An Information Based Theory of Stationary Action

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The quantization of energy proposed by Planck to account for the observed spectrum of black body radiation has associated with it a quantization of entropy. This in turn implies a quantization of observable information, directly implying observational uncertainty on the order of Planck's constant. The effect of that uncertainty is analyzed. In order to adhere strictly to the use of observable quantities, a probability measure is employed based on the distinguishability of statistical samples. This leads directly to the description of probability in terms of the absolute square of a complex amplitude. The Feynman rules may then be applied naturally for indistinguishable events without contradiction to the conventional rules for distinguishable events. This enables the straightforward calculation of the probability that a particle moves from one arbitrary point to another. The Feynman formulation of quantum phenomena and the principle of stationary action results when it is assumed that the classical action represents the measure of distinguishability. Parallel analysis on a Lorentz manifold yields the geodesic principle.

Introduction

Early efforts to understand black body radiation within the confines of classical physics focused on the entropy of electromagnetic radiation. In 1884 Boltzmann studied black body radiation in a perfectly reflecting enclosure. Treating the radiation pressure as that of a continuum gas he was able to define its entropy [1]. From that followed a theoretical basis for Stefan's empirically determined dependence of total radiated power on the fourth power of temperature.

Several years earlier Boltzmann had shown a correspondence between classical entropy and the discrete quantity that was later called the "statistical multiplicity"[2] of a molecular gas, though he made no use of this in his black body work. It was not until the mid-twentieth century introduction of information theory [3] that entropy could be understood as information lost due to the statistical treatment of trajectories in lieu of a more complete microscopic model of molecular states [4].

By 1900, Planck had developed a classical model of the entropy of a black body at temperature T in which electromagnetic dipole resonators operated in equilibrium with the radiant energy in Boltzmann's reflective enclosure. Comparing the latest empirical data to his model, he found it necessary to introduce the constant h limiting radiation energy to discrete multiples of $h\nu$ [5]. This quantization of energy has the effect of limiting the entropy to increments of $h\nu/T$ when expressed in the prevailing units, those of Boltzmann's constant k.

The appearance of entropy in discrete increments is consistent with the situation in statistical mechanics. In that case discrete values replace the continuum model since entropy is now based on the discrete statistical multiplicity of macro-states. As in the case of the continuum gas model, the entropy in the black body model will be interpreted as a paucity of accurate information, in this case due to some inherent natural limit.

Consider an ideal gas consisting of a single molecule. The Planck entropy implies an inability of the classical model to describe its trajectory in phase space with uncertainty less than h. It may be that a more accurate description of the trajectory is not possible. Alternately, the trajectory may be fully deterministic but some inherent limit in observational accuracy produces the uncertainty, even with perfectly accurate measuring equipment.

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In either case the description of observable quantities in the classical model, like that of a continuum gas is only approximate, with the action of observed natural phenomena differing from the classical description. The result is an observed stochastic component of order h in the molecular trajectory. In this situation, Planck's constant h is a more convenient unit of entropy.

Let η be a system dependent parameter, then let

$$I_1 = \eta h \tag{1}$$

represent the information in a hypothetical, more accurate and possibly fully deterministic model that supersedes the inaccurate part of the information in the classical description. Let H_1 represent the entropy of the more accurate model and H_0 the entropy of the classical model. Then [6]

$$H_1 = H_0 - I_1 \tag{2}$$

If the more accurate model is both fully deterministic and completely accurate

$$H_1 = 0 \tag{3}$$

Then

$$H_0 = I_1 = \eta h \tag{4}$$

The analysis that follows explicitly acknowledges statistical uncertainties in observations of physical systems. Following classical practice, we assume no explicit limit on the accuracy of measuring instruments. Also, as in the classical model, the explicit effect of a measurement is not assumed in advance to significantly affect its own result, nor the results of future measurements.

The existence of uncertainty requires that the inherently stochastic nature of the result of observation be incorporated in the analysis. The choice of probability measure can be of profound importance [7]. Following modern practice, careful attention is paid to ensuring our analysis is based strictly on what can be observed. To that end we employ a probability measure based on stochastic outcomes that are equal in statistical distinguishability from one another.

