can be applied to transform from z-basis to x-basis (50). A more radical change will be a transformation from position to momentum coordinate system, further indicating that a Hilbert basis is only a mathematical tool (5). One does not find any reason to map the choice of basis to the physical nature of the photon.

Hilbert space basis is a mere mathematical tool to predict the probability of a quantum observable and refuses to do anything beyond that (40). The basis can not determine the physical nature of the photon. The equation 4 that represents wave nature is not derived from Hilbert space mathematics: it is based on empirical observations. When the which-way information is unavailable, one observes interference; it represents that the system lies in a superposition of two states: we don't know which state it is in. One does not obtain any information about the quantum observables from the interference pattern: one obtains the observable probabilities when the wave becomes a particle upon measurement; this is the basic idea of the Complementarity principle.

Alice sends Bob an encryption key in quantum cryptography (34; 45) after sending the photons through one of the polarization choices. The photon will be in a superposition of different polarization choices and transmitted as a wave. Bob can not directly obtain the polarization (information) from the interference without a measurement: he needs to communicate with Alice to identify the correct filter. If Eve eavesdropped their communication, the wave would collapse and destroy the information.

8 Discussion

We have seen various problems in Qureshi's analysis of the delayed choice quantum eraser experiment. The screen is a single entity, irrespective of the design. The qubit detector will only act to block the photon's path. Superposition and entanglement are characteristics of quantum systems, not classical systems: the classical quantum distinction is critical to Bohr's complementarity principle. The negative sign coming from phase change does not affect the measurement result. Entanglement alone can not destroy interference. Hilbert basis can only predict probabilities and not decide wave-particle duality.

Let us explain the experiment using our corrections; Mach-Zehnder interferometer with a Pockels cell like which-way detector (2). Alice can also use a qubit detector as suggested by Qureshi (33); the qubit detector, due to dephasing effects (16), is likely to act as a block, similar to the Pockels cell arrangement. Alice must connect a switch with the qubit arrangement to place the qubit in the photon path and away.

$$\psi = \frac{1}{\sqrt{2}}\psi_A + \frac{1}{\sqrt{2}}\psi_B \tag{11}$$

Alice sends the photon through the first beam splitter. Equation 11 shows the quantum state after the photon passes the first beam splitter, where states A and B correspond to paths A and B of the Mach-Zhender arrangement. The experiment is designed so that when the photon reaches the second beam splitter, the photons have the same phase if they travel through both paths. The $P_x = |\psi_A\rangle\langle\psi_A| + |\psi_B\rangle\langle\psi_B|$. Alice switches on the qubit detector: the Hamiltonian. There is a probability of 0.5 for both the detectors showing no clicks $|\langle\psi_A|P_x|\psi\rangle|^2$ since the qubit impedes the photon's travel. There is a 0.5 chance for the photon to take path B. It will continue that way and reach the beam splitter: the state of the photon is now given by equation 12.

The photon has a quantum state, not the detector, contrary to equations 10 and 9. The projection operator now is $P_x = |\psi_{D1}\rangle\langle\psi_{D1}| + |\psi_{D2}\rangle\langle\psi_{D2}|$ after the photon reaches the beam splitter; the wave function is now 12

$$\psi = \frac{1}{\sqrt{2}}\psi_{D1} + \frac{1}{\sqrt{2}}\psi_{D2} \tag{12}$$

The detectors' placement ensures a way to get the photon path: and facilitates wave function collapse. The probability of clicks in D1 is $|\langle\psi_{D1}|P_x|\psi\rangle|^2 = 0.5$ and D2 $|\langle\psi_{D2}|P_x|\psi\rangle|^2 = 0.5$. Suppose the path information is unavailable: replace D1 and D2 with two Mach-Zehnder interferometers. In that case, the path is now ambiguous, and one will notice interference results on both Mach-Zehnder interferometers' detectors: further reiterating the role of path information.

