On the coupling constant 3/2 of traditional kinetic theory of gases $PV = E = 3/2k_BT$

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The traditional kinetic theory of gases, which states that $PV = E = \frac{3}{2}k_BT$, should be corrected to $PV = E = k_BT$, as the process that leads to the coupling constants of 3 and $\frac{1}{2}$ has changed and is no longer necessary.

I. KINETIC THEORY OF GASES

The Kinetic theory of gases deals with the volume, pressure, and energy related to the thermal behavior of gases in a closed space. The most basic theory that explains this is given as follows:

$$PV = E_k = \frac{3}{2}k_BT, \tag{1.1}$$

where P is pressure, V volume, E_k kinetic energy, k_B Boltzmann constant, and T temperature. Here, the coupling constant $\frac{3}{2}$ is considered an essential coupling coefficient that appears when deriving the ideal gas law.

However, regarding the coupling coefficient of 3, the text on the "Boltzmann constant" [1], the energy E_k is given by

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}k_BT. \tag{1.2}$$

Therefore, considering translational motion as a one-dimensional motion along a straight path[8], the equation (1.2) can be considered correct.

However, in reality, the velocity vector acting in a closed space acts perpendicular to the surface where the force acts. Therefore, the total acting area is not the cross-sectional area but the sum of the areas of the three surfaces. This is why the coupling constant 3 needs to be removed, and another reason is that the coupling constant $\frac{1}{2}$ also needs to be removed.

Therefore, the Kinetic theory of gases should be expressed as follows:

$$PV = E_k = k_BT. \tag{1.3}$$

II. DERIVATION

In a closed space ($V = L^3$), the momentum of a moving object is expressed by Newton's law of inertia as follows:

$$p = m_0\vec{v}, \tag{2.1}$$

where p is momentum, m_0 is the rest mass in the space, and $\vec{v}$ is the average velocity.

When external pressure is applied, the momentum changes in conjunction with the change in velocity as follows:

$$\Delta p = m_0\Delta\vec{v}. \tag{2.2}$$

The change in distance is the distance that the velocity v moves during the time Δt, which is given by:

$$\Delta L = \vec{v}\Delta t, \tag{2.3}$$

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[1] It is read as: Considering that the translational motion velocity vector v has three degrees of freedom (one for each dimension) gives the average energy per degree of freedom equal to one third of that, i.e. $\frac{1}{2}k_BT$.

where Δt is given by:

$$\Delta t = \frac{\Delta L}{\vec{v}}. \tag{2.4}$$

From this, the change in force (ΔF) can be obtained as follows:

$$\Delta F = \frac{\Delta p}{\Delta t} = \frac{m_0 \Delta \vec{v}}{\frac{\Delta L}{\vec{v}}} = \frac{m_0 \vec{v} \Delta \vec{v}}{\Delta L}. \tag{2.5}$$

The pressure (P) is given as follows:

$$P = \frac{F}{A}, \tag{2.6}$$

where A is the surface area of three sides of the closed space.

Rearranging Equation (2.5), we get:

$$\Delta P = \frac{m_0 \vec{v} \Delta \vec{v}}{A \Delta L}. \tag{2.7}$$

Integrating Equation (2.7), we obtain:

$$\begin{aligned} P &= \int m_0 \vec{v} d\vec{v} / \int A dL \\ &= \frac{E_k}{V}, \end{aligned} \tag{2.8}$$

where

$$\int m_0 \vec{v} d\vec{v} = \frac{1}{2} m_0 \vec{v}^2 = E_k, \tag{2.9}$$

and

$$\int A dL = \int 3L^2 dL = V. \tag{2.10}$$

As the result, the correct theory that satisfies the ideal gas law is as follows:

$$PV = E = k_B T. \tag{2.11}$$

In consideration of relativistic time, by substituting the following equation into equation (2.4):

$$dt = \frac{t_0}{\sqrt{1 - \frac{\vec{v}^2}{c^2}}}. \tag{2.12}$$

From (2.4) and the above, we have

$$dt_0 = \frac{dL \sqrt{1 - \frac{\vec{v}^2}{c^2}}}{\vec{v}}. \tag{2.13}$$

We can obtain the following equation from equations (2.5), (2.7) and (2.9),

$$\begin{aligned} E &= \int m_0 d\vec{v} \left(\frac{\vec{v}}{\sqrt{1 - \frac{\vec{v}^2}{c^2}}} \right) \\ &= m_0 c^2 \left(1 - \sqrt{1 - \frac{\vec{v}^2}{c^2}} \right). \end{aligned} \tag{2.14}$$

Here, if $v \ll c$, then we have

$$E_k = \frac{1}{2} m_0 v^2. \tag{2.15}$$

This is the exact expression for the traditional kinetic energy.

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[1] https://en.wikipedia.org/wiki/Ideal_gas_law
[2] https://en.wikipedia.org/wiki/Boltzmann_constant
[3] https://en.wikipedia.org/wiki/Newton%27s_laws_of_motion
[4] https://en.wikipedia.org/wiki/Kinetic_theory_of_gases
[5] https://en.wikipedia.org/wiki/Pressure
[6] https://en.wikipedia.org/wiki/Time_dilation
[7] https://vixra.org/abs/2302.0146 "On the mass-energy formula $E = mc^2$, the measurement of kinetic energy and temperature, Feb. 27, 2023.
[8] https://www.studysmarter.co.uk/explanations/physics/work-energy-and-power/translational-kinetic-energy/
[9] https://en.wikipedia.org/wiki/Momentum
[10] https://en.wikipedia.org/wiki/Energy
[11] https://en.wikipedia.org/wiki/Mass%E2%80%93energy_equivalence
[12] https://en.wikipedia.org/wiki/Work_(physics)
[13] https://en.wikipedia.org/wiki/Kinetic_energy
[14] https://en.wikipedia.org/wiki/Reaction_(physics)
[15] https://en.wikipedia.org/wiki/Power_(physics)
[16] https://en.wikipedia.org/wiki/Planck_relation