Deriving Measurement Collapse Using Zeta Function Regularisation and Novel Measurement Theory

Mark Stander[1*] and Benjamin Wallis[2]

[1*]Strategy, AECOM.
[2]School of Physics, Engineering and Technology, University of York.

*Corresponding author(s). E-mail(s): mark.stander@gmail.com;

Abstract

This paper shows how an application of zeta function regularisation to a physical model of quantum measurement yields a solution to the problem of wavefunction collapse. A realistic measurement ontology is introduced, which is based on particle distinguishability being imposed by the measurement process entering into the classical regime. Based on this, an outcome function is introduced. It is shown how regularisation of this outcome function leads to apparent collapse of the wavefunction. Some possible experimental approaches are described.

Keywords: Interpretations of quantum mechanics, Wavefunction collapse, Measurement problem, Zeta regularisation

1 Introduction and Contents of this Paper

This paper begins with some definitions which may serve as a reference for the reader. Section 2 is a background to the measurement problem. Section 3 states the assumptions of the theory. Section 4 develops the derivation of the collapse in mathematical terms. Section 4.1 describes the dimensions of the relevant Hilbert spaces. Section 4.2 introduces a realistic outcome function based on a counting function. Section 4.3 derives the counting function. Section 5 shows how regularisation is key to collapse. Section 6 shows how our derivation maps onto the measurement operator formalism. Section 7 aims to show how this theory might be experimentally validated. Section 7.1 gives an overview of recent collapse emission experiments. Section 7.2 is an overview of some approaches to show where the theory described in this paper differs from other collapse approaches. Section 8 gives an overview of results. Section 9 is a discussion of some open questions, problems and points of interest.

Definitions

In this section we will define the key terms and objects which we will be using, in the order they will introduced in the text. These can then act as a reference. In the text, we may reiterate these definitions as we give them additional context.

$\hat{O}$ is an arbitrary linear Hermitian measurement operator. Ψ is an arbitrary total wavefunction of a system. a_i is the probability amplitude of a wavefunction. ϕ is used to represent an eigenfunction onto which the wavefunction can collapse. λ is an eigenvalue. $\mathcal{H}$ is a general Hilbert space. $\mathcal{H}_\Psi$ is the Hilbert space of an arbitrary total wavefunction of a system, here typically referring to the total wavefunction of the many objects involved in the measurement process. d is the number of possible states following measurement of an isolated quantum particle, or its dimension, or number of single-particle basis states. m is the total number of interacting particles across the measurement process. n is an index which counts the number of particles in each composite system which interact in the measurement process, this index runs from 1 to k. k is the size of the largest system involved in the measurement process. c_n is the number of many particle systems of size n across the measurement process. C is the total number of n-particle systems. Ω_c is number of micro-states available for the distribution of the counting function, c_n.

2 Background to the Measurement Problem

The reader may use [1] [2], or any other of a number of undergraduate or elementary texts, for a basic treatment of the quantum measurement problem. See [3] for a recent survey. In terms of [4]'s characterisation of the problem, this paper aims to tackle the 'problem of definite outcomes'. However, for clarity, we will give a very brief overview of the measurement problem.

Quantum theory is meant to be a universal theory to explain all physical phenomenon. However, there appears to be two distinct time evolution phenomenon in quantum mechanics. Firstly, evolution of the wavefunction between measurements, as governed by the time-dependent Schrodinger equation. Secondly, quantum mechanics under quantum measurement. Under quantum measurement, the wavefunction appears to evolve non-linearly; that is, the total wavefunction will suddenly appear to collapse into a single eigenstate, with corresponding eigenfunction ϕ.

These two time evolution phenomena appear to be irreconcilable. In this paper we propose a reconciliation of these two phenomena in quantum mechanical terms.

3 Assumptions

We list some key assumptions below and give some explanation to those assumptions where that may prove useful. Assumptions 1 and 5 in particular are based on the model of the measurement process being the process in which a quantum system becomes non-isolated and interacts with other quantum and macroscopic objects, and the wider environment.

Assumption 1 Quantum objects in a measurement process can be modelled as distinguishable.

In our derivation we use the idea that classical mechanics assumes distinguishable particles. With this principle in mind, we examine the measurement problem as spanning both quantum and classical regimes. We therefore examine the measurement process through the standard quantum mechanical formalism but assuming that particles can be considered distinguishable. Due to this, we use general Hilbert space formalism and distinguishable-particle statistical models. In other words, we assume that the classical world imposes the principle of distinguishable particles onto the mathematical structure of quantum mechanics. This assumption may be weakened, see footnote [1].