Alice turns off the which-way detector, and the photon evolves in time as a wave function, as in equation 4. The photon wave travels through both paths in the form of 3. According to figure 3, the wave function has zero phases but for the one reflected from path B. So for detector D1, the two interfering wave functions are out of phase by π. So equation 4 becomes two components shown by one transmitted from path B, the other reflected from path A as in equations 13 and 14, for the zero and pi cases for ϕ in figure 3, respectively.

$$\psi = \frac{1}{2}(|\psi_A|^2 + |\psi_B|^2) + \psi_A\psi_B \cos\omega t \tag{13}$$

$$\psi = \frac{1}{2}(|\psi_A|^2 + |\psi_B|^2) - \psi_A\psi_B \cos\omega t \tag{14}$$

The single photon which has traveled both paths now has different components because of the phase change induced by the apparatus, three according to equation 13 and one according to equation 14 have a value of 0 and π respectively for ϕ in equation 4 as evident in figure 3. The components have different path lengths but represent the same wave function.

Consider a Young's double slit with the slits denoting the position to help with a spin analogy. Let us consider the origin at the middle of the two slits and the position of the slits (or the photon), $-x_1$ and x_1: this is analogous to path A and path B in the Mach-Zehnder interferometer. Let us consider the initial $|\uparrow\rangle$ state for the x_1 position: which-way detector on. One can write the $|\uparrow\rangle$ in the linear combination of eigenvectors of Pauli spin component σ_z for a one-half spin, the $|+\rangle$ and $|-\rangle$ and consider the time evolution with the positions as eigenvalues in equation 16.

$$|\chi(0)\rangle = \begin{pmatrix}1\\0\end{pmatrix} \tag{15}$$

$$|\chi(t)\rangle = \frac{1}{2}(e^{\frac{-ix_1 t}{\hbar}}\begin{pmatrix}1\\1\end{pmatrix} + e^{\frac{ix_1 t}{\hbar}}\begin{pmatrix}1\\-1\end{pmatrix}) \tag{16}$$

Equation 13 and 14's equivalent representation of interference in the spin basis: using the simplified form of equation 16 as in equation 17 can be given as $|\chi(t)\rangle\langle\chi(t)|$.

$$|\chi(t)\rangle = \begin{pmatrix}cos(\frac{x_1 t}{\hbar})\\ isin(\frac{x_1 t}{\hbar})\end{pmatrix} \tag{17}$$

Equation 17 says that the superposition evolves, causing interference when the which-way detector is turned off. One can add an instantaneous phase factor ϕ to equation 17 as in equation 4 and obtain plus and minus waves as in Equations 13 and 14. We observe that the phase shift is an instantaneous change induced by the apparatus and does not contribute to a different quantum state: it does not say anything about the position probability. The wave function *evolves* in the x-basis and is *measured* in the z-basis.

Consider the photon's initial state in the x-basis as in equation 18 and evolves in the z-basis 19. Equation 19 shows a result of the same form as 17. Representing the photon on one basis for measurement and expanding it on another is only a mathematical constraint. One cannot choose the physical nature of the photon through the Hilbert space basis choice. The path information availability causes the photon's choice of physical nature, but the effect can precede the cause: this is an empirical observation.

$$|\chi(0)\rangle = \begin{pmatrix}1\\1\end{pmatrix} \tag{18}$$

$$|\chi(t)\rangle = \frac{1}{2}(e^{\frac{-ix_1 t}{\hbar}}\begin{pmatrix}1\\0\end{pmatrix} + e^{\frac{ix_1 t}{\hbar}}\begin{pmatrix}0\\1\end{pmatrix}) \tag{19}$$

Further, one can transform the equation 11 can be transformed into the momentum-coordinate system: a direct consequence of the uncertainty principle. The action of the position operator on a position wave function can be written according to equation 20. As a result, one can write equation 11 as in equation ; it is not difficult to see that this is a sufficient replacement for the spin-based representation.

$$x_1|\psi_A\rangle = i\hbar\frac{\partial}{\partial p}|\psi_A\rangle \quad (20)$$

$$\psi = \frac{1}{\sqrt{2}}e^{\frac{-ipx_1}{\hbar}} + \frac{1}{\sqrt{2}}e^{\frac{-ipx_2}{\hbar}} \quad (21)$$

Qureshi's analysis (33) is motivated primarily by the need to explain the delayed choice quantum eraser experiment without causality violations. Carlo Rovelli has observed that entropy is the only concept in physics that follows causality; many areas, including special relativity, can allow causality violations (35). Feynman's path integral formulation, essential to quantum field theory, can allow causality violations (11). Avoiding causality violations cannot be the only motivation behind an analysis; Qureshi does not specify what problems other than causality violations exist in the state-of-the-art analysis of the delayed choice quantum eraser experiments.