Distinguishing one experimental outcome from another is necessarily a matter of distinguishing between their probability distributions. The distinguishability of probability distributions has been studied by Wootters [8]. It is measured by the quantity *statistical distance* on a probability space.

Consider two N sided loaded die with different loadings, where the differences in the probabilities of corresponding faces are $\delta p_1 \cdots \delta p_N$. The die are said to be distinguishable in n trials if

$$\frac{\sqrt{n}}{2}\left[\sum_{i=1}^{N}\frac{(\delta p_i)^2}{p_i}\right] > 1 \tag{5}$$

Then the statistical distance $\mathcal{S}$ between these dice on the appropriate probability space is defined by

$$\mathcal{S} = \lim_{n\to\infty}\frac{1}{\sqrt{n}} \times \begin{bmatrix}\text{the maximum number of intermediate outcomes each of} \\ \text{which is distinguishable (in } n \text{ trials) from its neighbors}\end{bmatrix} \tag{6}$$

Along with the stochastic analysis of what can be observed, the question remains: are these stochastic processes consistent with more deterministic, or even fully deterministic underlying natural processes, even if some of the variables necessary to make use of a more deterministic model are inherently hidden due to some natural limitation on their observability.

Analysis

Let $x = (x_0, x_1, x_2, x_3)$ represent the ordinary space and time of classical physics where x_0 represents time and x_1, x_2 and x_3 represent three-dimensional Euclidean space. Let us consider a particle that moves from start point A to end point B by an unknown trajectory through this space and time. Let $x(t)$ be an arbitrary trajectory with the same end points, where t is a time like parameter. We stipulate, in view of uncertainty, that for any value of the parameter t assigning a definite time and location on $x(t)$ there may be a nonzero probability that the particle can be observed at any time and location x. This defines the probability distribution $p(x, t)$. The set of all possible probability distributions constitutes a probability space.

Let $p(x,t)$ be piecewise differentiable with respect to t. The statistical distance between points on $x(t)$ in the physical space may then be expressed as the statistical distance $\mathcal{S}$ between corresponding points in the associated probability space [8].

$$d\mathcal{S}(t) = \frac{1}{2}\left\{\int_{-\infty}^{\infty} dx_0 \iiint_{-\infty}^{\infty} dx_1 dx_2 dx_3 \frac{1}{p(x,t)}\left[\frac{dp(x,t)}{dt}\right]^2\right\}^{1/2} dt \tag{7}$$

This expression may be simplified by the substitution $\zeta(x,t) = p^{1/2}(x,t)$. Then [8]

$$d\mathcal{S}(t) = \left\{\int_{-\infty}^{\infty} dx_0 \iiint_{-\infty}^{\infty} dx_1 dx_2 dx_3 \left[\frac{d\zeta(x,t)}{dt}\right]^2\right\}^{1/2} dt \tag{8}$$

This new expression defines an element of length $d\mathcal{S}$ in an infinite dimensional Euclidean ζ space.

Since $\zeta^2(x,t)$ is a probability, it's integral over all of space and time is unity for any value of the parameter t. Thus $d\mathcal{S}$ lies on the surface of an infinite dimensional unit hypersphere. Then statistical distance $\mathcal{S}$ between points on $x(t)$ is measured by the length of the arc traced on the surface of the hypersphere as the progress of parameter t traces out the trajectory in space and time between them [8].

Now let us divide the integral on the right-hand-side of (8) in two. Let $[da(t)/dt]^2$ represent the portion of the integral for which $x_0 < t$ and $[db(t)/dt]^2$ the portion for which $x_0 > t$.

$$\left[\frac{da(t)}{dt}\right]^2 = \int_{-\infty}^{t} dx_0 \iiint_{-\infty}^{\infty} dx_1 dx_2 dx_3 \left[\frac{d\zeta(x,t)}{dt}\right]^2 \tag{9}$$

$$\left[\frac{db(t)}{dt}\right]^2 = \int_{t}^{\infty} dx_0 \iiint_{-\infty}^{\infty} dx_1 dx_2 dx_3 \left[\frac{d\zeta(x,t)}{dt}\right]^2 \tag{10}$$

Then a represents events nominally in the past and b, the future. Let us form the complex quantity $a + ib = e^{i\theta}$ where $\theta = \tan^{-1} b/a$. The unit hypersphere is collapsed into the unit circle on the Argand plane, while the angle $\theta(t)$ measures statistical distance.

