

# The Cosmic Space Expansion Paradox: Limits of General Relativity in Application

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**ABSTRACT:** The motion of galaxies through the fabric of space-time, according to specific conditions, leads to an expansion in cosmic space and even an acceleration in its expansion. We also find that the properties of this expansion are consistent with the cosmological principle. We can also, according to the conditions of motion, reach the same formula of the Hubble-Lemaître law and the expansion of (FLRW) metric in general relativity, which expresses the expansion of the flat space-time fabric., and thus we get here a paradox in general relativity, where the metric expansion, which expresses the expansion of the flat space-time fabric, is also the one that expresses the motion of galaxies through the fabric of space-time according to specific conditions, This paradox does not represent an error in general relativity's perception of the cosmic model, but it reveals the limitations of general relativity in application, where we find that some of the results that can be obtained from general relativity are unacceptable, such as the motion of galaxies faster than the speed of light, the expansion of wavelengths of light with the expansion of space, the vacuum energy is the only source of dark energy

**Keywords:** expansion of cosmic space - Friedman's metric - Hubble's law - dark energy - acceleration of galaxies - inverse relativity - energy and time paradox - Lorentz volumes paradox - modified Lorentz transformations - Michael Girgis paradoxes

## 1 INTRODUCTION

The Friedmann–Lemaître–Robertson–Walker metric or FLRW metric [1] (Where each of the four scientists contributed to it) is the only metric that describes the expansion of cosmic space and the acceleration in space expansion of a homogeneous, isotropic universe in the equations of general relativity [2] [3], According to this metric, the expansion of the universe is an expansion in the fabric of space-time itself and not as a result of the motion of galaxies through the fabric of space-time. As for the motion of galaxies in the universe, it is considered an apparent motion as a result

of this expansion. While describing the Hubble-Lemaître law [1], which both the scientist Edwin Hubble contributed through his observations, which clarified the relation between the apparent motion of galaxies and the distance, and Georges Lemaître theoretically by considering this relation as an expansion in the fabric of space-time, but if the Hubble-Lemaître law and the expansion of FLRW metric describe the apparent motion (divergence) of galaxies through the expansion of space-time, can we do the opposite? Can we obtain the Hubble-Lemaître law through a real motion of galaxies according to specific conditions? Can we get the expansion of the FLRW metric through the motion of galaxies in the fabric of space-time and not in the expansion of the fabric of space-time? In other words, can the real motion of galaxies according to specific conditions create an expansion in cosmic space according to Hubble's law and (FLRW) metric, and if this Possible will all the results of general relativity in the cosmic model remain acceptable?.

## 2 METHODS

### 2-1 The motion Hypothesis of Galaxies

We assume that the cosmic space is a Euclidean space, i.e. a flat space, and we also assume that all galaxies in space move in real motion in diagonal vectors with acceleration or a constant and uniform jerk in the magnitude for all galaxies as a result of the release of its matter in the past from a central point, we can represent this type of motion in a spherical coordinate system [4], where the starting point represents the center of the sphere O (the center of the coordinate system), and diagonal vectors represent position vectors in the spherical coordinate system, as shown in Figure 1: 9

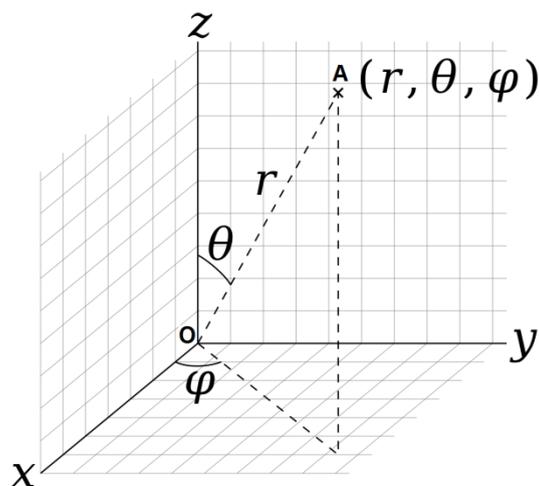


Figure 1: 9

If we have an observer at the center of the sphere O representing the spherical coordinate system, in this case his reference frame according to the previous hypothesis will appear to him that all galaxies move on the spherical coordinate  $r$  only, and the values of each of the coordinates  $\theta$  and  $\phi$  for each galaxy remain constant do not change with time, but the previous hypothesis includes two states of motion: the motion with uniform acceleration and the motion with uniform jerk for all galaxies