The aspects of reality are space and time. There are many studies on the study of the constituent aspects of space; quantum mechanics corresponds to a set of experiments of such constituent aspects. Theoretically and experimentally, quantum mechanics is well known to allow causality violations. One can say that just like space, time can also follow a separate set of rules for quantum mechanics in contrast with classical mechanics as posited by Bohr (6); our paper reiterates this fundamental aspect of the Complementarity principle. Studying the constituent aspects of time just as scientists have done for space would be necessary.

9 Conclusion

The biggest problem with Qureshi's analysis (33): the screen in quantum mechanical experiments can be designed as a single entity. This is because even if the analysis were considered in the many-worlds interpretation, where the classical-quantum distinction is nonexistent, the arguments of the analysis would not survive. Next, attributing quantum characteristics to instantaneous phase change aggravates this further: classical phase change does not affect the probabilities of path choice: this is a very poignant result. Because Qureshi envisions that the classical phase can contribute to a distinct state, he ignores the role played by Hilbert space in quantum mechanics. Hilbert space predicts probabilities of the photon observables in the particle nature of quantum mechanics; the basis is merely a skeletal structure to enable that. One cannot obtain this 'framework' from the wave evolution: which is equivalent to an undisturbed quantum system, not a measured one. There is a significant mathematical gap between wave and particle nature: wave nature can not be derived from particle nature.

Qureshi's central concept: experimenter chooses to show the which-way information: all of his arguments are designed to support this claim. It is convenient for Qureshi when he argues that entanglement alone is enough to destroy the interference: an attempt to downgrade the which-way information's role and upgrade the experimenter's active role. We emphasize the significance of the photon's random choice of wave-particle duality in the same experimental setup (23); we observe that the photon behavior is determined by the experimental setup alone. Since wave and particle detection can be random, the experimenter can not choose to show the wave nature in one instance and the particle nature in another and thus can not actively decide whether the photon will show wave or particle nature. The experimenter's only explicit role is to design the experiment to enable wave and particle detection. Qureshi's argument: entanglement alone can destroy interference is also negated by the above experiment, which uses an entangled photon pair: one for the screen and the other for the which-way detector (23).

The ideological basis for Qureshi's argument is classical-quantum entanglement. The classical-quantum distinction is critical to Complementarity. Photon entanglement with experimental apparatus does not allow collapse and duality. The which-way detector in state-of-the-art experiments have been classical systems, a quantum which-way detector proposed by Qureshi (33) must be implemented in practice; we observe that this is unlikely to follow Qureshi's expectations. We observe that the qubit which-way detector is unnecessary for the analysis: when Qureshi considers the detectors that constitute screen quantum mechanically. He can easily consider the same classical-quantum entanglement methodology for any macroscopic which-way detector already available in practice.

Qureshi is dogmatic to remove the notion of causality violations. Our primary intention has not been to pass a verdict on causality violations but to critically elucidate the issues in Qureshi's analysis in the purview of Copenhagen interpretation and Complementarity. We observe that Qureshi's attempts to maintain causality fails to follow the Copenhagen interpretation and is not cogent with empirical observations.

Acknowledgements

The author thanks the Amrita Vishwa Vidyapeeth University for its support.

References

[1] P Achuthan and Karuppath Narayanankutty. On mathematical modeling of quantum systems. In *AIP Conference Proceedings*, volume 1146, pages 105–112. American Institute of Physics, 2009. URL: https://doi.org/10.1063/1.3183535.

[2] CO Alley, O Jakubowicz, CA Steggerda, and WC Wickes. A delayed random choice quantum mechanics experiment with light quanta. In *Proceedings of the international symposium foundations of quantum mechanics in the light of new technology*, 1984. URL: https://inis.iaea.org/search/search.aspx?orig_q=RN:16072640.