$$d\mathcal{S}(t) = d\theta(t) \tag{11}$$

Now in order to express the probability $p[x(t)]$ of the test particle being found at a point on the trajectory $x(t)$ as an explicit function of the statistical distance $\theta(t)$ we may define the probability amplitude

$$\varphi[x(t)] \equiv \zeta[x(t)]e^{i\theta(t)} \tag{12}$$

Then

$$p[x(t)] = |\varphi[x(t)]|^2 \tag{13}$$

Representation of the probability in terms of a Hilbert space has the effect of including the real valued measure θ along with the real valued magnitude $|\varphi|^2$ in the statement of the probability [7].

<u>The Feynman Rules</u>

The identification of a probability with the absolute square of a complex quantity constitutes the Feynman *amplitude-probability rule* [9, 10].

While the Feynman rules are explicitly for the purpose of quantum mechanical calculation, the conditions assumed here in developing the probability amplitude consist of no more than a random variable with probability described by a function piecewise differentiable with respect to some parameter. This allows the Feynman rules to be treated as a feature of probability theory under these circumstances, when a measure based on statistical distinguishability is employed. Indeed, the peculiarities of quantum theory, depending upon what can and cannot be measured, may be regarded as depending upon distinguishability.

The conventional Laplace rule for the probability p_L of an event that may occur by any of m alternative means is

$$p_L = \sum_{i=1}^{m} p_i = \sum_{i=1}^{m} |\varphi_i|^2 \tag{14}$$

where p_i represents the probability of each of the alternatives. The Laplace rules are empirical in nature, verified by counting occurrences of the various alternatives [11]. We know from a century of experience with quantum phenomena that when the alternatives are *indistinguishable* the probability p_F is [9].

$$p_F = |\varphi_F|^2 = \left|\sum_{i=1}^{m} \varphi_i\right|^2 \tag{15}$$

This constitutes the *Feynman amplitude sum rule* [10, 12].

The conventional Laplace rule for the probability p_L that m events all occur is

$$p_L = \prod_{i=1}^{m} p_i = \prod_{i=1}^{m} |\varphi_i|^2 \tag{16}$$

where the values of p_i represent the probability of each event. Our experience with quantum phenomena informs us that when the m events are indistinguishable the probability p_F is [9].

$$p_F = |\varphi_F|^2 = \left|\prod_{i=1}^{m} \varphi_i\right|^2 \tag{17}$$

This constitutes the *Feynman amplitude product rule* [10 ,12]. While the two formulae yield identical probabilities, the latter establishes the phase $\theta_F = \sum \theta_i$ of the probability amplitude for the combined event.

Goyal, Knuth and Skilling have shown that the Feynman rules are a necessary result of any probability represented by a pair of real numbers [13]. Goyal and Knuth have further shown that the Feynman rules can coexist free of conflict with conventional probability [10]. Earlier Sykora reminded that while probabilities are routinely described in terms of a single number, a measure is also necessary. Though it is frequently not explicitly stated, the need for this second real number is never-the-less implied. He noted that clear statement of the measure has the salutary effect of eliminating ambiguities in statistical evaluation of observed result. [7]. In this case magnitude $|\varphi|^2$ and phase θ respectively describe the probability and a measure based on the distinguishability of alternatives. The complex probability amplitude allows the former to be conveniently formulated in terms a function of the latter.

Given that two real numbers are the minimum necessary to fully express a probability; given also that the Feynman rules are universally observed in nature when dealing with indistinguishable events; and given as well that the Feynman rules are the only two component alternative to the Laplace rules that are compatible with them, we elect to treat them as a natural part of probability theory.

Feynman [12,14] citing von Neumann [15] has shown that the introduction of a means of observation produces an arbitrary unknown phase shift in the probability amplitudes of previously unobservable events. Let $p_F = |\varphi_1 + \varphi_2|^2$ be the probability of an event with two indistinguishable alternative ways of occurring, signified by the two subscripts. Suppose now that a means of observing the alternatives is provided. The presence of the measuring equipment perturbs the phase of the probability amplitudes by arbitrary unknown amounts θ_1 and θ_2. Now $p_F = \left|\varphi_1 e^{i\theta_1} + \varphi_2 e^{i\theta_2}\right|^2$. The multiple observations required to observe the statistical frequency of these alternatives require that their phases be averaged over all angles. This results in reversion to the Laplace rule $p_L = |\varphi_1|^2 + |\varphi_2|^2$ as the sample size approaches infinity.

