

Complementarity principle cannot disallow causality violations

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Wave-particle duality of the photon is correlated with the availability its which-way information. The effects of this correlation are causality violating as evident in the infamous delayed choice quantum eraser experiment. It has been argued that causality violations are a result of not assigning quantum characteristics to macroscopic detectors. In this paper, we point out that this contradicts with the complementarity principle. However, the many-worlds interpretation, characterised by the universal wave function, where the entire universe is quantum mechanical readily accepts the idea of macroscopic quantum detectors. Thus an attempt to overcome retrocausality within the Copenhagen interpretation using macroscopic quantum detectors and attribution of ontological features to the Hilbert space formalism, would naturally take the narrative to the many worlds interpretation.

I. INTRODUCTION

The delayed choice quantum eraser(here after as DCQE), was devised as a thought experiment [1, 2] and fully realized in a concise form for the first time [3]. The experiment has been revolutionary as it challenges the way we perceive time. It is quite curious that an experiment can possibly demonstrate that the present is influenced not just by the past but by future events as well. The original idea behind the experiment [2] uses Bohr's complementarity principle to explain the quantum eraser experiment. A key consequence of this is another highly discussed idea of 'consciousness' causing the collapse of the quantum wave function to its complementary particle form [1]. Modern Scientific thinking has strongly espoused realism in general. Realism is the idea that scientific results are 'mind-independent.' However, certain quantum mechanical views have been trying to imply that quantum mechanics is not mind-independent [4] [5] [6] [7] [8]. At the very heart of such a narrative is the young's double slit experiment [9]. If the experimenter knows which slit the photon travels through, one will see a clump pattern corresponding to the particle nature of the photon. If the experimenter is unaware of the path, there will be an interference pattern on the screen. An additional paradox this observer-centric notion is associated with is back-action or, more formally, nontemporality, where present actions can be influenced by the future.

There have been some attempts outside the Copenhagen interpretation to explain the DCQE without back-action using the Relative States [10], consistent histories [11] and Bohmian mechanics [12]. The Complementarity principle as the fundamental reason for the DCQE proposed by [2] has been challenged by [13, 14], who consider momentum kick in the context of the uncertainty principle. Accordingly, the uncertainty principle is the basic idea behind the DCQE. But here again, the narrative is nontemporal. Additional considerations like the widely discussed improper mixed states [15–17], while still maintaining the basic concepts within the Copenhagen interpretation have been used to disprove temporal non-locality [18]. Tabish Qureshi [19] attempts to denounce back action within the original complementarity principle narrative [2]. However, there is one additional consideration, a macroscopic detector that is quantum mechanical. At the end of our study, we conclude that if complementarity was the first principle in the phenomenology to explain the DCQE, nontemporality is an inalienable part of it. We also conclude that Qureshi's attempt to grant quantum mechanical characteristics to the macroscopic instruments and ontological significance to the Hilbert space formalism will make the Many Worlds Interpretation a much more suitable alternative to the Copenhagen Interpretation.

The paper is designed as follows, we start with the basic ideas postulated in [19]. This is followed by rewriting the experimental implementation of DCQE [3] in the same formulation as [18, 19]. Here, we explain why [19] is beyond the scope of the Copenhagen interpretation and must be modified to find its place in Everettian quantum mechanics.