[3] Ulrik L Andersen, Oliver Glöckl, Stefan Lorenz, Gerd Leuchs, and Radim Filip. Experimental demonstration of continuous variable quantum erasing. *Physical review letters*, 93(10):100403, 2004. URL: https://link.aps.org/doi/10.1103/PhysRevLett.93.100403, doi:10.1103/PhysRevLett.93.100403.

[4] Lon Becker. That von neumann did not believe in a physical collapse. *British Journal for the Philosophy of Science*, 55(1), 2004. URL: https://doi.org/10.1093/bjps/55.1.121.

[5] Mani Bhaumik. Is the quantum state real in the hilbert space formulation? *Quanta*, 9(1):37–46, 2020. URL: http://quanta.ws/ojs/index.php/quanta/article/view/142, doi:10.12743/quanta.v9i1.142.

[6] Niels Bohr. Causality and complementarity. *Philosophy of Science*, 4(3):289–298, 1937.

[7] Niels Bohr et al. *The quantum postulate and the recent development of atomic theory*, volume 3. Printed in Great Britain by R. & R. Clarke, Limited, 1928. URL: https://www.informationphilosopher.com/solutions/scientists/bohr/Como_Nature.pdf.

[8] Richard N Boyd. On the current status of the issue of scientific realism. *Methodology, Epistemology, and Philosophy of Science: Essays in Honour of Wolfgang Stegmüller on the Occasion of His 60th Birthday, June 3rd, 1983*, pages 45–90, 1983. URL: https://doi.org/10.1007/978-94-015-7676-5_3.

[9] Bernard d'Espagnat. Reply to ka kirkpatrick. *arXiv preprint quant-ph/0111081*, 2001. URL: https://arxiv.org/abs/quant-ph/0111081.

[10] Paul Adrien Maurice Dirac. *The principles of quantum mechanics*. Number 27. Oxford university press, 1981. URL: https://global.oup.com/academic/product/the-principles-of-quantum-mechanics-9780198520115?cc=cn&lang=en&.

[11] John F Donoghue and Gabriel Menezes. Arrow of causality and quantum gravity. *Physical Review Letters*, 123(17):171601, 2019. URL: https://journals.aps.org/prl/pdf/10.1103/PhysRevLett.123.171601.

[12] Jonathan Simon Eakins. Classical and quantum causality in quantum field theory. or," the quantum universe". *arXiv preprint gr-qc/0408037*, 2004. URL: https://eprints.nottingham.ac.uk/10069/1/sophie.pdf.

[13] Hugh Everett III. " relative state" formulation of quantum mechanics. *Reviews of modern physics*, 29(3):454, 1957. URL: https://link.aps.org/doi/10.1103/RevModPhys.29.454, doi:10.1103/RevModPhys.29.454.

[14] Johannes Fankhauser. Taming the delayed choice quantum eraser. *arXiv preprint arXiv:1707.07884*, 2017. URL: https://arxiv.org/abs/1707.07884.

[15] RP Feynman. Feynman lectures on physics. vol. 3. quantum mechanics. le cours de physique de feynman. vol. 3. mecanique quantique. 1979. URL: https://www.osti.gov/etdeweb/biblio/5667847.

[16] Jay Gambetta, Alexandre Blais, David I Schuster, Andreas Wallraff, L Frunzio, J Majer, Michel H Devoret, Steven M Girvin, and Robert J Schoelkopf. Qubit-photon interactions in a cavity: Measurement-induced dephasing and number splitting. *Physical Review A*, 74(4):042318, 2006. URL: https://rsl.yale.edu/sites/default/files/physreva.74.pdf.qubit-photon_interactions_0.pdf.

[17] Philippe Grangier, Gerard Roger, and Alain Aspect. Experimental evidence for a photon anticorrelation effect on a beam splitter: a new light on single-photon interferences. *Europhysics Letters*, 1(4):173, 1986. URL: https://iopscience.iop.org/article/10.1209/0295-5075/1/4/004/meta.

[18] George Greenstein and Arthur Zajonc. *The quantum challenge: modern research on the foundations of*

quantum mechanics. Jones & Bartlett Learning, 2006.