## 2-2 The expansion in Spherical Coordinates

The first case is the motion of all galaxies with a constant and uniform acceleration in magnitude on the spherical coordinate  $r$ , starting from the center point O with a uniform initial velocity equal to zero, and therefore the change in the position of any galaxy within the spherical coordinate system with time relative to the observer O, is as follows

$$\begin{array}{lll}
 \theta = cost & \dot{\theta} = 0 & \ddot{\theta} = 0 \\
 \phi = cost & \dot{\phi} = 0 & \ddot{\phi} = 0 \\
 r \neq cost & \dot{r} \neq cost & \ddot{r} = cost
 \end{array} \quad (1.9)$$

Because galaxies move in Euclidean space, i.e. flat, so here we can use the equations of motion in a straight line [3] [4] to describe the motion of any galaxy on the r-coordinate, let it be galaxy A, for example, we write the velocity equation in the following differential formula

$$\frac{dr}{dt} = \dot{r} \quad (2.9)$$

Where  $\dot{r}$  is the velocity of the galaxy A on the r-coordinate in the direction of the vector  $\overrightarrow{OA}$

$$dr = \dot{r} dt \quad (3.9)$$

Because all the galaxies here are moving with acceleration, so the velocity of the galaxy on the r-coordinate is variable, Therefore, we cannot integrate the previous equation before calculating the change in velocity

$$\frac{d\dot{r}}{dt} = \ddot{r} \quad (4.9)$$

Where  $\ddot{r}$  is the acceleration of galaxy A on the r-coordinate in the direction of the vector  $\overrightarrow{OA}$

$$d\dot{r} = \ddot{r} dt \quad (5.9)$$

By the definite integral of both sides of the equation from 0 to t

$$\int_0^{\dot{r}} d\dot{r} = \int_0^t \ddot{r} dt \quad (6.9)$$

$$\dot{r} = \ddot{r} t \quad (7.9)$$

Substitute from 7.9 into 3.9

$$dr = \ddot{r} t dt \quad (8.9)$$

By the definite integral again for both sides of the equation from 0 to t

$$\int_0^r dr = \int_0^t \ddot{r} t dt \quad (9.9)$$

$$r = \frac{1}{2} \ddot{r} t^2 \quad (10.9)$$

Substitute from 7.9 into 10.9

$$r = \frac{1}{2} \ddot{r} \frac{\dot{r}^2}{\ddot{r}^2} \quad (11.9)$$

Rearrange the equation

$$\dot{r} = \sqrt{2 \ddot{r} r} \quad (12.9)$$

The equation shows that the velocity of the galaxy A is a function of the position, that is, at each position of the galaxy A on the r-coordinate in the direction of the vector  $\overrightarrow{OA}$  the galaxy has a different velocity magnitude, and we can generalize that to all galaxies because they are similar in conditions of motion (same starting point, initial velocity, and the magnitude of acceleration)

$$\dot{r} = g(r) \quad (13.9)$$

This means that the galaxies on the r-coordinate that have the same direction of the vector, but do not have the same position, will have a different magnitude of velocity, that is, a relative velocity that leads to an expansion of the distance between them with time, and therefore we can say here that the spherical coordinate  $r$  is expanding as the distance between each two galaxies on this coordinate increase with time. To calculate the rate of expansion of the r-coordinate, we differentiate Equation 12.9 with respect to the radius.

$$\frac{d\dot{r}}{dr} = \frac{1}{2} \frac{\sqrt{2 \ddot{r}}}{\sqrt{r}} \quad (14.9)$$

The left side of the equation represents the coordinate expansion rate  $r$ , called the expansion coefficient and denoted by the symbol  $H_r$

$$H_r = \frac{1}{2} \frac{\sqrt{2\dot{r}}}{\sqrt{r}} \quad (15.9)$$

Equation 15.9 shows that the expansion coefficient  $H_r$  at a uniform scale of the radius is a constant value, that is, all galaxies that lie on the surface of a sphere of radius  $r$  expand at the same rate relative to the center point or the observer O, but on a variable scale  $r$ , we find that the expansion rate is proportional to Inversely with the square root of the radius, This means that equal distances along the r-coordinate do not correspond to the same difference in velocities on the curve of the function, i.e. the spherical coordinate  $r$  is not symmetrical in expansion

