CLOCKS IN QUANTUM SCALE

TOMASZ KOBIERZYCKI
KOBIERZYCKITOMASZ@GMAIL.COM
APRIL 12, 2023

Abstract. In this paper I do present a model of quantum clock that follows a field equation. From that follows possible model of quantum gravity thus space-time in quantum scale.

Contents

1. Quantum Clock

On quantum scale time and space has to behave another way than they do in classical picture of space-time. Quantum clock is idea that measure of distance in space and time depends on rotation in complex plane of given complex tensor field. Full rotation goes back to same event, each clock carries how rotation changes compared to some units. So clock that needs one unit of distance to complete rotation it means that all events happen at one tick of a clock. If it's less than one it takes number of ticks to go back to starting point- and only some events happen at same time. This clock has to be quantized, it can only have a natural number of ticks to do full rotation. If n is number of ticks it takes to do a full rotation in complex plane it has to be a natural number. How long distance it takes for event to happen depends on it's energy. More energy complex tensor field has faster it rotates in complex plane so it reaches more events in one tick of a clock, each event can be mapped into complex plane as a point spanned by rotation of complex tensor field. It has to be a tensor field in order to be agreed with energy tensor that will have two indexes, vector field will only have one index so it wont match degrees of freedom of energy tensor that will have sixteen components in four dimensional complex space-time.

Clock defined this way has invariant part that is probability, let me denote a complex field as $\Psi_{\mu\nu}$ there is invariant probability that comes from it that can be written as a scalar:

$$\Psi_{\mu\nu}\Psi^{\mu\nu} = \Psi \tag{1.1}$$

This field is a complex four dimensional field so it transforms by four matrix that rotate it, it can be easy shown that it leads to spin two:

$$U^{\alpha}_{\mu}\left(\frac{\varphi}{2}\right)U^{\beta}_{\nu}\left(\frac{\varphi}{2}\right)\Psi_{\alpha\beta}\Psi^{\alpha\beta}U^{\mu}_{\alpha}\left(\frac{\varphi}{2}\right)U^{\nu}_{\beta}\left(\frac{\varphi}{2}\right) \tag{1.2}$$

One part of field transforms as spin one and another part transforms as spin one. I will get a scalar field as final result that has sixteen parts that sum, each part transforms as particle with spin two, but field itself from fact that it's a scalar field does not transform.It means that field itself acts as spin one field but it has two possible states of spin one, depending on using covariant or contravariant form of that field. Probability of finding particle in the field is equal to integral over space of that scalar It has to be normalized so probability of finding particle is equal to one at whole space:

$$\int \Psi_{\mu\nu}\Psi^{\mu\nu} d^3x = \int \Psi d^3x = 1 \tag{1.3}$$

2. Field Equation

There are two field equations for covariant and contravariant part of complex tensor field, they relate how covariant and contravariant derivative is connected to energy tensor that represents energy of matter field:

$$\left(\nabla_\mu \nabla_\nu + Q_{\mu\nu}\right) g_{\mu\nu} \Psi^{\mu\nu} = 0 \tag{2.1}$$

$$\left(\nabla^\mu \nabla^\nu + Q^{\mu\nu}\right) g^{\mu\nu} \Psi_{\mu\nu} = 0 \tag{2.2}$$

Where contravariant derivative is defined by using of metric tensor to raise an index:

$$\nabla^\mu = g^{\mu\alpha} \nabla_\alpha \tag{2.3}$$

Metric tensor is now a complex object that is defined in terms of complex numbers, otherwise it's same as normal metric tensor definition:

$$g_{\mu\nu} = \frac{\partial \xi^\alpha}{\partial z^\mu} \frac{\partial \xi^\alpha}{\partial z^\mu} \eta_{\alpha\beta} \tag{2.4}$$