Assumption 2 $\dim \mathcal{H}_\Psi$ represents a measurement outcome counting function.

This assumption is based on the following fact:

$$\hat{O}\left|\Psi\right\rangle = \sum_{i}^{\dim \mathcal{H}_\Psi} a_i(\hat{O}\left|\phi_i\right\rangle) = \sum_{i}^{\dim \mathcal{H}_\Psi} a_i(\lambda_i\left|\phi_i\right\rangle) \tag{1}$$

[1] This assumption can be weakened with an additional argument treating indistinguishable spaces as subspaces of the most general Hilbert space. These subspaces undergo collapse by imposition of a dimensional reduction due to contraction of the ambient spaces' dimension.

By definition, a linear operator on a finite dimensional vector space will preserve that spaces' number of dimensions. This may be seen as a consequence of the Spectral Theorem of Hermitian Matrices, see [5] for example. We are not claiming that there are $\dim \mathcal{H}_\Psi$ distinct eigenvalues. Rather the number of outcomes being counted correspond to the number of eigenvectors the initial state may be projected into, even when those eigenvectors might share the same eigenvalues, e.g. $\lambda_1 = \lambda_2 = \lambda_3$.

Assumption 3 We are able to model the number of many particle systems counting function, c_n, using the method of maximum entropy.

Assumption 4 The dimension of a single quantum particle, d, can be treated as a finite constant.

In some cases it is possible to define d, such as in the case of magnetic spin projection directly measured on a single Cartesian axis, where $d = 2$. This is a truncated, or finite, Hilbert space. However, Hilbert spaces of infinite dimension are necessary in quantum mechanics [6] [7]. We treat finite dimensional Hilbert spaces as good approximations to the calculations for the more general, infinite, case. We assume a finite constant, d, as an approximation of the infinite dimension of the more general Hilbert space in deriving our counting arguments.

Assumption 5 We can model the size of the largest system involved in the measurement process, k, as $k \to \infty$.

To elaborate, this assumption is to consider the size k, when the size of a macroscopic object such as the environment, as representing infinitely many constituent microscopic particles. This is an approach commonly used in statistical physics. The mathematical technique for modeling macroscopic systems by considering them as an infinite composition of microscopic particles is called taking the thermodynamic limit, or macroscopic limit. See [8] for a review of the thermodynamic limit in contemporary statistical physics. [9] gives a good overview of the thermodynamic limit, provides a number of references and demonstrates that the thermodynamic limit is essential to a non-paradoxical understanding of modern statistical physics.

In terms of a quantum mechanical approach, see [10] for a recent paper applying the thermodynamic limit. [10] use similar assumptions to those used in this paper in their approach to tackle the measurement problem. [10] considers 'infinite tensor products of usual Hilbert spaces', as this paper does. These infinite tensor product spaces are described as the 'large Hilbert spaces [that] are typically the ones expected to describe the quantum properties of macroscopic systems', similar to how these spaces are interpreted in this paper. [10] identifies von Neumann's [11] as being the 'seminal' article on this topic, and "translates the main elements of von Neumann's paper on infinite tensor products into the mathematical language currently used in quantum mechanics". An advanced mathematical treatment of the foundations of quantum theory at the thermodynamic limit are also discussed in [12], their Chapter 8, where their goal is describing a "classical limiting system".

4 Derivation of the Collapse

4.1 Hilbert Space Dimensions

For a basic composite system the Hilbert space is defined as:

$$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B \tag{2}$$

and:

$$\dim(\mathcal{H}_{AB}) = \dim(\mathcal{H}_A) \times \dim(\mathcal{H}_B) \tag{3}$$

For a many body system of m particles the Hilbert space is as follows:

$$\mathcal{H}^{\otimes m} \tag{4}$$

with dimension:

$$\dim(\mathcal{H}^{\otimes m}) = d^m \tag{5}$$

4.2 A Realistic Outcome Function

We introduce the idea of a counting function, c_n, which will count the expected number of complex systems of size n, all of which interact through the measurement process. Since each system of size n particles contributes towards the multiplicity of the number of outcomes according to d, we may therefore state our realistic outcome counting function:

$$\dim \mathcal{H}_\Psi = \prod_{n=1}^{k} d^{nc_n} \tag{6}$$

with k as the largest n size system involved in the measurement process.