### 2-3 The accelerated Expansion in Spherical

The second case is the motion of all galaxies with a constant and uniform jerk in magnitude on the spherical coordinate  $r$ , starting from the center point O with a uniform initial acceleration also equal to zero, and therefore the change in the position of any galaxy in the spherical coordinate system with time relative to the observer O, is as follows

$$\begin{array}{llll} \theta = cost & \dot{\theta} = 0 & \ddot{\theta} = 0 & \dddot{\theta} = 0 \\ \phi = cost & \dot{\phi} = 0 & \ddot{\phi} = 0 & \dddot{\phi} = 0 \\ r \neq cost & \dot{r} \neq cost & \ddot{r} \neq cost & \dddot{r} = cost \end{array} \quad (16.9)$$

In the previous case, we were able to integrate equation 5.9, because the acceleration was a constant value, but here the acceleration is variable because the galaxies move in a constant jerk, so to perform the integration on the previous equation, the change in acceleration must be calculated first

$$\frac{d\ddot{r}}{dt} = \dddot{r} \quad (17.9)$$

$$d\ddot{r} = \dddot{r} dt \quad (18.9)$$

Where  $\ddot{r}$  is the galaxy jerk A on the r-coordinate in the direction of the vector  $\overrightarrow{OA}$ , and by the definite integral of both sides of the equation

$$\int_0^{\ddot{r}} d\ddot{r} = \int_0^t \ddot{r} dt \quad (19.9)$$

$$\dot{r} = \ddot{r} t \quad (20.9)$$

By substituting from 20.9 into 5.9 and by definite integration of both sides of the equation

$$\int_0^{\dot{r}} d\dot{r} = \int_0^t \ddot{r} t dt \quad (21.9)$$

$$\dot{r} = \frac{1}{2} \ddot{r} t^2 \quad (22.9)$$

Substitute from 22.9 into 3.9

$$dr = \frac{1}{2} \ddot{r} t^2 dt \quad (23.9)$$

By the definite integral again for both sides of the equation from 0 to  $t$

$$\int_0^r dr = \int_0^t \frac{1}{2} \ddot{r} t^2 dt \quad (24.9)$$

$$r = \frac{1}{6} \ddot{r} t^3 \quad (25.9)$$

Substitute from 20.9 into 25.9

$$r = \frac{1}{6} \ddot{r} \frac{\dot{r}^3}{\ddot{r}^3} \quad (26.9)$$

$$\dot{r}^3 = 6 \ddot{r}^2 r \quad (27.9)$$

$$\dot{r} = \sqrt[3]{6 \ddot{r}^2 r} \quad (28.9)$$

Equation 28.9 shows that the acceleration of galaxy A here is also a function of position, that is, at every position of galaxy A on the  $r$ -coordinate in the direction of the vector  $\overrightarrow{OA}$  The galaxy has a different acceleration magnitude, and we can generalize that to all galaxies because they are similar in conditions of motion (same starting point, initial acceleration, and the magnitude of jerk)

$$\dot{r} = k(r) \quad (29.9)$$

This means that the galaxies on the  $r$ -coordinate that have the same direction of the vector, but do not have the same position, will have a different magnitude of acceleration, that is, a relative acceleration that leads to an acceleration in the expansion of the distance between them with time,

and therefore we can also say here that the spherical coordinate  $r$  is expanding with acceleration, to calculate the rate of acceleration in the  $r$ -coordinate expansion, we differentiate Equation 28.9 with respect to the radius

$$\frac{d\ddot{r}}{dr} = \frac{1}{3} \frac{\sqrt[3]{6\ddot{r}^2}}{\sqrt[3]{r^2}} \quad (30.9)$$

The left side of the equation represents the rate of acceleration of the expansion of the  $r$ -coordinate, called the expansion acceleration coefficient and denoted by the symbol  $M_r$

$$M_r = \frac{1}{3} \frac{\sqrt[3]{6\ddot{r}^2}}{\sqrt[3]{r^2}} \quad (31.9)$$

Equation 31.9 shows that the expansion acceleration coefficient  $M_r$  at a uniform scale of radius  $r$  is also a constant value, meaning that all galaxies lying on the surface of a sphere of radius  $r$  have the same rate of expansion acceleration relative to the center point or the observer O, but on a variable scale  $r$ , We find that the rate of acceleration of expansion decreases with the increase of  $r$ , This means that equal distances along the  $r$ -coordinates do not correspond to the same acceleration difference on the curve of the function, i.e. the spherical coordinate  $r$  is also not symmetrical in the expansion acceleration

Because the  $r$ -coordinate here is expanding with acceleration, therefore it will have a different expansion rate than the expansion rate calculated in the first case. To calculate the expansion rate for the  $r$ -coordinate in the second case, we must first obtain the relation between velocity and distance in the second case.