In view of this one may argue that the Feynman rules be treated as the more fundamental empirical rule of probability applying to any inherently indistinguishable alternatives lying on space and time continua, that are piecewise differentiable with respect to time, while the conventional Laplace rules becomes derivative of them. Until the possibility of an observation is present, only an amplitude exists without a corresponding frequentist probability. Where observation is possible, a frequency may be observed and a frequentist probability emerges. The real valued probability is independent of phase, and the phase of the probability amplitude is lost with the emergence of a probability. It appears then that the Feynman rules are not supplemental to the Laplace rules but rather a natural replacement for them.

Von Neumann's arbitrary phase shift was determined based on the properties of the Dirac-von Neumann model of quantum phenomena. Given the undeniable long-term success of that model, the von Neumann phase shift has been treated here as an empirical result.

To make proper use of the Feynman rules it becomes necessary to define all cases when events are inherently indistinguishable. Clearly that is the case when no means of measurement is present. In addition, in the more general case measurement results are indistinguishable when the statistical distance between them is less than unity. Kok [16] has shown this to be a consequence of the Kramer-Rao bound.

Based on experience with quantum phenomena, for events to be indistinguishable uncertainty must be inherent [17]. For that to be the case there must be some mechanism, discussed below, whereby any variables describing underlying physical processes are either inherently random, or are inherently hidden from direct observation.

<u>The Path Integral</u>

Let $\Delta\theta = \int_{x(t)} [d\theta(t)/dt]\, dt$. This is the statistical distance traced by a particle as it traverses the path $x(t)$. Let m be an arbitrary integer and let $\delta\theta = \Delta\theta/m$ so that $x[\theta(t)]$ is divided into m equally distinguishable segments. Then the probability amplitude for the j^{th} segment is $\varphi_j = \mathcal{A}_j^{(m)} e^{i\delta\theta}$ where $\mathcal{A}_j^{(m)} = \left[p_j^{(m)}\right]^{1/2}$ while $p_j^{(m)}$, dependent on the value of m, is the probability for the j^{th} segment were a measurement possible.

The probability amplitude $\varphi_{\mathcal{P}}[x(t)]$ for a test particle following an arbitrary path $\mathcal{P} = x(t)$, when the individual points cannot be observed, is the product of the probability amplitudes that the particle is found at each of the m intervals on $x[\theta(t)]$.

$$\varphi_{\mathcal{P}}[x(t)] = \lim_{m\to\infty} \prod_{j=1}^{m} \left[\mathcal{A}_j^{(m)} e^{i\delta\theta}\right] = \mathcal{A} e^{i\Delta\theta} \tag{18}$$

where $\mathcal{A}^2 = \left[\lim_{m\to\infty} \prod_{j=1}^{m} \mathcal{A}_j^{(m)}\right]^2$ is the probability that the path $x(t)$ is followed when observation of the path is possible.

We, know based on many decades of empirical experience verifying the Feynman formulation of quantum mechanics, that the proper expression for $\varphi_{\mathcal{P}}[x(t)]$ is [9]

$$\varphi_{\mathcal{P}}[x(t)] = \text{Const}\, e^{iS/\hbar} \tag{19}$$

where S is the classical action. Then the substitution

$$S/\hbar = \Delta\theta \tag{20}$$

yields the Feynman result. The classical action S corresponds to the statistical distance $\mathcal{S}$ traced by a particle as it traverses $x(t)$ while the classical Lagrangian corresponds to the rate of change of statistical distance with time.

The probability that a test particle goes from start point A to end point B by any path is then computed according to the Feynman amplitude sum rule with the path integral replacing the summation in (15).

$$p(BA) = \left| \int_A^B \varphi(BA)\, \mathcal{D}x(t) \right|^2 \tag{21}$$

Quantum Gravity

The derivation of the Feynman formalism for ordinary quantum mechanics developed here does not rely on the Euclidean nature of the physical space to which those rules are applied. A parallel derivation can be applied to a particle trajectory on a Lorentz manifold provided an uncertainty is postulated in the observed *four-space location* of a particle on the manifold. This leads directly to the geodesic principle of general relativity in lieu of Hamilton's principle. The equivalence of these two principles in the flat space limit [18] establishes a correspondence between the two forms of uncertainty.