4.3 The Approach for Deriving c_n

The goal of this subsection can be simply summarised as being an attempt to find the distribution for c_n (i.e. the number of n-particle systems, for each n, which all interact during the proposed measurement process) which best represents the current state of knowledge about a system. This is the distribution with the maximum entropy. See [13] (their Chapter 3.3), [14] (their Chapter 6.1) or a number of other elementary statistical mechanics texts, for similar approaches. We must also account for the additional number of ways that each

n-particle system can interact with the total measurement system, Ψ in a classical, distinguishable particle way. This is based on the fact that there are n ways for each n-particle system to interact with the total measurement system.

4.3.1 Derivation of c_n

Lemma 1

$$c_n = \frac{n}{e^{\alpha}} \tag{7}$$

Proof Ω_c is the total number of micro-states available for the distribution of the counting function, c_n. We want to find c_n such that Ω_c is maximised. We also define the total number of n-particle systems, C, as:

$$\sum_{n=1}^{k} c_n = C \tag{8}$$

Since we are interested in placing C distinct objects into k bins, it is clear that the combinatorial function which describes the total number of arrangements (neglecting, for now, to account for the n ways each particle interacts with the total system) is the multinomial coefficient [15] [16].

To account for the n ways that each n size particle can interact with the wider Ψ, we are introducing a 'degenerate multiplicity' of adding n sub-boxes to each c_n. This 'degenerate multiplicity' term is clearly n^{c_n}. We can now state Ω_c as:

$$\Omega_c = \frac{n^{c_n} C!}{c_1! c_2! ... c_k!} \tag{9}$$

We want to maximize Ω_c with respect to c_n. We take logarithms since $\max log(f) = \max f$, and this simplifies calculations.

We have:

$$\ln(\Omega_c) = \sum_{n}^{k} c_n \ln(n) + \ln(C!) - ln(c_n!) \tag{10}$$

We also use Stirling's approximation,
$\ln(x!) = x \ln(x) - x$.

Therefore:

$$\ln(\Omega_c) = \sum_{n}^{k} c_n \ln(n) + \ln(C!) - c_n \ln(c_n) + c_n \tag{11}$$

We use the method of Lagranian multipliers, with the constraint that $\sum c_n = C$, therefore adding the
$\alpha(C - \sum c_n)$ term. We therefore now have a function f which is to be maximised:

$$f = \sum_{n}^{k} \Big(c_n \ln(n) - c_n \ln(c_n) + c_n \Big) + \alpha(C - \sum_{n}^{k} c_n) + \ln(C!) \tag{12}$$

Bringing all the summed terms together:

$$f = \sum_{n}^{k} \Big(c_n \ln(n) - c_n \ln(c_n) + c_n - \alpha c_n \Big) + \alpha C + \ln(C!) \tag{13}$$

We now take the partial derivative in order to find the function c_n which maximises Ω_c. Note we use the fact that each n index in the sum is acted upon only by the corresponding n index in the partial derivative. Different indexed sum terms disappear since they are constants with respect to that n index derivative. We therefore have:

$$\frac{\partial f}{\partial c_n} = \ln(n) - \ln(c_n) - 1 + 1 - \alpha = 0 \tag{14}$$

it is clear using the second derivative test that this is a maximum. So:

$$c_n = \frac{n}{e^\alpha} \tag{15}$$

□

5 Collapse: Regularisation of the Outcome Function

Main Result Under measurement conditions

$$\dim \mathcal{H}_\Psi = 1 \tag{16}$$

Proof Bringing c_n from Lemma 1 into Equation 6, we have:

$$\dim \mathcal{H}_\Psi = \prod_{n=1}^{k} d^{n^2 e^{-\alpha}} \tag{17}$$

Using the counting function from equation 17 we can then examine what we would expect to happen under conditions of measurement. Under these conditions, we want to increase $k \to \infty$. See Assumption 5. We therefore have the following for the maximum number of possible outcomes following measurement:

$$\dim \mathcal{H}_\Psi = \prod_{n=1}^{k\to\infty} d^{n^2 e^{-\alpha}} \tag{18}$$

Taking logarithms of both sides:

$$\log \dim \mathcal{H}_\Psi = e^{-\alpha} \log d \sum_{n=1}^{\infty} n^2 \tag{19}$$

Using Zeta function regularization to assign a value to the divergent sum [17]:

$$\sum_{n=1}^{\infty} n^2 = 0 \tag{20}$$

we find that:

$$\log \dim \mathcal{H}_\Psi = 0 \tag{21}$$

and so

$$\dim \mathcal{H}_\Psi = 1 \tag{22}$$

□

Therefore the maximum number of possible outcomes from a quantum measurement following interaction with the environment is one. This is a model of wavefunction collapse as it shows how the non-linear projection into a measured, and single, state might occur. This will be further examined below in terms of measurement operators.