Substitute from 25.9 into 22.9

$$\dot{r} = \frac{1}{2} \ddot{r} \left[ \left( \frac{6r}{\ddot{r}} \right)^{\frac{1}{3}} \right]^2 \quad (32.9)$$

$$\dot{r} = \frac{1}{2} \ddot{r} \left( \frac{6r}{\ddot{r}} \right)^{\frac{2}{3}} \quad (33.9)$$

$$\dot{r} = \frac{1}{2} \ddot{r} \sqrt[3]{\left( \frac{6r}{\ddot{r}} \right)^2} \quad (34.9)$$

By cubed both sides of the equation

$$\dot{r}^3 = \frac{1}{8} \ddot{r}^3 \frac{36 r^2}{\ddot{r}^2} \quad (35.9)$$

$$\dot{r}^3 = \frac{36}{8} \ddot{r} r^2 \quad (36.9)$$

$$\dot{r}^3 = \frac{36}{8} \ddot{r} r^2 \quad (37.9)$$

$$\dot{r} = \sqrt[3]{\frac{9}{2} \ddot{r} r^2} \quad (38.9)$$

Here we get the velocity function at position  $g(r)$ , but in the second case, we differentiate equation 38.9 with respect to the radius

$$\frac{d\dot{r}}{dr} = \frac{2}{3} \sqrt[3]{\frac{9 \ddot{r}}{2 r}} \quad (39.9)$$

Here we get the expansion rate or expansion coefficient of the r-coordinate in the second case

$$H_r = \frac{2}{3} \frac{\sqrt[3]{9 \ddot{r}}}{\sqrt[3]{2 r}} \quad (40.9)$$

Where we find here, in the second case, that the expansion coefficient of the r-coordinate is inversely proportional to The cube root of the radius when the galaxies move in a constant jerk, i.e. at the third derivative of the distance with respect to time, while in the first case or with the second derivative of the distance with respect to time, the coordinate expansion coefficient is inversely proportional to the square root of the radius, and we can generalize this result with the following formula

$$H_r \propto \frac{\sqrt[2]{\ddot{r}}}{\sqrt[2]{r}} \quad or \quad \frac{\sqrt[3]{\ddot{r}}}{\sqrt[3]{r}} \quad or \quad \frac{\sqrt[n]{f^{(n)}(t)}}{\sqrt[n]{r}} \quad (41.9)$$

This means that the change in the rate of expansion of the r-coordinates at the higher orders of the derivative of the distance with respect to time, that is, when  $n = 4 \text{ , } 6 \text{ , } 8$  is very small and can be neglected, because it is inversely proportional to a root of degree  $n$  for the radius, Thus, we can deal with the expansion coefficient as a constant value, where the integral of the equation is as follows.

$$\int_0^{\dot{r}} d\dot{r} = H_r \int_0^r dr \quad H_r \approx const \quad (42.9)$$

$$\dot{r} = H_r r \quad (43.9)$$

Here we get the same formula of Hubble-Lemaître's law, where equal distances expand at the same rate, that is, at the higher orders of the derivative of distance with respect to time for the motion of galaxies on the r-coordinate, we can get a symmetric expansion of the spherical coordinate  $r$

## 2-4 Expansion and Accelerated Expansion in the Euclidean Space Metric

As a result of spherical coordinate expansion according to the conditions of motion of galaxies in Euclidean space, that we imposed above in the first case and the second case. it will have an effect on the Euclidean space metric, and because we use spherical coordinates here, so we will write the Euclidean space metric by spherical coordinates [1] [2]

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2 \quad (44.9)$$

$$ds^2 = dr^2 + \frac{dr^2}{dr^2} r^2 (d\theta^2 + \sin^2(\theta) d\phi^2) \quad (45.9)$$