Let $x^* = (x_0^*, x_1^*, x_2^*, x_3^*)$ represent the four-dimensional Lorentzian manifold of general relativity, where x_0^* represents time and x_1^*, x_2^* and x_3^* represent the three spatial dimensions. Substituting x^* for x in (7) through (11) and more carefully specifying t as an affine parameter, we find the equivalent definition of statistical distance. Continuing in the same vein through (18), we find the equivalent expression for the probability amplitude of an arbitrary path in terms of the statistical length of the path $\Delta\theta$.

The Einstein Hilbert action for empty space is [19]

$$S_g = \kappa \int \sqrt{-g}\, R\, d^4x \tag{22}$$

Where κ is the Einstein gravitational constant, R is the Ricci scalar and g is the determinant of the metric tensor.

The scalar invariant integral carries the units of length squared. This allows us to define a quantum unit of gravitational action proportional to ${s_c}^2$ where s_c is a small fixed interval on the Lorentzian manifold characterizing uncertainty in spacetime location. The connection between a Planck scale increment of length and Planck's constant is firmly established in gravitational theory [20]. In the flat space limit $s_c = \sqrt{32}\,\pi\, l_P \approx 17.77\, l_P$ where l_P is the Planck length.

We may now omit (20) which identifies statistical distance with the classical action, and substitute s/s_c for $\Delta\theta$ where $s = \int_{x^*(t)} [dx^*(t)/dt]\, dt$ is the length of $x^*(t)$. Then the probability amplitude $\varphi_{\mathcal{P}}^*[x^*(t)]$ for a particle to follow an arbitrary path $x^*(t)$ from A to B is

$$\varphi_{\mathcal{P}}^*[x^*(t)] = \mathcal{A}^* e^{is/s_c} \tag{23}$$

As the path integral (21) yields Hamilton's principle, the new path integral

$$p^*(BA) = \left| \int_A^B \varphi_{\mathcal{P}}^*(BA)\, \mathcal{D}x^*(t) \right|^2 \tag{24}$$

yields the geodesic principle. The foundation of general relativity is recovered directly. The principle of stationary action follows in the flat space limit [18]. The result is the relativistic equivalent to the Feynman space time formulation of non-relativistic quantum mechanics [14] providing a plausible and equally general model of quantum gravity subject to empirical validation.

The association of uncertainty with the geometric quantity spacetime-location provides some additional clarity. The presence of a yet to be defined source of uncertainty in the geometry of spacetime provides a manifest source of the limitations of classical physics. It remains unclear whether randomness in spacetime location is inherent to spacetime, or is due to some inherent limit on observational accuracy. Both cases are discussed below.

In either case inherent uncertainty in the four-space location of the measuring devices can account for some or all of observed uncertainty. It also provides a plausible explanation for the von Neumann phase shift. The measurement provides a change in available information limiting the location of the measured particle to a volume of spacetime $\approx s_c{}^4$. The phase of the incoming particle vanishes while a new arbitrary phase characterizes the ongoing particle after measurement.

The Black Body Spectrum Revisited
When viewed in the laboratory frame, the uncertainty of location in spacetime must appear as a small apparently random motion. This in turn results in a small zero-point energy. As early as 1913 employing purely classical analysis, Einstein and Stern showed that the assumption of zero-point energy $h\upsilon$ in Planck's dipole oscillators led to the Planck spectrum *without the independent assumption of energy quantization* [21].

Milonni has extended that analysis [21] noting each Planck oscillator is in equilibrium with an associated field mode of Planck's cavity. Employing the same zero-point energy, the equipartition theorem requires the oscillator and field each have energy $h\upsilon/2$.

Let us now momentarily assume that deterministic physical laws resembling the laws of classical physics continue to govern even at the microscopic level, subject to some undefined source of uncertainty in observed spacetime location. Since the apparently, or actually random motion of the electron and the field at the same location are identical, we would expect no coupling between the zero-point motion of the dipole oscillator and the zero-point component of the field under this assumption.