[19] David J Griffiths and Darrell F Schroeter. *Introduction to quantum mechanics*. Cambridge university press, 2018.

[20] Robert B Griffiths. Consistent histories and quantum delayed choice. *Fortschritte der Physik: Progress of Physics*, 46(6-8):741–748, 1998. URL: https://plato.stanford.edu/archives/sum2019/entries/qm-consistent-histories/.

[21] Narayanan Kutty Karuppath and Achuthan Panajikunnath. Quantum nonlocality, einstein-podolsky-rosen argument, and consciousness. *NeuroQuantology*, 8(2), 2010. URL: https://www.jneuroquantology.com/index.php/journal/article/view/289/287.

[22] Manpreet Kaur and Mandip Singh. Quantum double-double-slit experiment with momentum entangled photons. *Scientific Reports*, 10(1):11427, 2020. URL: https://www.nature.com/articles/s41598-020-68181-1.

[23] Yoon-Ho Kim, Rong Yu, Sergei P. Kulik, Yanhua Shih, and Marlan O. Scully. Delayed "choice" quantum eraser. *Phys. Rev. Lett.*, 84:1–5, Jan 2000. URL: https://link.aps.org/doi/10.1103/PhysRevLett.84.1, doi:10.1103/PhysRevLett.84.1.

[24] KA Kirkpatrick. Indistinguishability and improper mixtures. *arXiv preprint quant-ph/0109146*, 2001. URL: https://arxiv.org/abs/quant-ph/0109146.

[25] Jarrett Leplin. *Scientific realism*, volume 323. Univ of California Press, 1984.

[26] Xiao-Song Ma, Johannes Kofler, Angie Qarry, Nuray Tetik, Thomas Scheidl, Rupert Ursin, Sven Ramelow, Thomas Herbst, Lothar Ratschbacher, Alessandro Fedrizzi, et al. Quantum erasure with causally disconnected choice. *Proceedings of the National Academy of Sciences*, 110(4):1221–1226, 2013. URL: https://doi.org/10.1073/pnas.1213201110.

[27] Xiao-song Ma, Johannes Kofler, and Anton Zeilinger. Delayed-choice gedanken experiments and their realizations. *Reviews of Modern Physics*, 88(1):015005, 2016. URL: https://link.aps.org/doi/10.1103/RevModPhys.88.015005, doi:10.1103/RevModPhys.88.015005.

[28] Efstratios Manousakis. Founding quantum theory on the basis of consciousness. *Foundations of Physics*, 36(6):795–838, 2006. URL: https://doi.org/10.1007/s10701-006-9049-9.

[29] N David Mermin. Is the moon there when nobody looks? reality and the quantum theory. *Physics today*, 38(4):38–47, 1985. URL: https://doi.org/10.1063/1.880968.

[30] Ulrich Mohrhoff. Restoration of interference and the fallacy of delayed choice: Concerning an experiment proposed by englert, scully, and walther. *American Journal of Physics*, 64(12):1468–1475, 1996. URL: https://doi.org/10.1119/1.18411.

[31] Mehwish Nawaz, Hinna Tariq, Manzoor Ikram, et al. Delayed-choice tunable quantum eraser under cavity qed mach-zehnder setup. *Optics Communications*, 399:13–15, 2017. URL: https://doi.org/10.1016/j.optcom.2017.04.046.

[32] Tao Peng, Hui Chen, Yanhua Shih, and Marlan O Scully. Delayed-choice quantum eraser with thermal light. *Physical review letters*, 112(18):180401, 2014. URL: https://link.aps.org/doi/10.1103/PhysRevLett.112.180401, doi:10.1103/PhysRevLett.112.180401.

[33] Tabish Qureshi. Demystifying the delayed-choice quantum eraser. *European Journal of Physics*, 41(5):055403, 2020. URL: https://iopscience.iop.org/article/10.1088/1361-6404/ab923e, doi:10.1088/1361-6404/ab923e.