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II. BASIC ARGUMENTS

The idea proposed by [19] to denounce back action can be simply summarised. According to [19], the paradox of retrocausality arises because we do not recognize that it is the measurement basis that decides the availability of the which way information. To understand this in some detail, let us consider the simplified version of [3] fig1, because, in this context, the critical details of the experimental implementation are not absolutely necessary. In any case, those details have been covered elsewhere, say, for example, [2, 3, 10, 19, 20]. There are two sources, one in front of each slit of a young's double slit fig1. Each source produces a photon, which is entangled by the entangler into a pair of entangled photons that pass through the double slits. One of the entangled photons travels to the screen, showing either interference or a clumped pattern. The other goes to the detector which will tell us which source produced the photon under consideration or not give us any information at all. Evidently, there is a correlation between this which-way information and the result on the screen. The different colors in the box in fig1 represent the different configurations of the mirrors and detectors in [3], corresponding to the different states(path information) of the entangled photon pair. The orange indicates the photon originated from the atom placed near slit A and vice versa for B, where we use blue instead of orange. This arrangement is the representation of the DCQE using the double slit [3] in the methodology of what Qureshi described for the Mach-Zehnder interferometer version[19, 21]. If the path information is available, you will find clicks in both the detectors. However if the lower box(which-way detector) in fig1 does not tell us the exact path, instead lies in a superposition state of both path A and path B, then only one detector will show clicks. This corresponds to the interference pattern of bright and dark fringes. This is indicated as a combination of up and down arrows in the lower box fig1. For the particle pattern one will see clicks on both the detectors(screen) and for interference pattern only one of the detectors(screen) will show clicks(top box in fig1).

All is hunky-dory when you keep the screen and the which-way detector at the same distance. However, what becomes interesting is when the distance between the screen and the slit is shorter than the distance between the which-way detector and the slit. When one of the entangled photons hits the screen, there is no which-way information available and you would expect an interference pattern. Surprisingly you see a particle pattern. Even more surprisingly, the path information is available in the which-way detector, much later after the particle pattern has been registered on the screen. This can mean only one thing, future actions have influenced the present.

But, Tabish Qureshi [19], says you don't have to freak out. What is happening is that measurement influences the appearance and disappearance of the which-way information. According to Qureshi, the effect measurement has on the result is evident in the mathematical basis used to represent the measurement. In this way, measurement causes the apparent paradox and thus there is no retrocausality. When the which way information is available, the mathematical representation corresponding to it is the z-basis. And when the information disappears, the mathematical formalism is the x-basis or the plus-minus basis. Consider that the screen is closer to the source than the which way detector and if the plus-minus basis is measured, one loses the ability to recover the which-way information. Hence the particle detection is not observed. On the other hand, if you measure using the z-basis, the which-way information appears. And consequently the particle detection takes place once again. Therefore, irrespective of the relative positions of the screen and detector with respect to the source, it is the measurement basis which decides the result. Thus Qureshi concludes that there is no question of causality violation. In addition, Qureshi further claims that if one observes the screen data carefully one can obtain the information of which basis was used in the measurement. This he cites as an evidence to his claim that the measurement basis influences the result rather than any other factor.

III. THE QUANTUM ONTOLOGY

One of the common conceptions in quantum mechanics is the notion of interference resulting from the photon traveling through both slits and this resulting in interference. However, there is no experimental evidence of the fact that the photon travels through both slits. We only indirectly infer that the photon travels through both slits. Many pragmatic physicists and chemists prefer to accept the notion of a photon traveling through both slits purely based on convenience [22]. One can call it the popular Shut Up And Calculate(SUAC) [23]. The most intuitive way of understanding this would be to consider probabilities in a quantum superposition. We only have probabilities of which slit the photon would travel through. The interference resulting is an interference of probabilistic amplitudes rather than the typical classical wave; this is a well-known fact. It is a lack of knowledge of the *potential* path information about which slit the photon passes through that results in interference [24]. There is no direct empirical evidence that the photon does travel through both slits. This is because there is no availability of photon trajectory. We do not know what really happens, but the information similar to the availability of the path decides the wave or particle nature.

In [25–27], it is explained how the mathematical formulation of quantum mechanics can be used only to predict the probability of the evolution of things and fully understand the evolution of things themselves. Moreover, the development of Hilbert space formalism was conceived purely with the idea of ensuring compliance with the probability interpretation or the statistical interpretation of quantum mechanics [28]. This was necessary because the classical physics based canonical transformation approach did not ensure compliance with the probabilistic nature of quantum mechanics [28]. Consequently, there is no physical meaning similar to that of classical physics for the Hilbert space quantum formulation, just the prediction of probabilities. The basis in the Von Neuman’s Hilbert space formulation does not represent any physical change and only predicts probabilities. The most intuitive way to understand this is to consider the fact that the Hilbert space is a complex space. There is no way one can represent ontology using complex or imaginary numbers. To throw some more insight into this, a well known mathematical process is the Fourier transform, where one takes a given equation, transforms them into a complex space, where it is easier to solve. Then the solution is inverse transformed into the real number space. The complex space is clearly only a mathematical or epistemological construct. In quantum mechanics the default space itself is complex, hence does not represent any ontology.