Returning to Milonni, still employing purely classical analysis he demonstrates that when there is no interaction between the random components of the oscillator and the field, black body spectral density is

$$\rho(\upsilon) = \frac{8\pi h\upsilon^3/c^3}{e^{h\upsilon/kT} - 1} + 4\pi h\upsilon^3/c^3 \qquad (25)$$

in agreement with quantum electrodynamic theory. Though short of proof, this hints that even at the unobservable level the concept of spacetime location remains valid and fully deterministic laws resembling the classical ones may prevail. These restrictions further appear to limit the observed randomness to variations in the spacetime metric.

Discussion
The present analysis leads naturally to the Feynman formulation of quantum phenomena under the assumption that our ability to know the state of physical phenomena is inherently imperfect. It relies on a revised form of probability theory that employs the Feynman rules as the empirically determined rules of probability for indistinguishable events, reducing to the Laplace rules when events can be distinguished.

A similar point of view can be applied to the Dirac-von Neumann formalism. Consistent with the "ψ-epistemic" view [22], the probability amplitude represents the state of information available about the system. When the amplitude is defined this way, its collapse does not represent a change in the physical system. Instead, it indicates the state of available information about the system has changed as the result of a measurement. A frequentist probability has emerged, while simultaneously the phase of the probability amplitude has vanished.

The probability amplitude may be regarded as representing the available information about a conditional probability [10] based on the state of information prior to measurement. The amplitude after measurement represents a new conditional probability based on the new state of information generated by the measurement.

Goyal's analysis has shown [23] that the logic of the Feynman formalism is equivalent to that of Dirac-von Neumann when it is supplemented with a *no-disturbance postulate*. This posits that there exists a class of *trivial measurements* which have no effect on the probability amplitude. Trivial measurements are defined by the property that they yield no new information about the system being measured. This principle is a natural consequence of the epistemic interpretation of the probability amplitude. With no change in information the conditional probabilities of subsequent outcomes are unchanged.

The no-disturbance postulate resides uncomfortably alongside the notion that uncertainty is caused by the process of measurement. The apparent conflict is eliminated with the adoption of the ψ-epistemic viewpoint coupled with the proposed entropic origin of uncertainty. The fact that the no disturbance principle requires a change in available information to change the probability amplitude is a strong indicator of the latter's epistemic nature.

A defining feature of the ψ-epistemic view is that a change in ψ does not necessarily imply a change in reality as in the generally prevailing ψ-ontic view [22].

The entropy and information considerations that motivate the present analysis allow for fully deterministic underlying spacetime locations of both particles and waves that are beyond our ability to accurately observe, hence appearing stochastic in nature. Alternatively, they also allow for a spacetime location inherently stochastic in nature, not just appearance. Similarly, nothing in the ψ-epistemic viewpoint prevents the existence of fully deterministic, nor of inherently stochastic spacetime [22].

Wave Particle Duality

The empirically derived Laplace rules of probability among distinguishable alternatives have been shown here to lead naturally to the description of particle probabilities in terms of a complex probability amplitude with wavelike characteristics. With the adoption of the more general Feynman rules, also empirically derived, these probability amplitudes add as do the amplitudes of physical wave phenomena when observations of intermediate events are not possible. This imparts wavelike properties to classical particles resulting strictly from the novel rules of probability.

Planck's derivation of the black body spectrum was based almost entirely on a combination of classical mechanics and electromagnetic theory. It deviated only with an unexplained quantization of electromagnetic radiation. Planck demonstrated that the observed black body spectrum was consistent with the effect of energy quantization on the entropy of radiation.

Particle like behavior of classical electromagnetic waves with entropy empirically imposed by the observed black body spectrum were famously explored as early as 1905 [24, 25]. Einstein's classical analysis of the empirically determined high frequency portion of the black body spectrum found its entropy matched that of an ideal molecular gas with particle energy concentrated in a narrow band around the value $h\nu$. Particle like behavior in classical wave phenomena thus explained the photoelectric effect twenty years before the advent of modern quantum theory.

