Space and time in relativity

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Abstract

The nature of space and time in current relativity physics is defined by Einstein's special relativity theory (SRT), with length contraction and time dilation in moving inertial systems as fundamental elements in SRT's structure of space and time. Both are essentially ad hoc concepts, supported by a wealth of complex and exacting experiments. This paper instead starts by considering the simplest possible relativistic experiment – the propagation of a beam of light, as seen from the principle of relativity point of view. It is found that the principle of relativity by itself transforms space and time coordinates of an optical wave between inertial systems, and secures the covariance of the Maxwell equations – both otherwise the prerogative of the Lorentz transformation. When analyzed by those coordinate transformations defined by the principle of relativity, crucial relativistic experiments that provide the experimental support of SRT's concepts of space and time are explained without recourse to length contraction or time dilation. Consequently, time dilation and length contraction of rigid bodies inside moving inertial systems do not occur in a real world ruled by the principle of relativity. In particular, experiments show that time proceeds equally fast in a moving inertial system as in a stationary system. Concepts of space and time in current special relativity theory therefore are not compatible with the principle of relativity. A consistent special theory of relativity would have its basic structure of space and time defined by the principle of relativity, not by the physical interpretation of the Lorentz transformation. That shall simplify relativistic concepts, to make relativity theory more transparent and true to relativity physics, possibly opening up for further advances directly from the basis of relativity.

Introduction

This paper engages the principle of relativity [1,2] for a study of space and time in special relativity physics. As is well known, in current special relativity theory [2] (SRT) the structure of space and time is defined through a physical interpretation [2,3] of the Lorentz transformation [2,4] (LT), in terms of length contraction and time dilation inside moving inertial systems. The present paper instead starts from an analysis of the propagation of light, based directly on the principle of relativity. That is followed by corresponding analyses of certain relativistic experiments crucial to the support of the conceptual basis of SRT. On that background, and based on the experiments, the paper shall discuss how concepts of space and time in relativity physics relate to the principle of relativity.

Consider an inertial system K_2 with coordinates ξ,η,ζ moving with its ξ-axis in the positive x-axis direction of another, 'stationary' inertial system K_1 – with coordinates x,y,z – at a velocity v as seen from K_1, and with its η - and ζ - axes parallel to the y- and z-axes, respectively, of K_1. A wave of light propagates along the ξ-axis in K_2 as $\xi = c\tau$, where c is the universal velocity of light [2], τ is the time of flight for the light wave and ξ is the path length traveled by the light wave during time τ as measured in K_2. When observed from K_1, and according to the principle of relativity, the propagation of the light wave shall be given by $x = ct$, where x and t are the path length and flight time of the light wave measured in K_1. Consequently, the principle of relativity by itself makes $\xi = c\tau$ transform into $x = ct$. That crucial insight corresponds with what can be seen from figure 1: For light propagating along the positive ξ-axis in K_2, path length ξ and flight time τ in K_2 transform separately of each other into path length x and flight time t in K_1 by a common factor $H(v) > 1$, as

$$x = H(v)\xi \qquad (1)$$

$$t = H(v)\tau, \qquad (2)$$

and transform by $H(v)^{-1} < 1$ for light propagating in the negative ξ-axis direction. The question is what effects those coordinate transformations might have in relativity physics.