There are certain key points about Bohr’s complementarity principle elucidated in [25, 26], which contradict with [19] claim of the quantum nature of the macroscopic detector. The detectors are classical in nature, and the statistical results of quantum mechanics are a direct consequence of the distinction between the classical nature of the macroscopic detector and the quantum nature of the photon. Hence there is no such thing as macroscopic detector states, in the complementarity principle. Detectors are classical in nature. They are merely a tool to observe or measure the duality of the quantum entity. Mathematical formalism has one job, and that is to predict the probabilities of quantum states. In the end, the preparation of the apparatus has total control over temporality [2]. If the apparatus is prepared to allow retrocausality, the result will indeed be retrocausal [2]. That is the basic idea of the complementarity principle.

The whole concept of conferring quantum status to the classical apparatus had been considered by Von-Neuman in his original formulation itself [27]. In an attempt to develop his well known composite systems formalism, Von-Neuman comes up with a concept of psycho-physical parallelism, an idea which has many resemblances to the relative states or the Many Worlds interpretation [27, 29, 30]. The composite systems formulation is the idea of microscopic macroscopic entanglement which is key to understanding the quantum which-way detectors in [19]. However, Stapp [25] has indicated that the composite systems formulation does not comply with the Copenhagen interpretation. The whole idea of assigning quantum attributes to classical apparatus has been vehemently criticised [25]. The key points in [25] are strongly relevant to our discussion:

1. It is practically impossible to conceive collapse for macroscopic systems. One can not say that the world exists in a superposition state and it collapses when we perceive it. In other words, one can not say that the moon exists only when one looks at it [31].
2. There is no clarity in the distinction between the classical and quantum states. Nor is there any clear ontology defined for the quantum to classical transition. In fact this is an active area of research, a fine example of it is the idea of quantum Darwinism [32]. However this falls into the ambit of Everettian quantum formulation [5].

IV. MATHEMATICAL NUANCES OF THE DCQE

In [3] the path taken by the photon is key to the understanding of the experiment. However, in [19] the emphasis is on the quantum state of the which-way detector.

We will first look at the [3] version. There is a clear emphasis on the path and the entanglement in the biphoton amplitude [33–35] is abstracted. For the sake of clarity the main components of the mathematical formulation in [3] are repeated here.

$$\begin{aligned}\Psi(t_0, t_1) &= A(t_0, t_1^A) + A(t_0, t_1^B), \\ \Psi(t_0, t_2) &= A(t_0, t_2^A) - A(t_0, t_2^B),\end{aligned}\tag{1}$$

$$\Psi(t_0, t_3) = A(t_0, t_3^A), \quad \Psi(t_0, t_4) = A(t_0, t_4^B),\tag{2}$$

Here the indices represent the path taken by the photon, upper index shows the information about the atom of origin (path A or B) and the lower indices represent the which-way detector. Contrast this with [19], where the which-way detector is given a quantum characteristic in the form of a qbit. In the case of [3], the biphoton Amplitude abstracts the entanglement of the photons that reach the screen and the which-way detectors. In [19] the two

entangled photons are represented explicitly. This helps Qureshi in assigning quantum mechanical characteristics to the which-way detector as he treats them as a single qbit fig1. Although Qureshi designed his analysis for the Mach-Zhender interferometer version of the DCQE [21], as he himself notes [19], the analysis is compatible with the double-slit version [3] as well.

$$|\Psi_F\rangle = \frac{1}{2} [(|D_1\rangle + |D_2\rangle)|\uparrow\rangle + (|D_1\rangle - |D_2\rangle)|\downarrow\rangle]. \quad (3)$$