Now I will add new scalar quality, that comes form field equation that scalar quality is number of clock ticks:

$$\nabla_\mu \nabla_\nu g_{\mu\nu} \Psi^{\mu\nu} = \Phi_{\mu\nu} \tag{2.5}$$

$$\nabla^\mu \nabla^\nu g^{\mu\nu} \Psi_{\mu\nu} = \Phi^{\mu\nu} \tag{2.6}$$

$$\Phi = \Phi_{\mu\nu} \Phi^{\mu\nu} = g_{\mu\nu} g^{\mu\nu} \nabla_\mu \nabla_\nu \Psi^{\mu\nu} \nabla^\mu \nabla^\nu \Psi_{\mu\nu} \tag{2.7}$$

This has also a probability build into it, so it's number of ticks time probability. Field equation reads that how field does change it's equal to it's energy tensor. This tensor can be understand as a tensor that says what is amount of energy stored in given rotation of on of tensor field in complex plane. Field equation itself is a tensor equation there parts $g^{\mu\nu} \Psi_{\mu\nu}$ and $g_{\mu\nu} \Psi^{\mu\nu}$ are parts of tensor field. Field equation can be rewritten:

$$g_{\mu\nu} \left(\nabla_\mu \nabla_\nu + Q_{\mu\nu}\right) \Psi^{\mu\nu} = 0 \tag{2.8}$$

$$g^{\mu\nu} \left(\nabla^\mu \nabla^\nu + Q^{\mu\nu}\right) \Psi_{\mu\nu} = 0 \tag{2.9}$$

Now it's clear that both terms in bracket act on a tensor complex field. Those two are complex field equations, when measured they turn into real fields. That turning into real fields gives a gravity effects and defines how distance is measured.

3. Gravity

Gravity field will tell what is shortest distance in space-time, first I define property of event that is it's energy times probability:

$$Q\Psi = Q_{\mu\nu}Q^{\mu\nu}\Psi_{\mu\nu}\Psi^{\mu\nu} \tag{3.1}$$

Now I will set a new event that I want to calculate distance to:

$$\frac{Q\Psi}{\Phi} = Q' \tag{3.2}$$

$$\frac{Q_{\mu\nu}Q^{\mu\nu}\Psi_{\mu\nu}\Psi^{\mu\nu}}{g_{\mu\nu}g^{\mu\nu}\nabla_\mu\nabla_\nu\Psi^{\mu\nu}\nabla^\mu\nabla^\nu\Psi_{\mu\nu}} = Q'_{\mu\nu}Q'^{\mu\nu} \tag{3.3}$$

Number Q' says how much ticks of a clock it takes to travel to event Q. It it's bigger than one it means that it will take distance equal to that number to travel to that event, it has to be natural number. If it's less than one it means that this event happens at once tick of a clock, so it's contained in that tick. Now to to calculate path object takes I need to vary this field for given path:

$$\delta\left(\int_P \frac{Q\Psi}{\Phi}\right) = 0 \tag{3.4}$$

$$\delta\left(\int_P \frac{Q_{\mu\nu}Q^{\mu\nu}\Psi_{\mu\nu}\Psi^{\mu\nu}}{g_{\mu\nu}g^{\mu\nu}\nabla_\mu\nabla_\nu\Psi^{\mu\nu}\nabla^\mu\nabla^\nu\Psi_{\mu\nu}}\right) = 0 \tag{3.5}$$

This creates gravity in this model. Now I want to calculate probability of that path it's equal to integral taken on that path:

$$\int_P \Psi_{\mu\nu}\Psi^{\mu\nu} \tag{3.6}$$

Combining them both I will get what is probability of given path, but I need to sum all possible paths that vary to zero:

$$\sum_i \delta\left(\int_{P_i} \frac{Q\Psi}{\Phi}\right) = 0 \tag{3.7}$$

Now probability of given path is equal to that integral for that path. So putting it all together, before measurement particle moves in all possible paths after it does only in one. Probability of given path is given by integral for that path:

$$\sum_i \int_{P_i} \Psi\delta\left(\int_{P_i} \frac{Q\Psi}{\Phi}\right) = 0 \rightarrow \delta\left(\int_{P_j} \frac{Q\Psi}{\Phi}\right) = 0 \tag{3.8}$$