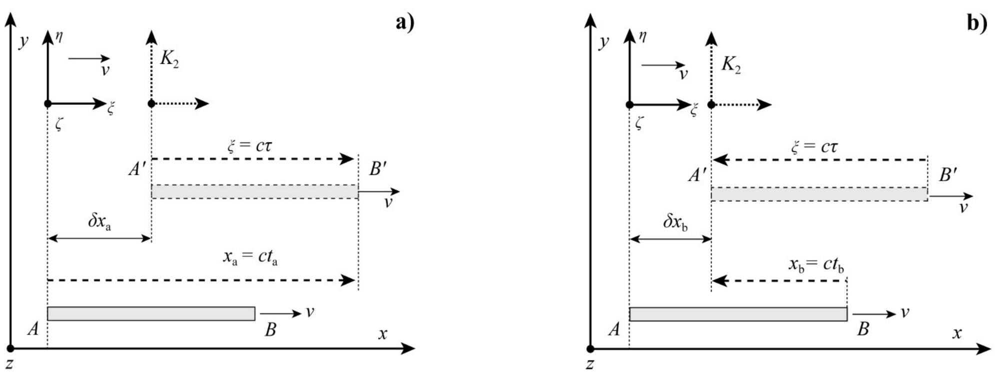

Figure 1. Observed light wave coordinates

System K_2 moves at a velocity v in relation to stationary system K_1. In a) a wave of light propagates as $\xi = c\tau$ from A to B in K_2. When observed from K_1 the light reaches B in the position B' in K_1 after a path length $x_a = ct_a > \xi$ in K_1, when K_2 has moved a distance δx_a in K_1. In b) light propagating from B to A in K_2 meets A in the position A' as seen from K_1, after a path length $x_b = ct_b < \xi$.

Covariance of the Maxwell equations

With space and time coordinates of electromagnetic fields transforming by equations (1) and (2) between inertial systems K_1 and K_2, the corresponding partial differential operators transform as

$$\partial/\partial x = (\partial\xi/\partial x)\partial/\partial\xi = H(v)^{-1}\,\partial/\partial\xi \qquad (3)$$

$$\partial/\partial t = (\partial\tau/\partial t)\partial/\partial\tau = H(v)^{-1}\,\partial/\partial\tau. \qquad (4)$$

These can be applied to the Maxwell equations for the electromagnetic field in empty space,

$$curl\boldsymbol{E}(x,y,z,t) = -\,\partial\boldsymbol{B}(x,y,z,t)/\partial t \qquad (5)$$

$$curl\boldsymbol{B}(x,y,z,t) = c^{-2}\,\partial\boldsymbol{E}(x,y,z,t)/\partial t, \qquad (6)$$

where $\boldsymbol{E}(x,y,z,t)$ and $\boldsymbol{B}(x,y,z,t)$ are the electric and magnetic field vectors, respectively, represented in the space and time coordinates of 'stationary' system K_1. For a plane wave propagating in the positive x-axis direction in K_1, as in figure 1a, with $\boldsymbol{E}(x,y,z,t)$ oscillating in the xy-plane and $\boldsymbol{B}(x,y,z,t)$ in the xz-plane, $curl\boldsymbol{E}$ reduces to $(\partial E_y/\partial x)\boldsymbol{k}$ and $curl\boldsymbol{B}$ reduces to $-(\partial B_z/\partial x)\boldsymbol{j}$, where E_y is the y-component of $\boldsymbol{E}(x,y,z,t)$ and B_z is the z-component of $\boldsymbol{B}(x,y,z,t)$, with $\boldsymbol{j}$ and $\boldsymbol{k}$ being the unit vectors in y- and z- axes directions, respectively. Equations (5) and (6) thereby reduce to

$$(\partial E_y/\partial x)\boldsymbol{k} = -(\partial B_z/\partial t)\boldsymbol{k} \qquad (7)$$

$$-(\partial B_z/\partial x)\boldsymbol{j} = c^{-2}(\partial E_y/\partial t)\boldsymbol{j}. \qquad (8)$$

From these and relations (3) and (4) follows that the principle of relativity makes Maxwell's equations for the field be represented in the ξ,η,ζ,τ coordinates of 'moving' system K_2 as

$$curl\boldsymbol{E}(\xi,\eta,\zeta,\tau) = -\,\partial\boldsymbol{B}(\xi,\eta,\zeta,\tau)/\partial\tau \qquad (9)$$

$$curl\boldsymbol{B}(\xi,\eta,\zeta,\tau) = c^{-2}\,\partial\boldsymbol{E}(\xi,\eta,\zeta,\tau)/\partial\tau\,. \qquad (10)$$