6 Measurement Operators and Selection Criteria

6.1 Single Eigenstate Selection

Take an arbitrary linear Hermitian measurement operator, $\hat{O}$ acting on our total system, Ψ. Since, by definition, as above, it is clear that

$$\hat{O}\,|\Psi\rangle = \sum_{i}^{\dim \mathcal{H}_\Psi} a_i(\hat{O}\,|\phi_i\rangle) = \sum_{i}^{\dim \mathcal{H}_\Psi} a_i(\lambda_i\,|\phi_i\rangle) \tag{23}$$

Upon collapse, however, the cardinality of the set of possible eigenstates must reduce to

$$\mathbf{card}(\{\lambda_i\,|\phi_i\rangle\}) = \dim \mathcal{H}_\Psi = 1 \tag{24}$$

So clearly upon collapse there is only one eigenfunction and eigenvalue, as expected.

6.2 Born Rule

This single eigenstate is selected from the possible set of eigenstates and this outcome is selected with a probability defined by the Born rule. In bra-ket notation, the probability of measuring an eigenvalue, λ_i, that corresponds to an outcome relating to an isolated system is:

$$|\langle\phi_i|\Psi\rangle|^2 = |a_i|^2 \tag{25}$$

The laws of quantum physics dictate the probabilities associated with the outcomes, and the possible eigenstates, and so the selection process, through the Born rule and the measurement operators, is physically realistic.

7 Approaches to Experiments

[18] gives an overview of some possible experimental tests of some popular collapse models, [19] provide an overview of non-interferometric collapse experiments and discuss avenues for future experiments. [20] describes some the philosophical, theoretical and experimental aspects of collapse models. The experimental methods described may be suitably altered to allow for a direct test of the approach described here, and the theoretical and philosophical aspects are relevant.

7.1 Emission Experiments: the Dioso-Penrose Approach and Direct Validation

A recent experimental test has ruled out a parameter-free version of the gravity-collapse, Diosi-Penrose model [21] [22] [23] [24], testing for emissions based on a proposed random diffusion process [25]. This emission process has been derived from the fluctuations the Dioso-Penrose model would predict. The model suggested in this paper does not explicitly involve a random emission process (although we do recognise that neither did the Dioso-Penrose model). It would be interesting to understand how the theorem described in [26] should apply to our model. This theorem proves that given certain assumptions, all collapse theories should induce a diffusion. Understanding the specifics of this theorem, and its application to the regularisation model presented in this paper, might provide a direct route to validation of the approach described in this paper. Performing the calculations involved in determining the diffusion radiation that might be observed is beyond the scope of this paper.

7.2 Differentiating Tests Against the GRW Model

An approach to potentially differentially validate this model of measurement against other proposed collapse models would be to highlight key differences between collapse theories, and test these differences experimentally. The GRW model is a well known collapse model[2] and is a suitable model for this differential validation.

The GRW model has two parameters: the collapse strength, $\tau_{collapse}$, and the spatial correlation collapse function, r_c.

Fist, let us examine the $\tau_{collapse}$ parameter. $\tau_{collapse}$ gives the collapse rate and is measured in collapses per second. Numerically, GRW suggested $\tau_{collapse,GRW} = 10^{-16}s^{-1}$, [29], while Adler later suggested a value of $\tau_{collapse,Adler} = 10^{-8}s^{-1}$ [30]. The model proposed in this paper does not explicitly have any time parameters associated with the principle theory, and so a differentiating test for our model against the GRW model might be to test for whether collapse is associated with time, or whether, as our model suggests, it is determined solely by the sequence of interacting particle systems, and complexity of those systems. For example, this model would suggest that a small number of particles, kept sufficiently isolated, will not undergo collapse without further interaction. The GRW approach suggests otherwise, however.