Dividing both sides of the equation by  $dt^2$

$$\frac{ds^2}{dt^2} = \frac{dr^2}{dt^2} + \frac{dr^2}{dt^2} \frac{r^2}{dr^2} (d\theta^2 + \sin^2(\theta) d\phi^2) \quad (46.9)$$

$$d\dot{s}^2 = d\dot{r}^2 + d\dot{r}^2 \frac{r^2}{dr^2} (d\theta^2 + \sin^2(\theta) d\phi^2) \quad (47.9)$$

Dividing both sides of the equation by  $dr^2$

$$\frac{d\dot{s}^2}{dr^2} = \frac{d\dot{r}^2}{dr^2} + \frac{d\dot{r}^2}{dr^2} \frac{r^2}{dr^2} (d\theta^2 + \sin^2(\theta) d\phi^2) \quad (48.9)$$

$$\frac{d\dot{s}^2}{dr^2} = H_r^2 + H_r^2 \frac{r^2}{dr^2} (d\theta^2 + \sin^2(\theta) d\phi^2) \quad (49.9)$$

$$\frac{d\dot{s}^2}{dr^2} = H_r^2 \left( 1 + \frac{r^2}{dr^2} (d\theta^2 + \sin^2(\theta) d\phi^2) \right) \quad (50.9)$$

$$d\dot{s}^2 = H_r^2 (dr^2 + r^2 (d\theta^2 + \sin^2(\theta) d\phi^2)) \quad (51.9)$$

Substitute from 44.9 into 51.9

$$d\dot{s}^2 = H_r^2 ds^2 \quad (52.9)$$

Compensate for the value of  $H_r^2$

$$\frac{d\dot{s}}{ds} = \frac{d\dot{r}}{dr} \quad (53.9)$$

We conclude from equation 53.9 that the expansion coefficient of the Euclidean space metric by spherical coordinates is not equal to zero, but rather equal to the expansion coefficient of the spherical coordinates  $r$ , that is, the expansion of the spherical coordinate  $r$  according to the conditions of motion in the first or second case leads to the expansion of Euclidean space at the same rate, when the distances are equal. That is, when  $ds = dr$ , we find that  $d\dot{s} = d\dot{r}$ , and in this case we can generalize the formula for the spherical coordinate expansion coefficient on the Euclidean metric, written in the following formula

$$H_s = \frac{d\dot{s}}{ds} \propto \frac{\sqrt[2]{\ddot{s}}}{\sqrt[2]{s}} \quad or \quad \frac{\sqrt[3]{\ddot{s}}}{\sqrt[3]{s}} \quad or \quad \frac{\sqrt[n]{f^{(n)}(t)}}{\sqrt[n]{s}} \quad (55.9)$$

By dividing both sides of equation 47.9 by  $dt^2$  again, then by dividing by  $dr^2$  and following the same steps, we get to the following equation

$$d\ddot{s}^2 = M_r^2 ds^2 \quad (56.9)$$

Compensate for the value of  $M_r^2$

$$\frac{d\ddot{s}}{ds} = \frac{d\ddot{r}}{dr} \quad (57.9)$$

We get the acceleration coefficient in the expansion of the Euclidean space metric that is also equal to the acceleration coefficient in the expansion of the spherical coordinates  $r$  for equal distances, that is, when  $ds = dr$  we find that  $d\ddot{s} = d\ddot{r}$ . Thus, also in the upper derivative of the distance with respect to time, we get a symmetrical expansion in the Euclidean space metric, where equal distances in the Euclidean space extend at the same rate, that is, the rate of expansion will appear to be similar at every point in space and in every direction as well, and no point is distinguished from the other. Where any point in Euclidean space will appear to the observer who is at it to be the center of expansion as well, not necessarily the point that we assumed at the beginning, and this result agrees with the cosmological principle, so we can write the expansion of the Euclidean space metric in the upper derivative of the distance with respect to time in the following formula

$$ds^2 = (a(t))^2 (dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2) \quad (58.9)$$

Where  $a(t)$  is known as the cosmic scale factor, which is a function of time, and by adding the time dimension to the equation

$$ds^2 = -c^2 \tau^2 + (a(t))^2 (dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2) \quad (59.9)$$

Here we get the cosmic space expansion metric as a result of the motion of galaxies through space-time fabric according to the previous conditions of motion, and it is also the same FLRW metric in general relativity that expresses the expansion of the flat space-time fabric, Which represents a paradox in general relativity, where the motion of galaxies through space-time fabric according to specific conditions is mathematically equivalent to describing the expansion of the fabric of space-time itself by obtaining the same Hubble-Lemaître law formula and the same FLRW metric expansion of the flat space-time fabric. This paradox does not represent a mistake in the cosmic space of general relativity, but it does not guarantee getting same results of general relativity, as we find many results in the cosmic model that have become unacceptable in our model.