With the benefit of modern information theoretic insights not available in 1905, the present analysis allows us to associate the entropy of the black body spectrum with our inability to account for the apparently, or actually random spacetime locations of both resonator and field. As a result, information about the energy of the radiation field as well as the corresponding entropy at the location of the particles are quantized, rather than the field energy itself. The quantization of field entropy due to the resulting uncertainty mimics that of a particulate gas. This accounts for the appearance of photons.

While the present analysis takes no exception to the notion that light consists of photons in all observable phenomena, it also allows for the existence of unobservable fully deterministic underlying behavior in which electromagnetic effects are not quantized, as in the Einstein-Stern-Milonni black body analysis. Though this is an isolated result, it suggests that deterministic physical processes may continue to operate in the quantum regime even though the location of events in spacetime may suffer inherent limitations in observability. Alternately, there may be a random component in spacetime itself. Further investigation is called for before generalizations can be made with confidence, as was the case with Planck's isolated black body result. Let us examine both cases.

Fully Deterministic Spacetime

The general relativistic model contains within itself a mechanism whereby an observer with only local information will observe a small zero-point energy. Correspondence between Newtonian mechanics and general relativity occurs when energies of objects under observation are suitably small, and there are no variations in the spacetime metric due to events outside the range of observation [26]. The latter of these conditions precludes from consideration a background level of broadband gravitational radiation.

If such background exists then, it can be expected to impose on the classical picture a small apparently random source of zero-point energy. Even in the full general relativistic model, background gravitational radiation that appears stochastic to an observer with only local knowledge must add an apparently random component to the predictable trajectories of ponderable masses.

The full nature of such a stochastic background of gravitational radiation is an open question [27]. Neither a spectrum nor a characteristic time for the gravitational background is known. That this is the source of uncertainty is of course speculation. If this is the case, the general relativistic model is the not-fully-predictable but fully deterministic model. The lack of predictability stems from our inability to know the background gravitational radiation in anything but stochastic terms.

In the fully deterministic spacetime model, the preexisting laws of general relativity, including classical mechanics, are assumed to be in place while quantum phenomena are caused by an additional small stochastic component in the knowledge of the observer. This model is characterized by fully deterministic location in fully deterministic spacetime with fully deterministic laws of physics, impaired by an inherent limit on observability of deterministic spacetime locations.

Spacetime With a Random Component

In the alternative to the deterministic case, the preexisting laws of mechanics are not assumed. Let us call this the bootstrap model in recognition of how these laws may come into being. In this model uncertainty is assumed due to fully random variations in the spacetime metric not tied to any underlying deterministic process. This possibility introduces an intriguing problem in our understanding of the laws of physics.

The macroscopically deterministic laws of mechanics are generally described by differential equations. It is in the nature of these equations that macroscopic behavior follows directly from behavior at the microscopic level, yet we are assuming randomness at the microscopic level. How then could it come to be that differential equations provide a near perfect description of a highly ordered universe at the macroscopic level?

We have argued that the Feynman rules constitute a more plausible empirically justified axiomatic basis for probability theory than the Laplace rules in that, at least when working on a time differentiable manifold, they cover a broader range of phenomena including both indistinguishable and distinguishable events. In the face of randomness at the microscopic level, these rules provide a mechanism for the laws of mechanics to emerge from randomness. The path integral assures that the observability of particles that just happen to follow paths of near stationary length in spacetime will be coherently enhanced, while the observability of those that do not will be coherently suppressed. Thus, both the principle of stationary action, and the geodesic principle are bootstrapped into existence on the strength of the Feynman rules.

Despite randomness at the microscopic level, a system at least macroscopically described by deterministic differential equations may be observed. In this way the general relativistic model, indicative of what can be observed in a universe highly random at the microscopic level, may arise out of that randomness.

A central feature of the present discussion is the concept of inherent uncertainty. It posits that there exists a class of observational results that are inherently unpredictable by any theory. This holds true even though there may be deterministic hidden variables describing some or all the underlying phenomena. In either case observational results appear random. In the fully deterministic proposal, the randomness is only apparent, resulting from the observer's inability to distinguish deterministic undulation of the spacetime metric from motion with respect to the metric. In the alternative the spacetime metric has a truly random component. As a result of these alternatives, the precise definition of inherent uncertainty remains for now uncertain itself.

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