Simultaneity

In Einstein's classic thought experiment on simultaneity of events in different inertial systems [5], a traveler is situated at the midpoint of a railway carriage passing through a train station at a velocity v. Just as the traveler is opposite a person on the station platform, the rear end of the carriage is opposite a point A by the railway and the front end opposite a point B, and a short flash of light is emitted from each of said points. Situated midway between A and B, the person on the station platform would see the light pulses as simultaneous. We want to know how the traveler sees the light pulses. Let the length of the carriage be $2L$. As shown by figure 1 and equation (1), the person on the station platform shall observe a path length $H(v)L$ for light from point A to reach the traveler, and a shorter path $H(v)^{-1}L$ for light from point B. That might lead the person on the platform to conclude that the light pulse from point B reaches the traveler before the pulse from point A, because the traveler moves towards B and away from A [5-7]. However, in order to learn how the *traveler* sees the light pulses, observations from the station platform have to be transformed back into the carriage. That happens by the inverse transformations, i.e. by the factor $H(v)^{-1}$ for light from point A and by $H(v)$ for light from point B. As a result, both light paths reaching the traveler have the length L; according to the principle of relativity the traveler, too, shall see the light pulses as being simultaneous.

Analysis of real experiments

Analysis of concrete physical experiments requires precise mathematical expressions for $H(v)$ and $H(v)^{-1}$. These can most conveniently be found by means of the LT [2,4],

$$\xi = (x - vt)/[1 - (v/c)^2]^{1/2} \qquad (11)$$

$$\tau = (t - vx/c^2)/[1 - (v/c)^2]^{1/2}, \qquad (12)$$

where the space and time coordinates are subject to $x = ct$ and $\xi = c\tau$. From $t = x/c$ set into equation (11) and $x = ct$ into equation (12) follows

$$\xi = x\,(1 - v/c)/[1 - (v/c)^2]^{1/2} \qquad (13)$$

$$\tau = t\,(1 - v/c)/[1 - (v/c)^2]^{1/2}. \qquad (14)$$

This shows explicitly how space and time coordinates of electromagnetic waves transform individually and equally between inertial systems K_1 and K_2 by the common factor

$$H(v)^{-1} = [(1 - v/c)/(1 + v/c)]^{1/2} \qquad (15)$$

or its inverse.

The most important and fundamental experimental tests of SRT seek to answer the question whether or not length contraction and time dilation are real physical phenomena. Here we shall examine those classic experiments that serve to define and support the concepts of space and time in SRT. Length contraction of a moving rigid body was introduced [8,9] purely *ad hoc* in order to explain the Michelson-Morley experiment [10], subsequently tested in other versions of the experiment [11,12]. In its original format, the experiment comprises an interferometer with two arms of equal length L at right angle to each other. Light enters from a source and into the arms through a beam splitter, is reflected by mirrors at the far ends of the arms and meets again behind the beam splitter. One arm points in the longitudinal direction of the earth's motion around the sun at velocity v, to test any effect of motion on the length of the light path in the longitudinal arm as referred to the path in the transverse arm. When observed from a system stationary with the sun, the back-and-forth light path in the interferometer's transverse arm is seen as having a length $2L[1 - (v/c)^2]^{-1/2}$ (given only to second order in v/c in [10]). Equations (1) and (15) show that the back-and-forth optical path length through the *longitudinal* arm would be seen by the stationary observer as $H(v)L + H(v)^{-1}L = 2L[1 - (v/c)^2]^{-1/2}$, exactly equal to the observed path length in the transverse arm. Consequently, the stationary observer would agree with one moving with the interferometer about the null result of the experiment, says the principle of relativity.