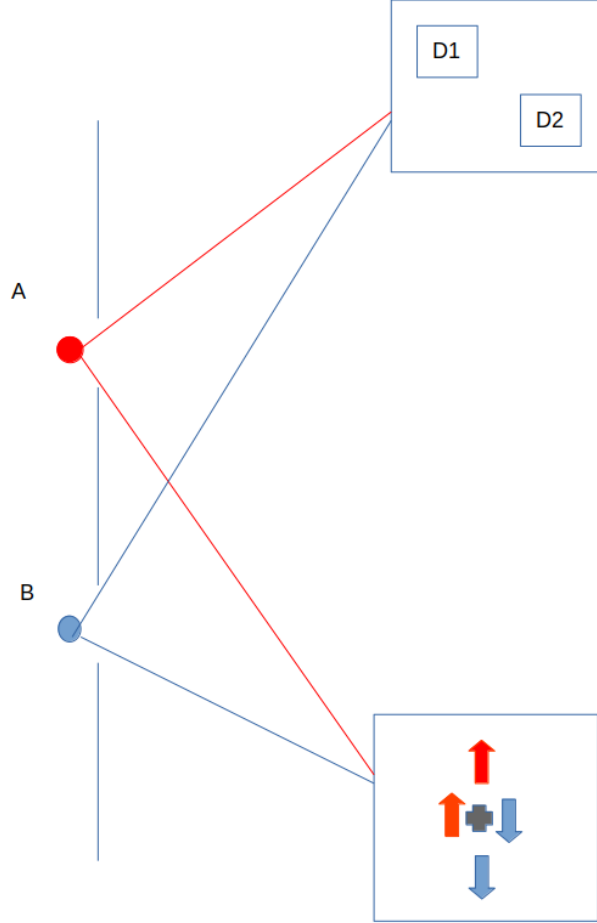


FIG. 1: The double-slit version [3] of the DCQE based on the quantum which-way detector model for the Mach-zehnder version. [19, 21]. The box on the top represents the screen and the bottommost box represents the which-way detector.

In eqn3, the $\uparrow$ and $\downarrow$ represent the which-way qbit. In this case where the which way information is available, the detector screens D_1 and D_2 are both equally probable and when many photons strike the detectors. One can see hits on both the detectors and evinces the particle nature of the photon. In our analysis, this means that the which-way box in fig1 gives the path information (either the up or down arrow). In [19] the qbit represents the case the photon passes through either slit A or B. However, there is a need to explicitly include the case that the which-way information is not available and the which-way detector lies in a superpositional state (combination of both the arrows fig1). Mathematically this can be represented as in [19]:

$$|\Psi_F\rangle = \frac{1}{2} [|D_1\rangle(|\uparrow\rangle + |\downarrow\rangle) + |D_2\rangle(|\uparrow\rangle - |\downarrow\rangle)]. \quad (4)$$

In this case the which-way information is unavailable, which we represent using the superposition of the up and down arrows implying that the detector lies in a superposition. This way of representation is more apt than in [19], where the superposition is not explicitly represented 1. It is noteworthy that the final result is very much similar in

both the formalisms [3, 19]. It comes down to the question of whether the detector can be represented as a quantum system or not.

The composite systems formulation has its origins in the psycho-physical parallelism [27] and has taken the form of relative states [5], this has been explained in the work [36]. More specifically one can consider [37]. Here quantum entanglement is explained in the Many Worlds Interpretation formalism. The key to the idea is the entanglement of the quantum system with the measurement apparatus. The Many Worlds Interpretation has been applied to the DCQE by Sean Carroll in his unpublished work [38]. In this case although Sean brings up the Universal Wave function, he does not say anything in detail about the splitting of the worlds. The branching or splitting of the worlds is a key aspect of the Many Worlds Interpretation. However, Sean does recognize that the Copenhagen interpretation will conceptualize a retrocausal narrative.