The first result of the space-time fabric expansion model is that galaxies can move away from us at a speed greater than the speed of light, but this result is not allowed in the model of the motion of galaxies through the fabric of space-time, where the motion of galaxies here is subject to the second postulate of special relativity [3], and therefore the maximum speed that galaxies can theoretically reach is the speed of light.

The second result of the space-time fabric expansion model, the vacuum energy is the dark energy, while in the model of the motion of galaxies, the expansion or acceleration in the cosmic space depends on the kinetic energy of the galaxies that the galaxies can acquire from vacuum energy or from another source

The third result of the space-time fabric expansion model, the expansion occurs at any level, where the expansion occurs between particles of a star gas or between electrons and the nucleus of an atom, due to vacuum energy, but due to the large electromagnetic forces, this expansion does not appear, while the expansion of cosmic space in the model of the motion of galaxies only appears between galaxies without the need to explain why it does not appear between the vacuum of molecules and the vacuum of atoms

The fourth result of the space-time fabric expansion model is the wavelength expansion of light or the redshift [1] beside the relativistic Doppler Effect, but in the model of galaxy motion the redshift depends on the relativistic Doppler Effect only [3]

### 3 RESULTS

We can obtain an expansion of cosmic space and acceleration in the expansion also through the motion of galaxies in the fabric of space-time according to three conditions. The first condition is the launch of all galaxies or, more precisely, the matter that made up galaxies in the past from the same spatial point in diagonal vectors. The second condition is the launch of all galaxies with unified initial velocities or accelerations. The third condition is that all galaxies move with a constant and uniform acceleration or jerk, The expansion in cosmic space due to previous conditions of motion is characterized at the upper derivative of distance with respect to time, by the following, Equal distances expand at an almost constant rate, that is, the expansion in space is the same relative to all points of space and in all directions of space, and this corresponds to the cosmological principle, The expansion of the cosmic space resulting from the conditions of motion fulfills the Hubble-Lemaître law, and also achieves the expansion of the (FLRW) metric used in general relativity, and this represents a paradox in general relativity because the (FLRW) metric expresses the expansion of the fabric of space-time, Not motion through the fabric of space-time, In other words, we cannot here determine whether the cosmic space expansion resulting from an expansion in the fabric of space-time or from the motion of galaxies through the fabric of space-time according to specific conditions of motion. This paradox does not represent an error in general relativity's perception of the cosmic model, but it reveals the limits of the results that can be obtained from general relativity such as moving faster than the speed of light or the expansion of wavelengths as a result of the expansion of cosmic space, or that vacuum energy is the only source of dark energy

## 4 DISUSSIONS

The model of cosmic space expansion through the motion of galaxies relied mainly on the hypothesis of a flat cosmic space or plane, so we cannot get the same results from a cosmic space with a positive or negative curvature, but we find that this hypothesis is consistent with the results of current astronomical observations, It also depended on the upper derivative of distance with respect to time., but we also find that the expansion coefficient is directly proportional to the root of the derivative, where the degree of the root is equal to the order of the derivative, and this means that the rate of symmetric expansion or in the higher orders is very small. This result also agrees with the current astronomical observation results, where the value of the Hubble constant is very small at the level of short distances.

The model of cosmic space expansion through the motion of galaxies ignored the effect of gravity on the motion of galaxies, but the purpose here is not to calculate the actual expansion rate of cosmic space, but the purpose is to reveal the possibility of obtaining an expansion in cosmic space through the motion of galaxies in the fabric of space-time and not the expansion of the fabric itself

The formula of the Hubble-Lemaître law does not reveal to us limits to the speed that galaxies can reach in space, although the motion of galaxies through the fabric of space-time is subject to the second postulate of special relativity, that is, the maximum speed of galaxies should theoretically be the speed of light or the speed very close to the speed of light, and this is not clear in the formulation of the Hubble-Lemaître law, the proposed explanation here is the collapse of the equation or the Hubble-Lemaître law when the galaxies reach the speed of light

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