Contrary to length contraction, the concept of time dilation inside moving inertial systems has an entirely theoretical origin [2,3], supported by various experimental tests and technical applications [11,13-15]. Botermann et al [16] made the highest resolution ever of an Ives-Stilwell [17] kind of experiment, with $^7Li^+$ ions circulating in an experimental storage ring at a third of light velocity. One laser beam, in a direction parallel to the ions' motion, excited one of two neighboring transitions in the $^3S_1 \rightarrow {}^3P_2$ hyperfine structure of the moving $^7Li^+$ ions, at a resonant optical frequency f_p = 777210326.98 MHz measured in the laboratory. Another laser beam, in the anti-parallel direction, excited the second transition in the moving $^7Li^+$ ions into resonance at a frequency measured in the laboratory as f_a = 384225534.98 MHz. Transition frequencies for the ions at rest were given as f_1 = 546455143.0 MHz and f_2 = 546474960.7 MHz for the first and second of said transitions, respectively. According to the principle of relativity, resonant laser light emitted in the parallel direction of the moving ions will be seen from the ions' system as having a frequency lower than in the laboratory by a factor $H(v)^{-1}$. Correspondingly, laser light in the anti-parallel direction will be received by the moving ions at a frequency higher than in the laboratory by the factor $H(v)$. The ions' velocity was given only coarsely as v = 0.338c. A more precise value consistent with the measurements can be found from SRT's formula for the relativistic Doppler effect of light, shown in [16],

$$f_D = (f_0/\gamma) / [1 - (v/c)\cos\varphi], \qquad (16)$$

where f_D is the equivalent of f_p or f_a, f_0 corresponds to f_1 or f_2, v is the moving ion's velocity, $\gamma = 1/[1 - (v/c)^2]^{1/2}$ is the '*time dilation factor*' in SRT and φ is the angle of observation. With the frequency pairs f_p / f_1 or f_a / f_2 – and $\varphi = \pi$ or 0 – set into equation (16) follows $v = 0.338377222c$ for the ions' velocity. That gives $H(v)^{-1}$ = 0.703098151 and $H(v)$ = 1.422276532, and the transition frequencies in the moving ions become calculated as $f_{mp} = H(v)^{-1}f_p$ = 546455143.8 MHz and $f_{ma} = H(v)f_a$ = 546474961.3 MHz, respectively. Inside the measurements' stated resolution of 4 x10^{-9}, f_{mp} and f_{ma} are equal to the ions' rest-frame transition frequencies f_1 and f_2; according to the principle of relativity, transition frequencies in the moving ions are the same as for the ions at rest.

Discussion and conclusions

The present analysis demonstrates that the principle of relativity has intrinsic and fundamental functions in special relativity *physics*, that are currently unrecognized and inactive in relativity *theory*. Most basically, the principle of relativity makes space and time coordinates of electromagnetic fields in empty space transform separately and similarly between inertial systems. That opens up for more direct and transparent analyses of relativistic processes and events. In addition, the principle of relativity by itself secures the covariance of Maxwell's equations. This altogether reduces the original importance of the LT in relativity theory.

When analyzed by the principle of relativity, the thought experiment [5] shows that light paths reaching the traveler from light sources *A* and *B* are equally long; both observers see the light pulses as being simultaneous. According to the principle of relativity, therefore, if two events located at different positions are physically simultaneous in a stationary inertial system, they are physically simultaneous in a moving inertial system, too. That would change current concepts of simultaneity [5], enabling the synchronization of clocks between inertial systems. Flight times L/c of the light pulses received by the moving traveler from either of points *A* and *B* equal the flight time L/c for the light pulses to reach the person on the railway station: Time proceeds at the same pace in the train compartment as on the stationary railway station, says the principle of relativity, time dilation is not present inside the moving compartment. Likewise, path lengths of light pulses from points *A* and *B* to the moving traveler have the same length *L* as the light paths from *A* and *B* to the person on the station platform – half the length of the compartment; there would be no length contraction of the moving train compartment.