We will try a slightly more rigorous implementation of the Many Worlds interpretation of the DCQE using [37]. Let the photon be denoted as P. Here the pair of entangled photons. We will show how disadvantages in the formalism of [19] in eqn3 and eqn4 can be remedied when analyzed in the purview of the Many Worlds Interpretation.

$$|\Psi_I\rangle = \frac{1}{2}(D_1^1() + D_2^1()) \otimes P^1(\uparrow) \otimes P^2(\uparrow) \otimes \frac{1}{2}(C_A^2() + C_B^2()) + \frac{1}{2}(D_1^1() + D_2^1()) \otimes P^1(\downarrow) \otimes P^2(\downarrow) \otimes \frac{1}{2}(C_A^2() + C_B^2()) \quad (5)$$

In eqn5 $P(\uparrow)$ says that the photon has its origin in the atom A. The upper indices 1 and 2 indicate the respective photon within the entangled pair. 1 for the screen and 2 for the which-way detector. In C_A and C_B the lower indices represent the respective mirror configuration corresponding to the respective path of the photon, a concise representation of the mirror arrangement in [3]. The upper indices also indicate the compatibility of entanglement. However in the case of $P^2(\uparrow)$ and $P^2(\downarrow)$, the entanglement is compatible only with C_A and C_B respectively. Hence the entanglement of $P(\uparrow) \otimes C_A(\uparrow)$ can be written concisely as $P(\uparrow\uparrow)$ and vice-versa. This is the same as the qbit detector described in [19]. However, we note that such a qbit detector can exist only with the macroscopic-microscopic entanglement as in the Many-Worlds Interpretation. This kind of entanglement plays a key role in the well known phenomenon of decoherence [39]. On the other hand, if the state of the photon $P(\uparrow)$ were to collapse, then such a representation becomes impossible. Thus the Copenhagen interpretation simply does not allow quantum representation of macroscopic detectors. Hence the Many Worlds Interpretation provides a different ontology: branching of the worlds.

$$|\Psi_I\rangle = \frac{1}{2} [(|D_1()\rangle + |D_2()\rangle)|P(\uparrow\uparrow)\rangle + (|D_1()\rangle - |D_2()\rangle)|P(\downarrow\downarrow)\rangle]. \quad (6)$$

The photon from the respective slits that carry the path information. The empty brackets in eqn6 indicate that they are still a part of the universal wave function and that they have not split yet. If the detection takes place for the photon coming from the slit A, the result would be an entanglement with the detector D_1 . In a similar pattern there are four such worlds as given below. In addition the screen and the which-way detectors $P(\uparrow\uparrow)$, along with the respective versions of the observer, also split into correlated worlds.

$$|\Psi_1\rangle = |D_1(\uparrow\uparrow)\rangle |P(\uparrow\uparrow)\rangle \quad (7)$$

$$|\Psi_2\rangle = |D_2(\uparrow\uparrow)\rangle |P(\uparrow\uparrow)\rangle \quad (8)$$

$$|\Psi_3\rangle = |D_1(\downarrow\downarrow)\rangle |P(\downarrow\downarrow)\rangle \quad (9)$$

$$|\Psi_4\rangle = |D_2(\downarrow\downarrow)\rangle |P(\downarrow\downarrow)\rangle \quad (10)$$

In each of the above cases, there will be a separate world or branching of the universal wave function. Note that the empty brackets have acquired a state and thus belong to a branch now. That is, every possibility branches into a distinct branch or reality. Not just that, further each of those distinct worlds will further split into a final pair of correlated worlds. Thus a total of eight worlds. And the bottom-most pair of worlds are correlated with each other, because of the entanglement as we show in fig2.

What really matters here is the fact that all of this narrative could be seen as a direct consequence of not tolerating duality and the collapse postulate. If you wish to eliminate the ontological concepts of collapse or duality, what one would get is plain mathematics. When one would like to develop a pure ontology using the Hilbert formalism, the most natural consequence is the Many Worlds Interpretation. The collapse postulate is actually a nonlinear addendum to the linear Hilbert space formalism. If you include collapse and duality, then the quantum characteristics of the detector are inconceivable. For this reason the [19] narrative must be modified in accordance with the Many Worlds Interpretation.