Another potential route for differential validation is to look at the spatial correlation function r_c parameter. A proposed value for r_c, according to GRW was $r_c = 10^{-7}$m [29]. This is the scale at which collapses become apparent. For distances $< r_c$ collapses are not apparent, for distances $> r_c$ collapses are apparent. This paper does not explicitly suggest that collapse should be dependent on length scales. Collapses would be apparent over all length scales,

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[2] [27] gives a good review of the history of Collapse Theories, some recent positions and a thorough bibliography. Of note, [27] mentions how the stochastic modification to the Schrödinger equation was first developed in [28].

so long as the criteria for complexity of systems interacting and sequence of interactions are met.

8 Overview of Results

The theory in this paper shows how a quantum system, under measurement, is projected into a single state at measurement and "collapses", given certain assumptions. We also discussed how this theory might be experimentally tested, either directly, or as a comparison against other collapse approaches.

9 Discussion

In this section we highlight some open questions and points of interest.

This approach avoids problems of other interpretations. The approach described, does not create any obvious conflicts with the existing mathematical framework, or require a conscious observer. This new interpretation of quantum mechanical measurement therefore avoids some of the problems associated with other interpretations, which have been widely discussed.

However, no clear line is drawn. An important thing to note is that this formulation suggests that it is unclear where the line between quantum and classical worlds may lie, exactly. We have found that it is in the (thermodynamic) limit, $k \to \infty$, where k is the size of the largest k-particle system involved in measurement, that this formulation produces a physically interesting result. However, it is unclear how to interpret this when trying to understand how large objects might be before they collapse. Perhaps this suggests that so long as there are a finite number of quantum particles in a system then wave-function collapse will not occur? We assume that $k \to \infty$ when the system interacts with the measurement environment but this k might identify the universe itself. We also acknowledge that taking the thermodynamic limit, while central to statistical physics (see [8], [9]), may have its own philosophical and interpretive difficulties. It is also clear that in the thermodynamic limit, the volume, V, should also $V \to \infty$ (while particles density is fixed). This agrees with the model of the measurement process involving larger and larger objects as the quantum system becomes non-isolated.

Quantum and classical physics are mixed. Another thing to note is that this formulation includes both quantum (counting the dimensionality of a many-body Hilbert space and resulting outcomes), statistical-mechanical (counting the number of ways for systems of particles to interact with other systems of particles) claims and classical claims, all of which are needed for the regularisation-based collapse to take place. This approach suggests that the number of quantum outcomes, at measurement, collapses to just one, but also that the number of ways that the system of particles (described classically) can interact collapses to just one. Our argument also relies on distinguishable particle statistics. We have worked on the assumption that since the measurement process spans classical and quantum worlds, then this distinguishable property is imposed, and so relevant in calculations. We treat the thermodynamic limit in the quantum scale as the same limit in which distinguishability becomes apparent at the interface between quantum and classical worlds. It is possible that this might be seen as a controversial claim, and acknowledge that more work can be done to examine the distinguishable property in physical terms. This is an open area of research in quantum foundations, see [31], for example.

This approach is highly reliant on regularisation. It might be argued that the approach described in this paper simply hides the mystery of the measurement problem inside the mystery of regularisation, and reveals nothing about either.

This approach might be useful at other scales. It would be interesting to investigate the theory of the scale changes in physical terms at different scales. For example, taking a simplified but realistic physical model at the quantum, atomic and larger levels, then examining these through the lens of the theory discussed.

This approach would benefit from experimental validation. In future, we would like to further develop this work to be able to validate or invalidate its theory, whether through direct experiment or through examination to understand if this theory is incompatible with existing quantum theory and experiments. It would be interesting to calculate the radiation emissions from random diffusion, which is predicted by [26] to directly test the model proposed here. It would also be interested to validate this model against the GRW model by looking at differences in predictions in regards to wavefunction collapse, with time components and length components being particularly of interest.

This approach would benefit from further theoretical work. We would also like to understand how this theory might work in the broader context of quantum field-theory, which has only been touched upon. In terms of theoretical validation, it would also be useful to understand the role that quantum decoherence might play, given its important role in the foundations of quantum physics. It would also be interesting to examine whether some of the ideas presented in this paper, such as the measurement ontology; outcome counting argument and regularisation approach to mediate wave-function collapse, might be usefully deployed in the frameworks outlined by other interpretations. For example, might the regularisation approach be useful as a potential mechanism in other objective collapse interpretations? It may also be interesting to further understand the c_n function.

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