Conclusions from the thought experiment are supported by the real physical experiments. The null result of the Michelson-Morley experiment, when analyzed by the principle of relativity coordinate transformations, implies that there are no changes in the physical length of the longitudinal arm of the interferometer as it moves. Contrary to current interpretations, therefore, the Michelson-Morley experiment provides concrete, physical evidence that length contraction of moving rigid bodies does not occur in the real world. Analysis of the Botermann et al experiment shows that transition frequencies in fast-moving ions are the same as for the ions at rest. Time in the moving ions' system runs as fast as in the stationary laboratory; time dilation in moving inertial systems is not a real physical phenomenon. Consequently, and confirmed by the experiment, real time is absolute and universal, shared by inertial systems at any velocity. That will question the concept of a four-dimensional spacetime [18] as a template for the overall structure of the real physical world.

In addition there is another ‘experiment’ that deserves to be discussed and explained: muons coming down to earth. Created in the upper atmosphere by cosmic rays, few if any muons would be able to reach down to ground inside their life time of about 2.2 μs even at light velocity. And yet they do. That fact is widely regarded as solid proof that time dilation does indeed exist in the real world. However, the muon ‘experiment’ is nothing but a real-life version of the thought experiment [5] discussed above: The muon is at rest inside an inertial system moving at high velocity, carrying with it a short-lived signal propagating inside that inertial system. Like the stationary observer on the station platform, and according to figure 1a, equation (1) and the principle of relativity, we – the stationary observers on the ground – would observe a lifetime signal that is $H(v)$ times longer than the duration of the muon’s life time signal inside its moving rest system. Velocities of those muons at ground level are measured to be upwards of $0.99c$ [19,20], with $H(v)$ in excess of 140. The muons would then be seen by us to fly an average of more than 90 km before decaying, enough for an abundance of them to arrive on earth without the assistance of time dilation.

In sum, and when interpreted by the principle of relativity, experiments currently understood to provide the experimental support for space and time in SRT now demonstrate the exact opposite: length contraction and time dilation do not exist in a world ruled by the principle of relativity, and so are not real physical phenomena. Space and time in moving inertial systems are the same as in stationary systems; concepts of space and time in SRT are not compatible with the principle of relativity. Consequently, current special relativity theory is not compatible with the principle of relativity. A self-consistent special relativity theory shall have its core structure and properties defined by the principle of relativity, with deeper conceptual roots and revised concepts of space and time.

The advantage of hindsight demonstrates where SRT’s problem lies: the LT does not have separate transformations of space and time coordinates of electromagnetic fields between inertial systems. Yet equations (13) - (15) show that the coordinate transformations defined by the principle of relativity are implicit in the LT, but were never made explicit and active in SRT. Instead SRT continues to apply classical Galilean transformations of space coordinates of electromagnetic fields, in combination with length contraction, as in the original explanation of the Michelson-Morley experiment, even after the LT was developed. Likewise, as shown for instance by SRT’s version of the relativistic Doppler effect – equation (16) above, SRT combines time dilation in moving systems with Galilean transformations of time-dilated coordinates of electromagnetic fields. This happens despite it having been well understood and acknowledged for more than a century [2,4] that Galilean transformations do not work for space and time coordinates of electromagnetic fields; that’s why the LT was developed. The coordinate transformations defined by the principle of relativity avoid such unphysical combinations; the principle of relativity controls and explains relativistic experiments on its own, without the need for further theory. Thus while the LT may itself be fully consistent with the principle of relativity – which remains to be found out, its physical interpretation is not. On the other hand there is no question that SRT provides mathematically precise calculations of relativistic experiments, while being based on concepts of space and time that do not exist in the real physical world. That has been misleading us for more than a century.

Analysis of real experiments as well as Einstein's thought experiment also suggests another aspect of the principle of relativity, originally described by Galileo in his thought experiment imagined to take place inside a ship's cabin [1]: Physical phenomena and events – and not only the laws of nature that guide them, are the same in moving inertial systems as in a stationary system. That would be another extension of the principle of relativity's power, beyond the role ascribed to it in today's relativity and could have wide implications.

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