In addition to that, it is also noteworthy that the Many worlds Interpretation attaches no importance to the which-way or path information. This is a direct consequence of its non-adherence to the Copenhagen interpretation. Hence for that reason, the [19] attempting to use which-way information along with quantum detector needs a lot

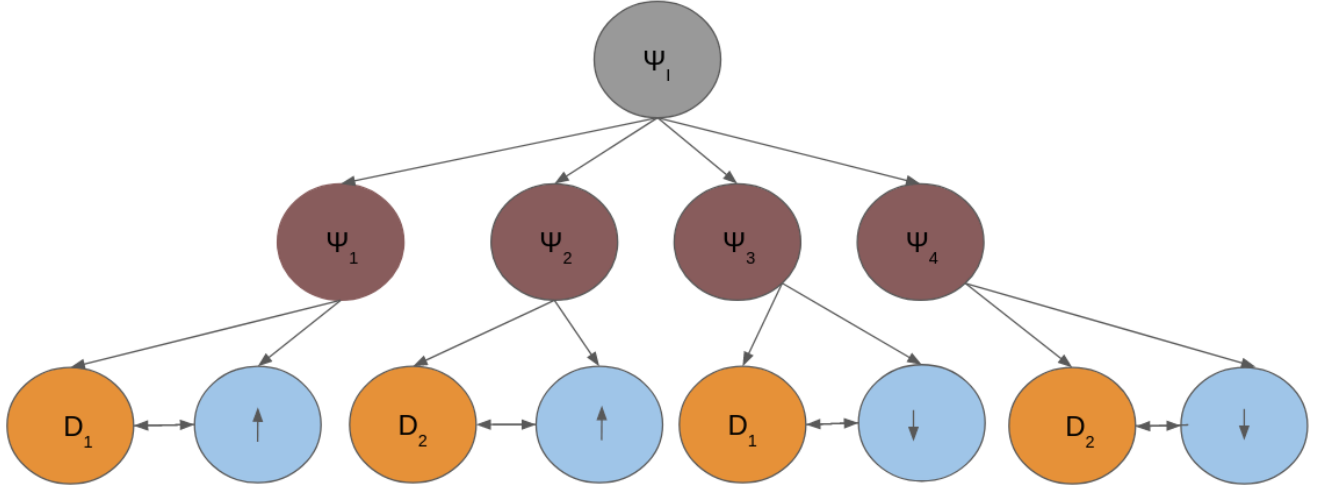


FIG. 2: The Branching of the Worlds 6. The double arrow indicates correlations.

of refinement. It is straight forward to see that if you rewrite eqn5 after eliminating all the macroscopic-microscopic entanglements, all that remains with a quantum mechanical characteristic is the path of the photon. This would lead to ontology as in eqn2. This is a direct consequence of Bohr's statistical interpretation where the distinction between the classical and quantum states is what leads to the statistical or probabilistic results [26]. Collapse is an inevitable consequence of such a paradigm. Either collapse or microscopic-macroscopic entanglement. Both these ontologies are mutually exclusive.

The which-way information is the key to appearance and reappearance of interference fringes [2]. The DCQE [3] is based on this particular concept. However, [13] posited that the disappearance of the fringes is due to the momentum kick imparted to the detector. In the case of the Many Worlds Interpretation, there is no uncertainty principle. This may seem amenable to the which-way information or complementarity principle narrative [2]. In addition in [2], it is noted that the arrangement of the apparatus or the setup can ensure causality violations. The many worlds interpretation also suggests that the experimental setup will have determine the result, but causality is maintained here [38]. However, the many worlds interpretation does not allow collapse nor does it have any scope for duality. In addition, there is an interesting claim made in both [19] and [38] that entanglement alone is enough to cause collapse. According [19], in the absence of the entanglement all the photons will hit the D_1 (wave). Once there is entanglement, there is a probability of $\frac{1}{2}$ for the photon to hit either detector(screen). In this case interference is lost merely because of the entanglement. However, this overlooks the fact that erasure of the which way information can bring back the interference. There is a clear correlation between the which way information and the result on the screen. Even in the presence of entanglement, it is well known that there is a loss of interference. This has a lot to do with the correlation with the which way information than entanglement. It is clear that the results of the DCQE experiment are dependent on the path information of photon [24]. In our analysis 2 using the model in [37] the correlation is recognized as a correlation between worlds.