[34] George L Roberts, Marco Lucamarini, James F Dynes, Seb J Savory, ZL Yuan, and Andrew J Shields. Modulator-free coherent-one-way quantum key distribution. *Laser & Photonics Reviews*, 11(4):1700067, 2017. URL: https://onlinelibrary.wiley.com/doi/full/10.1002/lpor.201700067.

[35] Carlo Rovelli. *The order of time*. Penguin, 2019. URL: https://www.penguinrandomhouse.com/books/551483/the-order-of-time-by-carlo-rovelli/.

[36] Wolfgang Rueckner and Joseph Peidle. Young's double-slit experiment with single photons and quantum eraser. *American Journal of Physics*, 81(12):951–958, 2013. URL: https://pubs.aip.org/aapt/ajp/article-abstract/81/12/951/1044320/Young-s-double-slit-experiment-with-single-photons?redirectedFrom=fulltext.

[37] Giuliano Scarcelli, Yu Zhou, and Yanhua Shih. Random delayed-choice quantum eraser via two-photon imaging. *The European Physical Journal D*, 44:167–173, 2007. URL: https://doi.org/10.1140/epjd/e2007-00164-y.

[38] Marlan O. Scully and Kai Drühl. Quantum eraser: A proposed photon correlation experiment concerning observation and "delayed choice" in quantum mechanics. *Phys. Rev. A*, 25:2208–2213, Apr 1982. URL: https://link.aps.org/doi/10.1103/PhysRevA.25.2208, doi:10.1103/PhysRevA.25.2208.

[39] Abner Shimony. Role of the observer in quantum theory. *American Journal of Physics*, 31(10):755–773, 1963. doi:10.1119/1.1969073.

[40] Henry Pierce Stapp. The copenhagen interpretation. *American journal of physics*, 40(8):1098–1116, 1972. URL: https://doi.org/10.1119/1.1986768.

[41] Ian Stewart. Grandfather paradox. *Nature*, 464(7293):1398–1398, 2010. URL: https://www.nature.com/articles/4641398a.

[42] Christopher G Timpson and Harvey R Brown. Proper and improper separability. *International Journal of Quantum Information*, 3(04):679–690, 2005. URL: https://doi.org/10.1142/S0219749905001493.

[43] Frank J Tipler. Nonlocality as evidence for a multiverse cosmology. *Modern Physics Letters A*, 27(04):1250019, 2012. URL: https://doi.org/10.1142/S0217732312500198.

[44] Frank J Tipler. Quantum nonlocality does not exist. *Proceedings of the National Academy of Sciences*, 111(31):11281–11286, 2014. URL: https://doi.org/10.1073/pnas.1324238111.

[45] Paul D Townsend, JG Rarity, and PR Tapster. Single photon interference in 10 km long optical fibre interferometer. *Electronics Letters*, 7(29):634–635, 1993. URL: https://www.infona.pl/resource/bwmeta1.element.ieee-art-000000211832.

[46] Lev Vaidman. Many-Worlds Interpretation of Quantum Mechanics. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Fall 2021 edition, 2021. URL: https://plato.stanford.edu/archives/fall2021/entries/qm-manyworlds.

[47] John Von Neumann. *Mathematical foundations of quantum mechanics: New edition*, volume 53. Princeton university press, 2018. URL: https://press.princeton.edu/books/hardcover/9780691178561/mathematical-foundations-of-quantum-mechanics.

[48] John Archibald Wheeler. The "past" and the "delayed-choice" double-slit experiment. In *Mathematical foundations of quantum theory*, pages 9–48. Elsevier, 1978. URL: https://doi.org/10.1016/B978-0-12-473250-6.50006-6.

[49] Eugene P. Wigner. The Problem of Measurement. *American Journal of Physics*, 31(1):6–15, 01 1963. doi:10.1119/1.1969254.

[50] Nouredine Zettili. Quantum mechanics: concepts and applications. 2009. URL: https://books.google.co.in/books?hl=en&lr=&id=TAjVyzm57YwC&oi=fnd&pg=PR5&dq=quantum+mechanics+zettili&ots=-nbLm3Jt6Y&sig=h19fPlgayULdi4RsMVhJtNB84Fs#v=onepage&q=quantum%20mechanics%20zettili&f=false.