V. THE PREFERRED BASIS PROBLEM

The preferred basis problem has been the Achilles heel of the Many Worlds Interpretation [40–42]. As explained in the previous sections, the Many Worlds Interpretation is dependent purely upon the Hilbert space formalism for its ontological arguments. But the basis states used to represent a quantum system can be manifold and are ambiguous. This is called the preferred basis problem. This is as strongly applicable to the [19] as the Many Worlds Interpretation. Here in [19], the claim is that there will be residual information left behind indicating which of the two different basis of the system will be evident in the residual information left behind.

$$|\Psi_F\rangle = \frac{1}{2} [|D_1\rangle|+\rangle + |D_2\rangle|-\rangle]. \quad (11)$$

According to [19] there is residual information left behind after the photon hits the screen. That is, the detectors

D_1 and D_2 carry the information about the state of the which way detector. The information about whether it was the $|+\rangle$ or the $|-\rangle$ is left behind. Typically in [19], one considers the x basis in eqn11 (For simplicity we are not including the normalization constants throughout this discussion). In [19] we have:

$$|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (12)$$

$$|-\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (13)$$

where $|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, which can be termed as the z states. Here the analogy is being drawn to quantum spin [19, 20]. Path A is treated as spin up, and vice versa for path B. But there are other basis which can be used to represent the same situation without any change in meaning. Let us write $|\uparrow\rangle = \begin{pmatrix} i \\ 0 \end{pmatrix}$ and $|\downarrow\rangle = \begin{pmatrix} 0 \\ i \end{pmatrix}$.

$$|+\rangle = \begin{pmatrix} i \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ i \end{pmatrix} \quad (14)$$

$$|-\rangle = \begin{pmatrix} i \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ i \end{pmatrix} \quad (15)$$

This is a typical example where the basis is represented using complex numbers. In fact, there are many such basis states possible. Even in [3] the basis representation is abstracted indicating that there can be many possible basis states to represent the quantum system under consideration. So the argument made by [19] is unlikely to be true, because the residual information is going to be ambiguous. There is no definite and specific information that can be possibly available. What is referred to as residual information of the basis-states in [19] is obviously and the interference. It does not carry any other information.

VI. CONCLUSION

The thought experiment by [19] is a very interesting analysis of the DCQE. It makes an attempt to nullify the retrocausal arguments of the experiment by giving quantum characteristics to macroscopic detectors. The whole idea of the DCQE in his analysis is based on duality as is the case with the original conception of the DCQE [2], except for the attribution of quantum characteristics to the macroscopic detector. In an attempt to explain the DCQE using a quantum bit detector, [19] extends beyond the philosophical boundaries of the wave particle duality and thus the Copenhagen interpretation. It enters into the realm of Everettian quantum mechanics. In such a scheme the DCQE may not be retrocausal. However, this design of his is not compatible with the Copenhagen Interpretation, which is the premise of his argument.

Throughout his work, Qureshi tries to attribute ontological characteristics to the Hilbert space formulation of quantum mechanics. His attempt to claim experimental proof for such an ontology suffers from the same fallacy as the Many Worlds Interpretation: the preferred basis problem. The most natural interpretation of quantum mechanics when you consider attributing ontological characteristics to the mathematical formulation of quantum mechanics is the Many Worlds Interpretation. In this paper using [37] we have shown clearly how the DCQE is interpreted in the Many Worlds Interpretation. There are many criticisms against the Many Worlds interpretation [41] including the preferred basis problem [40] which we have seen here in some detail.

The DCQE experiment if considered within the Copenhagen interpretation, must definitely be causality violating in nature. Any attempt to defy this rule is likely to take the narrative to some other interpretation. In this paper we have seen in detail an analysis where an attempt to give quantum characteristics to macroscopic detectors and ontological characteristics to Hilbert space formulation has taken the whole business to the Many Worlds Interpretation.

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